Escola Tècnica Superior d'Enginyers de Camins, Canals i Ports de Barcelona

MSc Computational Mechanics

Computational Structural Mechanics and dynamics

Plate Theory

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1 Shear locking effect

Analyze the shear locking effect on the Reissner Mindlin element and compare with the MZC element for the Simple Support Uniform Load square plate.
Figure. 1 show the boundary condition – strong simple support and loads applied on the plate with size 1×1.

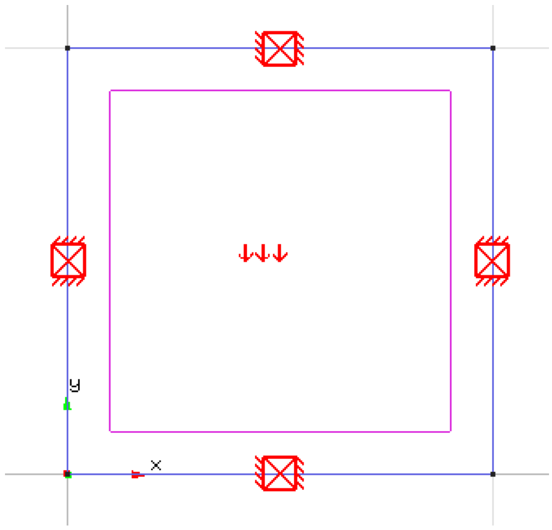
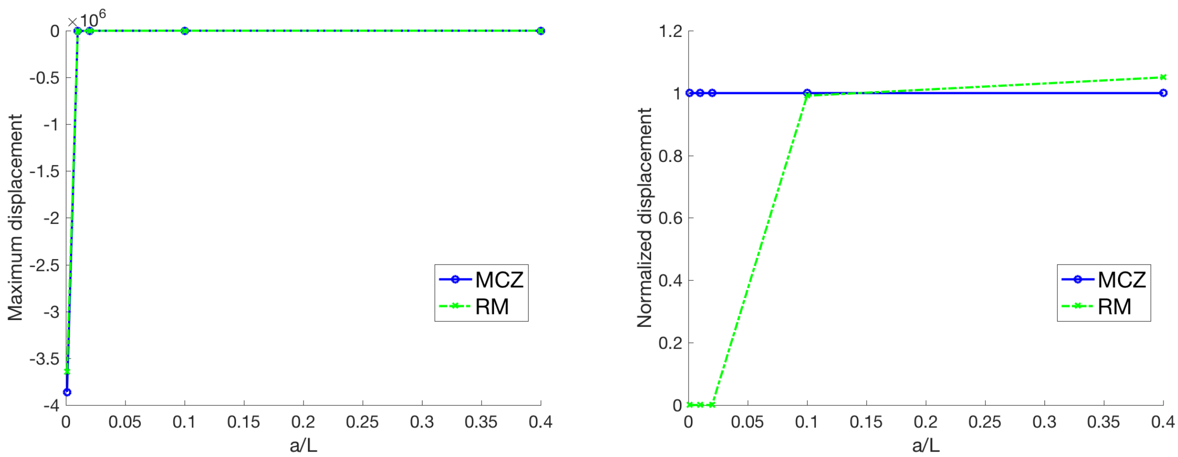


