Universitat Politècnica de Catalunya
Numerical Methods in Engineering
Computational Structural Mechanics and Dynamics

Shells

Assignment 8

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1.1   Statement

Analyze the following concrete hyperbolic Shell under self weight with all four edges clamped. Explain the behavior of all the stresses presented.

$$t = 0.1$$
$$X_1 = [0, 0, 2]$$
$$X_2 = [10, 0, -2]$$
$$X_3 = [10, 10, 2]$$
$$X_4 = [0, 10, -2]$$

1.2   Solution

First off we must define the domain. To do so I created a domain in GiD consisting in a flat plane [0,0]x[10,10]. I created a regular triangular mesh and exported it to a `.msh` file. I created a matlab file to read the mesh and then evaluated

$$z(x, y) = 4 \left(\frac{(10 - x)(10 - y)}{10} + \frac{xy}{10} \right) - 2 \tag{1}$$

to obtain the z-coordinate. Since the range of values of Z is relatively small, the mesh doesn't become terribly distorted. The choice of mesh was such that every two triangles formed a square. The domain is divided into 81 squares, hence 162 triangular elements.

It is worth noting that hyperboloids are doubly ruled surfaces. This means that for every point of the surface, there exist two straight lines that pass through such point and are contained entirely within the surface. In our case, if we look at the projection of the domain onto the XY plane, our lines are all parallel to either the x or y axes. This will allow to interpret the results by analogy to a grid of straight beams. A more visual approach is shown in figure 1. For every point there is a red and a blue line that go through it. This lines can be qualitatively treated as beams.
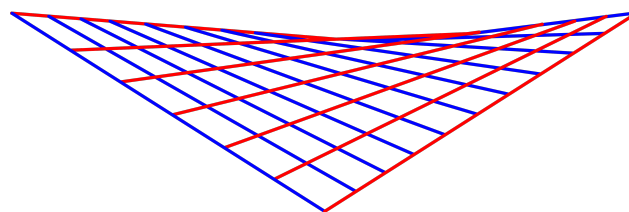

Figure 1: A hyperboloid is a doubly ruled surface.

Having the mesh, I needed the boundary conditions. I simply locked all five degrees of freedom for all nodes on the border $\partial\Omega$:

$$\begin{bmatrix} x \\ y \end{bmatrix} \in \partial\Omega \iff \{x = 0, 10\} \cup \{y = 0, 10\} \tag{2}$$

The physical parameters chosen are approximate values for concrete:

$$E = 30 \times 10^9 \text{ Pa}$$
$$\nu = 0.15$$
$$\gamma = 24 \times 10^3 \text{ N/m}^3$$

The last thing needed was running the program and seeing the results. The first thing to check was that the geometry was well defined, boundary conditions properly enforced and that the result seems qualitatively sensible. This can be seen figure 2. For a better view of the displacements one can check figure 3. As specified, the displacements at the boundaries are zero. Gravity acts towards the negative Z axis so displacements in this direction dominate.
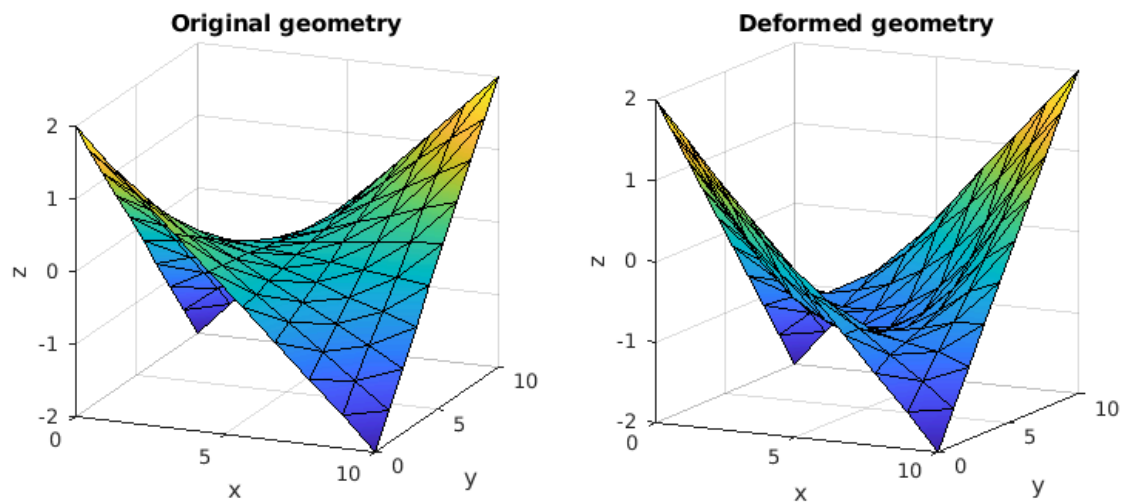

Figure 2: Original and deformed geometries. Vetrical axis is scaled by 2.5 for better contrast. Deformation is scaled again by 1000. Color indicates z-axis value.
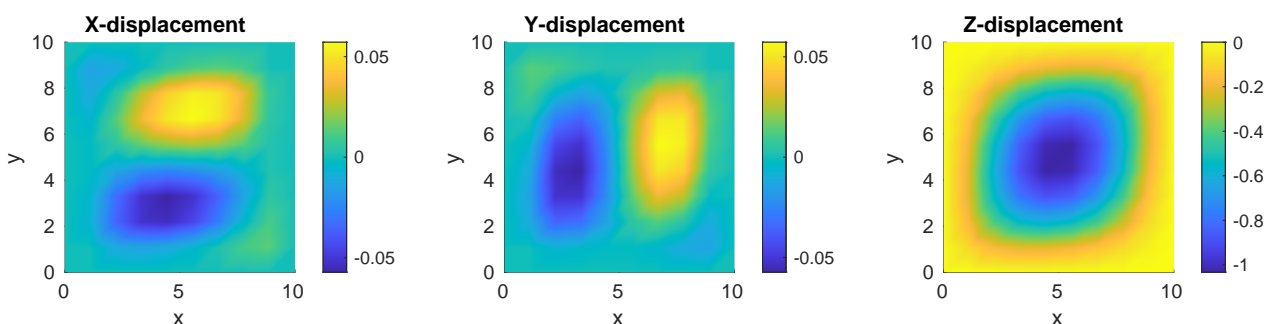

