## Assignment 7

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## Part a

In this part, one should compare the MZC element to the RM element. Therefore, a $5 \times 5$ mesh was made for a square plate (Figure 1). Each side of the plate is 1 m long. This plate is simply supported on each corner, with a uniform downwoard force of -1 N . This test was made for different values of the thickness. The properties of the material are the following: $E=10.92 M P a$ and $\nu=0.3$.


Figure 1: $5 \times 5$ Mesh

The values for different thicknesses are given in the table below:

| $\mathrm{t} / \mathrm{L}=\mathrm{t}$ <br> $(\mathrm{L}=1 \mathrm{~m})$ | Thickness $\mathrm{t}(\mathrm{m})$ | 0.001 | 0.01 | 0.02 | 0.1 | 0.4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MZC | Displacement <br> Rotation x/y <br> Bending Moment <br> $\mathrm{x} / \mathrm{y}$ | -23.8563 | -0.02386 | -0.00298 | $-2.38563 \mathrm{E}-05$ | $-3.72755 \mathrm{E}-07$ |
|  | Displacement | -22.352 | -0.02238 | -0.00281 | $-2.46919 \mathrm{E}-05$ | $-8.22639 \mathrm{E}-07$ |
|  | Rotation x/y | 53.951 | 0.053976 | 0.006756 | $5.53212 \mathrm{E}-05$ | $8.86508 \mathrm{E}-07$ |
|  | Bending Moment <br> $\mathrm{x} / \mathrm{y}$ | 0.145964 | 0.145757 | 0.145149 | 0.134518 | 0.122695 |

Table 1: Values for different thickness t

Knowing that the MZC element holds for values of $t / L \leq 0.1$, the last two columns of the table should not be considered ( 0.1 is a critical value for MZC).

Comparing the values with each other for each element, the values are more or less similar. This shows that both methods can be used for thin plates.

However, the values for displacement, rotation and bending moment for each thickness are a bit smaller in the RM element, and this is due to the fact that there is a transverse shear locking effect, that lowers the value of other components.

## Part b

For this part, one should do a patch test for the MZC element. To do so, a $2 \times 2$ mesh was made for a square plate of thickness $t=0.1 \mathrm{~m}$. Two cases were made (Case 1 and Case 2) that focus only on displacement that will be applied on each node. The equations are the following:

$$
z_{1}=-(x+y) 10^{-2} \quad z_{2}=-\frac{\left(x^{2}+x y+y^{2}\right) 10^{-3}}{2}
$$



Figure 2: $2 \times 2$ Mesh
After doing the calculations, the answer for the displacement for node 4 (Figure 2) should have a value close to the one calculated. The values for the boundary conditions and for node 4 are calculated below:

| Nodes | x | y | $\mathrm{z}_{1}$ | $\mathrm{z}_{2}$ |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 1 | -0.010 | -0.000500 |
| 2 | 0.5 | 1 | -0.015 | -0.000875 |
| 3 | 0 | 0.5 | -0.005 | -0.000125 |
| 4 | 0.5 | 0.5 | -0.010 | -0.000375 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 1 | 1 | -0.020 | -0.001500 |
| 7 | 0.5 | 0 | -0.005 | -0.000125 |
| 8 | 1 | 0.5 | -0.015 | -0.000875 |
| 9 | 1 | 0 | -0.010 | -0.000500 |

Table 2: Coordinates with Boundary Conditions for Each Node
The results of the displacement are shown below:

|  | Calculated | Result | \% Error |
| :--- | :--- | :--- | :--- |
| Case1 | -0.01 | -0.0100005 | $0.005 \%$ |
| Case2 | -0.000375 | -0.000483354 | $28.89 \%$ |

Table 3: Results for Node 4
Comparing the results to the calculated values, the displacements almost the same. The error in the second case is higher because the equation used for the boundary conditions is quadratic, and the shape functions are linear. Using a quadratic or a refined mesh will solve this problem.