Figure 1: Simple supported plate

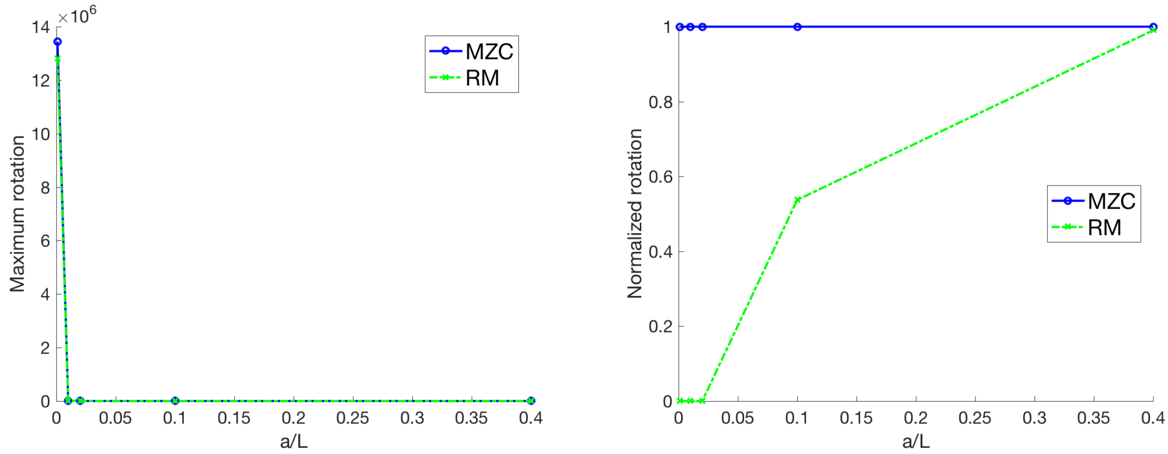
Figure 2 and Figure 3 demonstrates the different of the displacement and rotation field using these two methods. The normalized plot can give a more vivid view of the different between these methods, where the y axis is $exp(\frac{w-w_{MZC}}{w})$ w is the parameter we are insterested in. It can be observed that Reissner Mindlin works well for thick plates. But for thin plate, it will suffer from the transverse shear locking effect so the plate is too rigid compared with the results from the Kichhoff plate theory. For the thick plate, the Reissner Mindlin plate is more accurate but for thin plate Kichhoff plate is prefered.



(a) Maximum displacement

(b) Normalized maximum displacement

Figure 2: Maximum displacement in the Z direction



(a) Maximum displacement (b) Normalized maximum displacement

Figure 3: Maximum rotation in the X direction

Reissner-Mindlin plate theory is equivalent to Timoshenko beam theory while Kichhoff plate theory is equivalent to Euler Bernoulli beam theory. Therefore, for MZC plate elements, we don't have the information of the shear stress because it assumes the shear deformation is negligible.

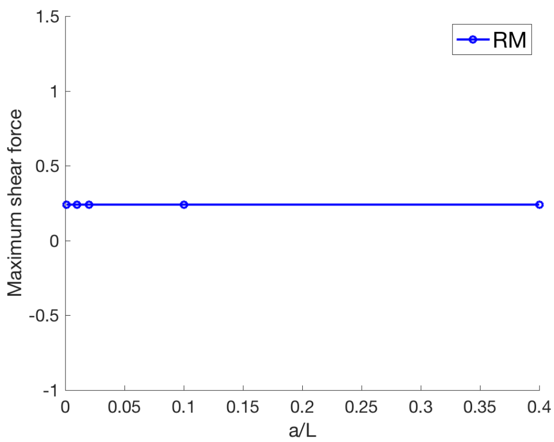


Figure 4: Maximum shear force with Reissner Mindlin elements

In conclusion, Reissner-Mindlin plate theory assumes that the points laying on a normal to the plate middle plane before deformation, remain on a straight line which is not necessarily orthogonal to the middle plane after deformation. It is equivalent to Timoshenko beam theory. It works for thick plates and some times thin plates. Similar to classical Timoshenko, Reissner-Mindlin plate elements suffer from the tranverse shear locking effect for thin plates. This can be avoided by applying the reduced integration method and appropriate elements. The normal orthogonality condition only holds for thin plates (thickness/ average side ratio:$t/L \leq 0.05$). For moderately thick ($0.05 \leq t/L \leq 0.1$) and very thick ($t/L \geq 0.10$) plates, the distortion of the normal during deformation increases. Reissner-Mindlin theory represents a better approximation to the actual deformation of the plate in these cases.

2 Patch test

The patch test in the finite element method is a simple indicator of the quality of a finite element. A broader definition of patch test (applicable to any numerical method, including and beyond finite elements) is any test problem having an exact solution that can, in principle, be exactly reproduced by the numerical approximation. Therefore, a finite-element simulation that uses linear shape functions has patch tests for which the exact solution must be piecewise linear, while higher-order finite elements have correspondingly higher-order patch tests. In this section a patch test mesh is defined and verified for the MCZ element with both linear and quadratic shape functions.