Figure 3: Displacements in milimiters

When it comes to stresses it is worth looking at figure 4 and seeing how shear stress is minimal, only present at the boundary due to the effects of the boundary reactions. This is no coincidence, hyperbolic structures are used for this exact reason, usually as roofs. We can observe the load map in figure 5. Unsurprisingly, it is

under tension on the most steep parts of the domain and under compression in the center.
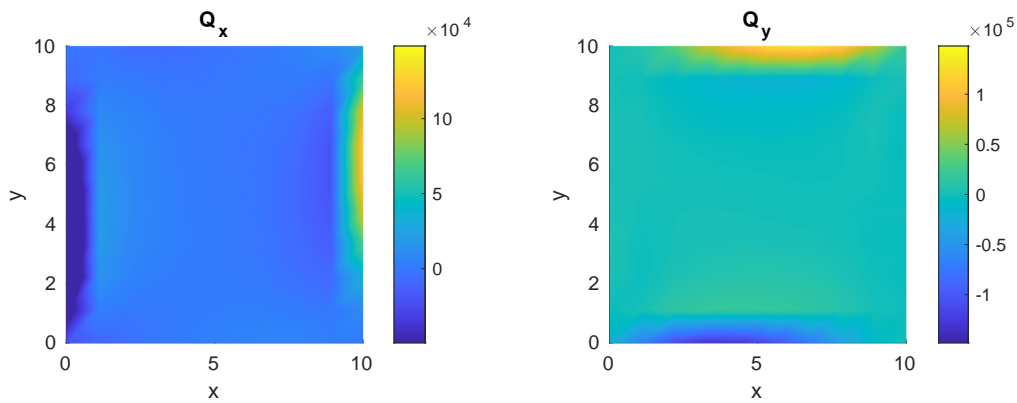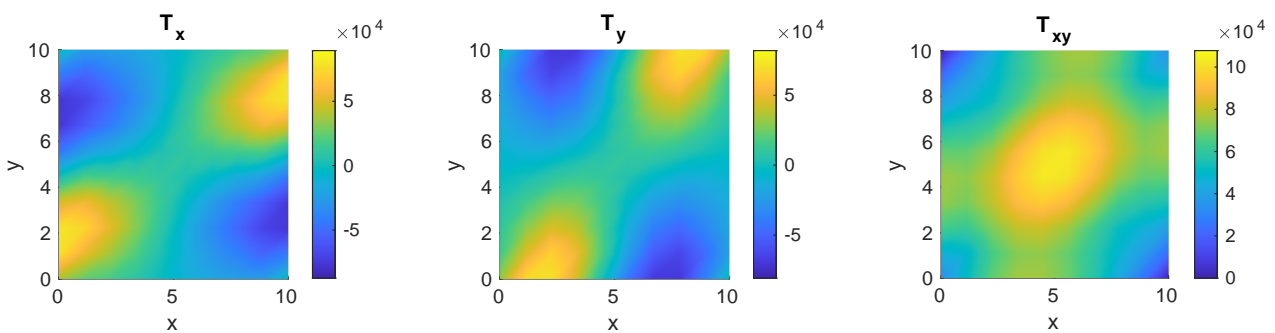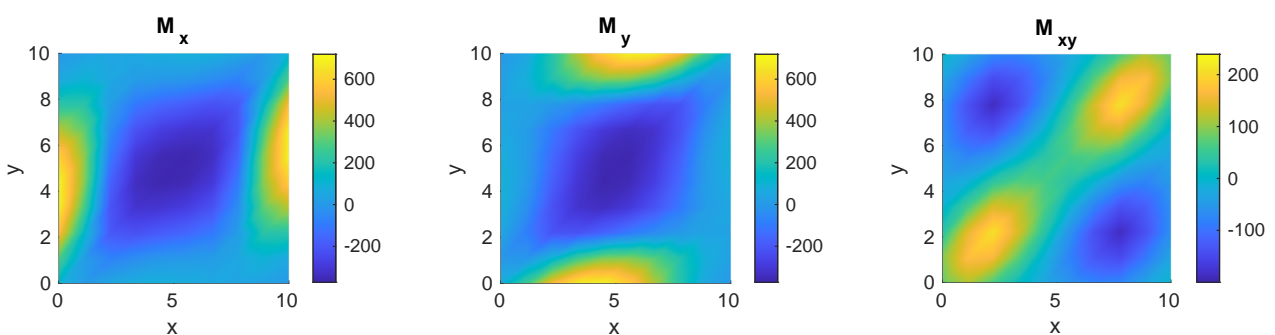

Figure 4: Shear map

Figure 5: Load map

Figure 6: Moment map

In figure 6 we can see the moment diagram. We see how it is generally positive at the edges and negative at the center. This can be easily explained if we go back to the idea of the doubly ruled surace previously mentioned. Each imaginary beam has a downwards distributed load (negative moment at the center) and is clamped at both ends (positive moment at the edges).