Universitat Politècnica de Catalunya
Master of Science in Computational Mechanics
Computational Structural Mechanics and Dynamics
CSMD
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Assignment 7 - Thick and Thin Plates Theories

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Assignment 7.1

a.1

For this problem the geometry is such that it has no symmetry with respect of the middle plane, thus, at the interface of two plates with different thickness shear effects may not be neglected as orthogonality condition in the deformed planes may not be fulfilled. Even though the dimensions are such that $t/width \leq 0.1$, the Reisser-Middlin theory is chosen. In order to avoid the shear locking effect a reduced integration of the shear stuffiness matrices in the structure section where $t/width$ is small enough should be employed. Quadrilateral elements and zero displacement w at supports may be applied.

a.2

For this problem, the geometry has a symmetry about the middle-plane, this way, the hypothesis of orthogonality conditions in the deformed section is reasonable. Also, $t/width \leq 0.1$ in all sections. Consequently the Kirchoff theory may be applied. Quadrilateral elements and zero displacement w at supports may be applied.

Assignment 7.2 - Patch test

In order to test the element convergence the Patch shown in Figure 1 is chosen:

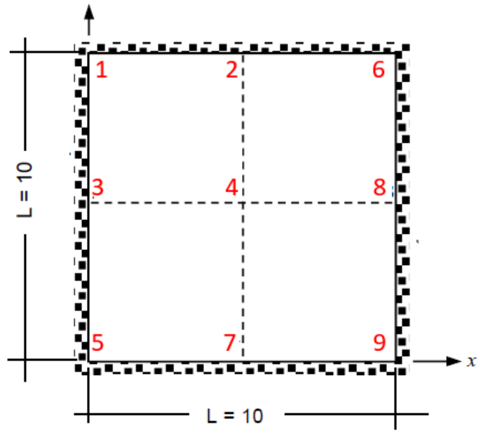

Figure 1: Chosen Patch

This patch consists of four (4) 4-noded quadrilateral C^1 continuity elements. Two displacement patch tests are performed in order to describe the procedure underlying this convergence requirement test. The first will test the free body motion, and the latter a arbitrarily quadratic displacement field will be imposed. For both cases, body forces are set to zero and node 4 is the free node where the displacement will be evaluated.

For the free body motion $w = 10^{-3}[m]$, $\theta_x = 0$ and $\theta_y = 0$ is imposed on the nodes 1-9 (except node 4). The results are summarized in Table 1.

$\mathbf{w_4}$ [m]	$\theta_{\mathbf{x_4}}$ **[rad]**	$\theta_{\mathbf{y_4}}$ **[rad]**	σ_x [Pa]	σ_y [Pa]	τ_{xy} **[Pa]**
1.0000×10^{-3}	$\approx 10^{-20}$	$\approx 10^{-20}$	$\approx 10^{-26}$	$\approx 10^{-27}$	$\approx 10^{-28}$

Table 1: Free body motion patch test results.

As it can be seen the free body motion is fullfilled as the displacements in node 4 are the same as the displacements imposed in the outter nodes. Also, the zero energy mode is achieved as stress/strains are zero inside the elements.

For the arbitrary imposed displacement field we choose a displacement field order such that the strain state is constant within the element. As the w is interpolated with a third order polynomial, w needs to be a second order one, thus we choose : $w = (1 + x + y + xy + x^2 + y^2) \times 10^{-3}$[m], $\theta_x = (1 + y + 2x) \times 10^{-3}$ [rad] and $\theta_y = (1 + x + 2y) \times 10^{-3}$ [rad] as imposed displacements on the nodes 1-9 (except node 4). The results are summarized in Table 2.

$\mathbf{w_4}$ [m]	$\theta_{\mathbf{x_4}}$ **[rad]**	$\theta_{\mathbf{y_4}}$ **[rad]**	σ_x [Pa]	σ_y [Pa]	τ_{xy} **[Pa]**
0.0860	0.0160	0.0160	-2.6000×10^{-9}	-2.6000×10^{-9}	-7.0000×10^{-9}

Table 2: Quadratic arbitrary displacement field.

The displacements found for node 4 agrees with the values of the quadratic displacement field at the node 4 coordinates: $w(5, 5) = 0.0860$ [m] , $\theta_x(5, 5) = 0.0160$ [rad], $\theta_y(5, 5) = 0.0160$ [rad]. Also, a constant stress/strain state over the elements were found as expected.

Routine changes

Routine changes in order to implement the patch test is shown below.

```
%PRESCRIBED DISPLACEMENT
X = coordinates(:,1);
Y = coordinates(:,2);
global W Tx Ty
W = (1e-3)*(1 + X + Y + X.*Y + X.^2 + Y.^2 );
Tx = (1e-3)*(2*X + Y + 1);
Ty = (1e-3)*(2*Y + X + 1);
Di = [W Tx Ty];
L = length(W);
Di = reshape(Di',3*L,1);
Di = [Di(1:9);Di(13:27)];

fixdesp  = [
    1, 1,  0.05;
    1, 2,  0.01;
    1, 3,  0.005;
    2, 1,  0.0875;
    2, 2,  0.0125;
    2, 3,  0.01;
    3, 1,  0.0125;
    3, 2,  0.005;
    3, 3,  0.0025;
    5, 1,  0.0;
    5, 2,  0.0;
    5, 3,  0.0;
    6, 1,  0.15;
    6, 2,  0.015;
    6, 3,  0.015;
    7, 1,  0.0125;
    7, 2,  0.0025;
    7, 3,  0.005;
    8, 1,  0.0875;
    8, 2,  0.01;
    8, 3,  0.0125;
    9, 1,  0.05;
    9, 2,  0.005;
    9, 3,  0.01];

fixdesp(:,3) = Di;
```