Assignment 9

Computational Structural Mechanics and Dynamics

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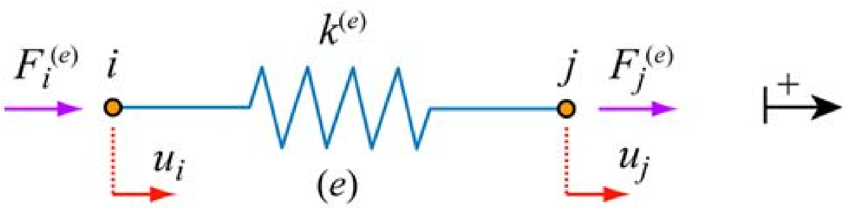

Contents

1 Assignment 9.1

Describe in extension how can be applied a non-symmetric load on this formulation?

In engineering applications one encounter quite often axially symmetric parts. It is important, therefore, to formulate a solving procedure, specific for this type of bodies. One begins with the introduction of a cylindrical system of coordinates (r, ϕ, z). When external loads are independent of the tangential coordinate (ϕ), the problem is axisymmetric. In the particular case, when the axially symmetrical loading is oriented in the tangential direction (ϕ), the problem turns into pure torsion. The latter case can be treated separately from the general axially symmetric loading inn the (r,z)-plane.
In the BEM formulation the problem becomes one-dimensional. The range of integration extends over the boundary meridian only. In the case of simple, symmetrical body forces, like gravity or centrifugal forces, as well as for symmetrical stationary fields of temperature, one can transform the corresponding domain integral into a boundary integral. General cases of non-symmetric external loads can be reduced into a series of one-dimensional problems of the above kind. To do that, one needs to develop the load function into a Fourier series.
Here, the zeroth component of the series corresponds to the axisymmetrical load. Each next component of the series allows analytical integration over the circumference and thus the reduction of the problem to a one-dimensional one. The solution of the original problem is presented then as a superposition of the partial solutions, which correspond to the components of the series [1].

Regarding the mathematical analysis, on the references section you can find the book where it is fully developed [2](page 303, chapter 9, section 6, "Axisymmetric shells with non-symmetrical load").

2 Assignment 9.2

Using thin beams formulation, describe the shape of the B(e) matrix and comment the integration rule.

Taking into account a thin shell element, we know that we'll work under the Kirchhoff's assumptions, so that our general deformation matrix B will only consist of the bending strain matrix ,B_b, and the membrane strain matrix ,B_m. The transverse shear no longer contributes to the internal work, so shear stresses will have to be calculated from the equilibrium equations. Obtaining the following form of B:

$$B^e = \begin{bmatrix} B_m^e \\ B_b^e \end{bmatrix}$$

Comments on integration:
When working with curved Kirchhoff shell elements, the element can suffer a membrane locking. The displacement tries to represent a zero membrane state without affecting the bending moment, therefore, a typical way of solving this issue is to implement a reduced integration by using only one Gauss point when calculating the stiffness matrix for the membrane. We may obtain a lack of precision.
This comments came from analyzing the book from Oñate [3] added in the references section.

References

[1] Stein, E., Wendland, W., *Finite Element and Boundary Element Techniques from Mathematical and Engineering Point of View*, 1998, University of Hanover.

[2] O. C. Zienkiewicz, Robert Leroy Taylor, R. L., *The Finite Element Method: Solid mechanics*, 2000, University of Wales, Swansea.

[3] Oñate, E., *Structural Analysis with the Finite Element Method. Linear Statics. Volume 2. Beams, Plates and Shells. Lecture Notes on Numerical Methods in Engineering and Sciences (First edit, Vol. 2)*, 2003, CIMNE, Barcelona.