2.1 Linear displacement field

The rectangular MZC elements are shown in Figure.5. Firstly, we carry on the patch test with linear displacement field $w = \frac{x+y-1}{10}$. The displacements of the nodes at the boundary of the patch are prescribed according to the analytical solution. Then the values of the displacement at the internal nodes are computed and compared to the exact ones. The material paramters are defined as follows:

$$\begin{aligned} young &= 2.1e + 11N/m^3 \\ poisson \quad ratio &= 0.2 \\ density &= 0 \\ thickness &= 0.1m \end{aligned} \tag{1}$$

The regular meshes we use is shown in Figure 5.

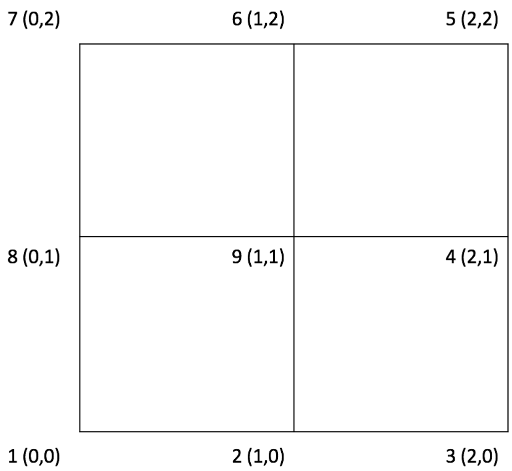


Figure 5: Rectangular MZC elements

The boundary conditions are imposed according to the analytical displacement field. They are shown as follows

node	Imposed Linear Displacement
1	-0.1
2	0
3	0.1
4	0.2
5	0.3
6	0.2
7	0.1
8	0

Table 1: Imposed linear boundary displacement

$\theta_x = 0.1$ and $\theta_y = 0.1$.
The displacement field is demonstrated in Figure.7. The numerical solution of displacement at the node 9 (1, 1) is 0.1m and the rotation in x and y direction is 0.1, which are the same as the exact solution. So, the rectangular MZC plate elements for the on imposing to the patch boundary nodes the linear displacement field successfully passes the patch test.

Another simulation is done with the irregular meshes shown in Figure.6 by moving node 9 in the original meshes a little.

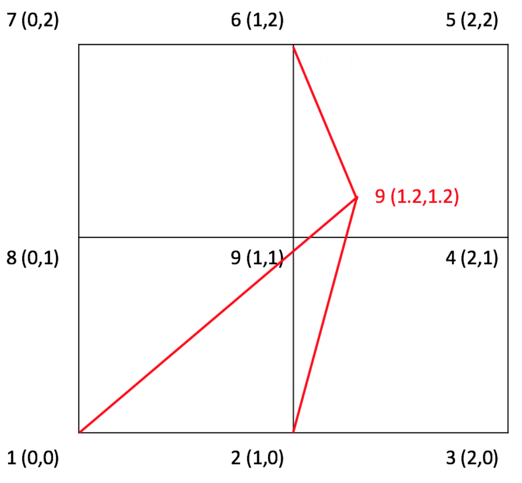
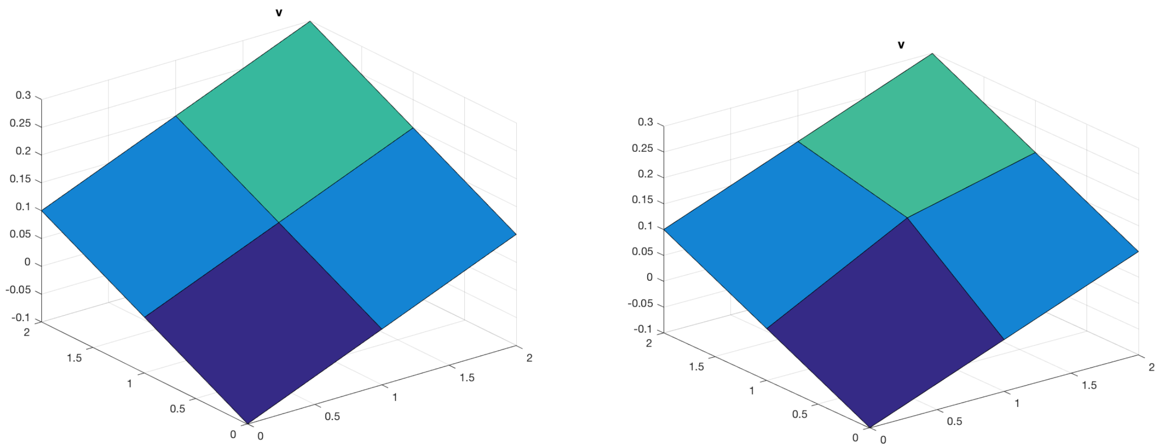


