SUMMARY

Assignment Plates

The aim of the assignment is to check the behaviour of MZC and RM elements for thick and thin plates, as well as to implement a patch test for the MZC element.

The selected problem to carry out the assessments is the simple supported plate under uniform load described below

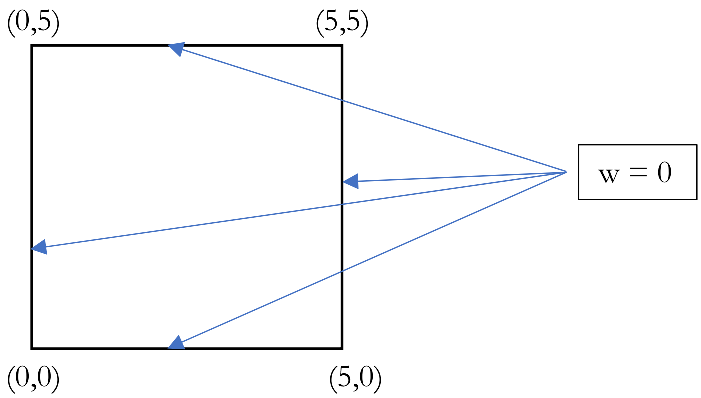

Figure 1: Geometry description

With a 5x5 mesh the discretization results as follows

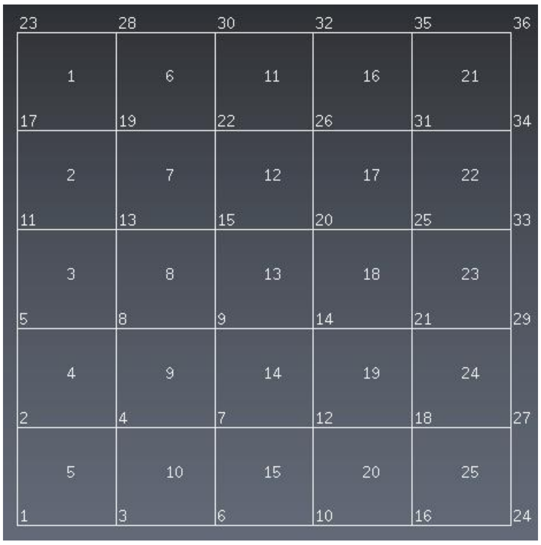

Figure 2: Meshing

The material data of the problem is the following

Young's modulus E	10.92
Poisson	0.3
Uniform load	1

Thickness
0.001
0.01
0.02
0.1
0.4

Units are not described because are not important in this assessment, which only is focused in relative results between different plates.

Mat-fem along with GiD are used to compute the results, as well as to carry-out the post-processing tasks.

Assessment of the shear locking effect on the Reissner-Mindlin (RM) element

As it is stated along the theory, RM plate elements suffer from shear locking effect. In other words, the numerical solution becomes stiffer for thin plates situations.

This defect appears due to the domination of the shear terms over the bending ones as the plate is thinner. It can be observed that the influence of the thickness on the shear and bending constitutive matrix is very different.

$$\hat{\mathbf{D}}_b = \frac{t^3}{12}\mathbf{D}_b \quad \text{and} \quad \hat{\mathbf{D}}_s = t\mathbf{D}_s$$

This difference reduces the bending stiffness matrix weight on the result much more faster than the shear one as the plate becomes thinner.

Results: Displacements and bending moment

In order to asses the locking effect, the results of the RM element are compared with the ones of the MZC element, which is known to have a good behaviour to predict the bending for thin plates (t/L<0.1).

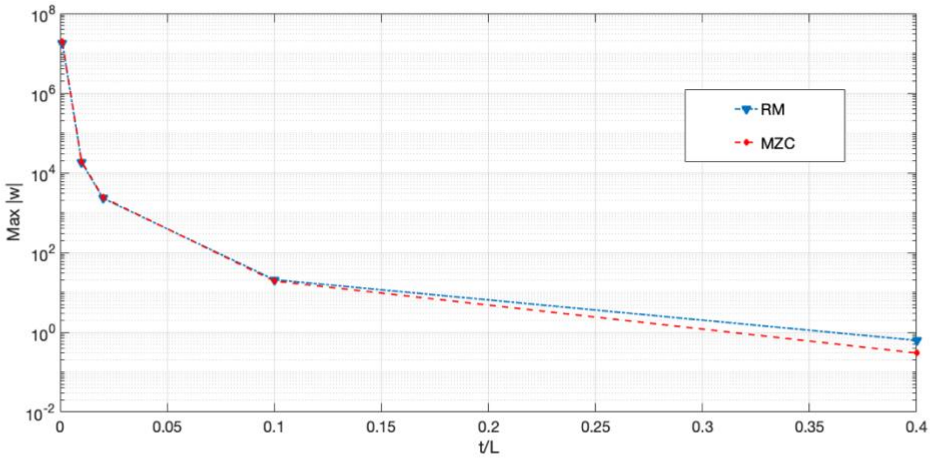

Figure 3: Maximum vertical displacements for RM and MZC plates

As can be seen at the previous plot, the behaviour is quite similar regarding the vertical displacements. However, the results start to diverge for t/L>0.1. In this case, the Kirchhoff model of the MZC is uncapable of reproduce the actual behaviour of the loaded plate.

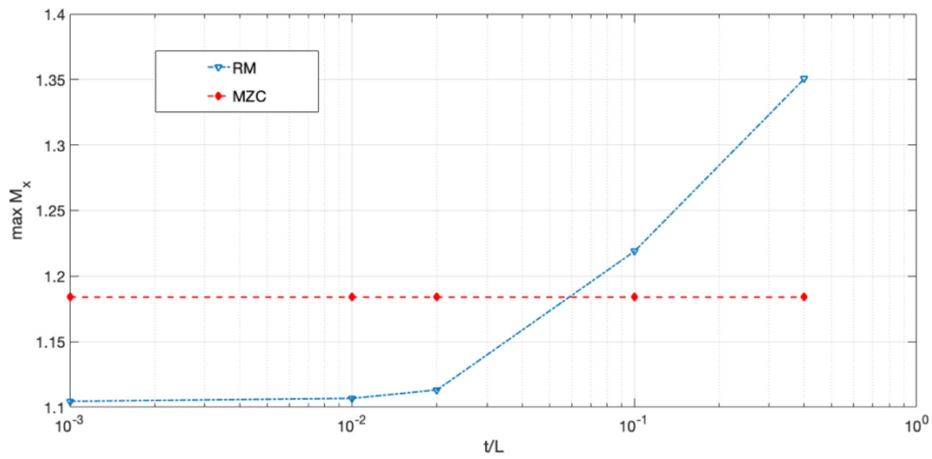

Figure 4: Maximum bending moment (x) for RM and MZC plates

As for the maximum bending moment (M_x), it is observed the locking effect for t/L<0.02, where the RM plate cannot compute correctly the bending value due to the shear locking.

Patch test for the MZC element

If the purpose is assessing the convergence of the MZC element, it turns out interesting to carry out a Patch test. It consists in applying a displacement field as initial data, and then, computing the results in order to check if the model is capable of reproduce the rigid body motions.

The selected patch is the following

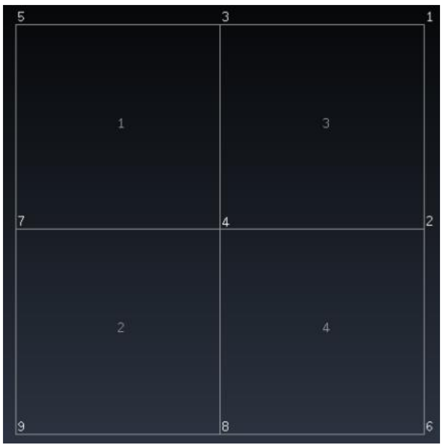

Figure 5: Meshing of the patch test

By prescribing the displacements at the boundaries, the inner node is used to check the results of the test.

Two displacement fields are applied:

1. $w(x,y) = 3 - x - 2y$	2. $w(x,y) = 1/2(x^2 + 2y^2 + 3xy)$

The first displacement field, is a linear one with arbitrary coefficients, and the second field is quadratic.

The results of the patch test are the following

Vertical displacement at node 5			
Displacement field	w (expected)	w (achieved)	error
1	0.000	2.78E-16	0.00%
2	3.000	3.65329949	21.78%

As can be seen, the patch test is satisfactory for the linear field case, and the MZC plate is capable of reproduce the rigid body motions. Therefore, the model will converge if it is conveniently refined. However, the error for the quadratic case is very large and the model is unable to reproduce a constant curvature field.

Another patch test is performed changing the position of the node 5, to asses if the MZC plate can converge for arbitrary quadrilateral shapes. The new position is (0.75 , 0.75).

The results for a linear displacement field are the following

Expected results at node 5			
Displacement field	w (expected)	w (achieved)	error
1	0.750	0.216	71.17%

This result shows that the MZC plate cannot guarantee good results for arbitrary quadrilateral shapes. Therefore, it is only a suitable method for rectangular elements.

References

Oñate E. Structural Analysis with the Finite Element Method Linear Statics Volume 2. Beams, Plates and Shells. In: Springer. 2013. p. 233–381. chapters 5,6.