Figure 6: Irregular quadrilateral elements

The exact solution sought at the node 9(1.2, 1.2) is 0.14m for the displacement and 0.12 for the rotation. The evaluated solution using the MZC plate elements for the patch test is 0.111613m for the displacement and 0.104044 for the rotation. So, the quadrilateral MZC plate elements for the on imposing to the patch boundary nodes the linear displacement field does not always successfully pass the patch test.

node	Imposed Linear Displacement	θ_x	θ_y
1	0	0	0
2	0.1	0.2	0.1
3	0.4	0.4	0.2
4	0.7	0.5	0.4
5	1.2	0.6	0.6
6	0.7	0.4	0.5
7	0.4	0.2	0.4
8	0.1	0.1	0.2

Table 2: Imposed quadratic boundary displacement



(a) Linear regular meshes

(b) Linear irregular meshes

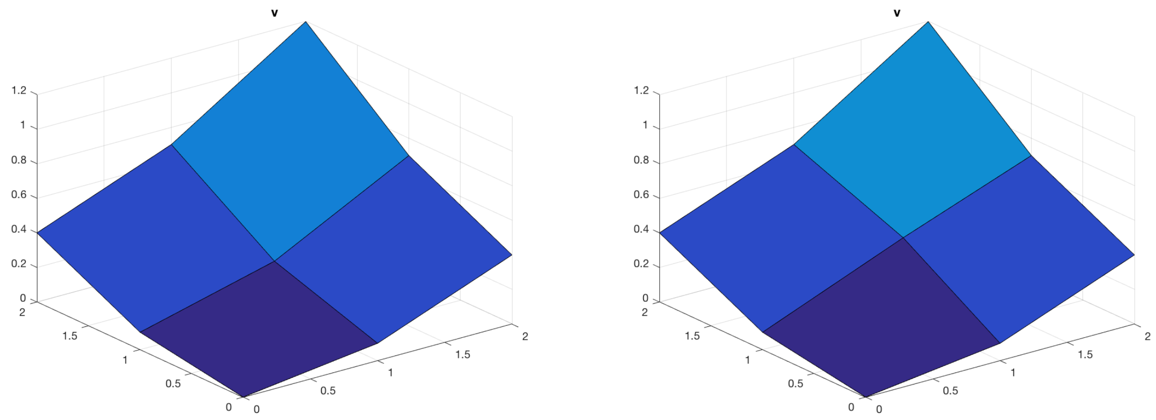
Figure 7: Patch test with linear analytical displacement field

2.2 Quadratic displacement field

Secondly, we carry on the patch test with quadratic displacement field $w = \frac{x^2+y^2+xy}{10}$. The rotation is $\theta_x = \frac{2x+y}{10}$ and $\theta_y = \frac{x+2y}{10}$. The displacements of the nodes at the boundary of the patch are prescribed according to the analytical solution, which is shown in Table.2. Then the values of the displacement at the internal nodes are computed and compared to the exact ones.

The numerical solution of displacment at the node 9 (1, 1) is 0.3m and the rotation is 0.3, which are the same as the exact ones. So, the rectangular MZC plate elements for the on imposing to the patch boundary nodes the quadratic displacement field successfully passes the patch test.
Similarly as the linear displacement field, another simulation is done with the meshes in Figure.6. In this case the displacement at the node 9 (1, 1) is 0.338591m and the rotation is 0.339244 which are different from the exact solutions 0.432m and 0.36.

In conclusion, it is found that on imposing to the patch boundary nodes the linear and quadratic displacement fields, only the rectangular MZC elements pass the patch test. So, the non-conforming MZC element converges for the rectangular elements. On the other hand, it does not converge for any other quadrilateral of random shape. The application MZC element plate is limited by the geometry of the structure.



(a) Quadratic regular meshes (b) Quadratic irregular meshes

Figure 8: Patch test with quadratic analytical displacement field