Assignment 7.1

The aim of this assignment is to compare two different plate formulation with Matlab codes, one representing the MZC formulation (based on the Kirchoff classical theory) and the other one representing the Reissner-Mindlin formulation.

The comparison between the two element types was performed using a 5mx5m square plate, discretized into a mesh of 25 square first order elements, as we can observe in the Figure 1.

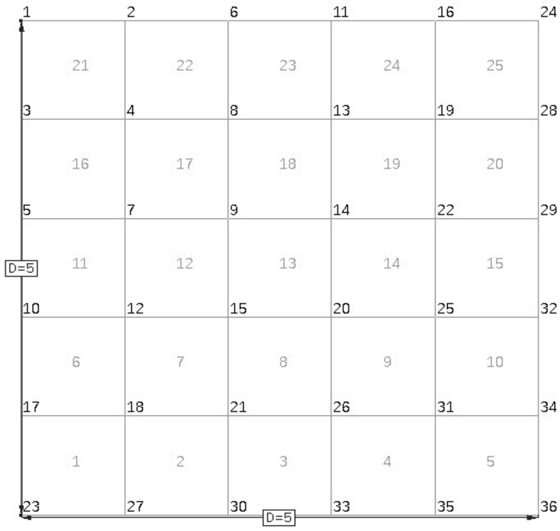


Figure 1.1 – Geometry and Meshing in GiD

Boundary conditions

The types of boundary conditions that are enforced in this problem are the following:

- Displacements Constraints / Linear Constraints: Movement in Z direction is prevented along the boundary of the plate (Figure 1.2).We allow the free rotation around X and Y direction.

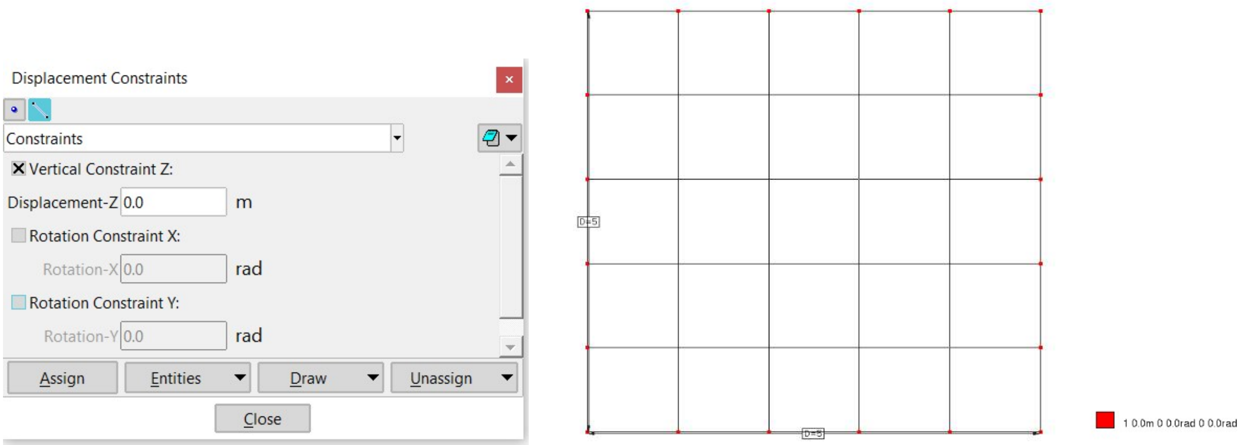


Figure 1.2 – Boundary Conditions

Loads

The way to load this model is the following:

- Assign Uniform Loads / Uniform Loads: We apply an uniform load of 1 N/m in Z direction in all the surface of the plate (Figure 1.3)

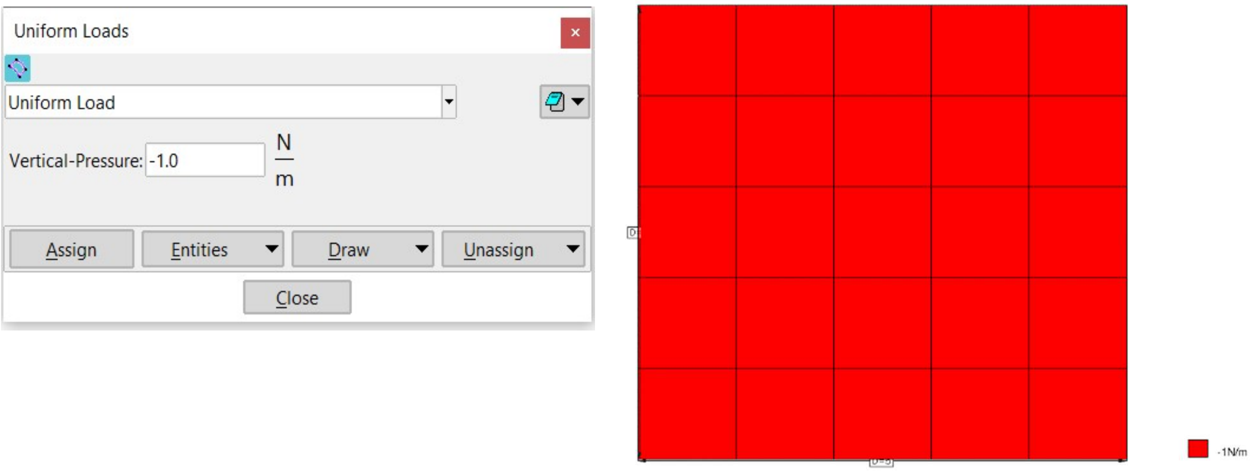


Figure 1.3 – Uniform Load

Material

We use a material with the following mechanical characteristics (Figure 1.4):

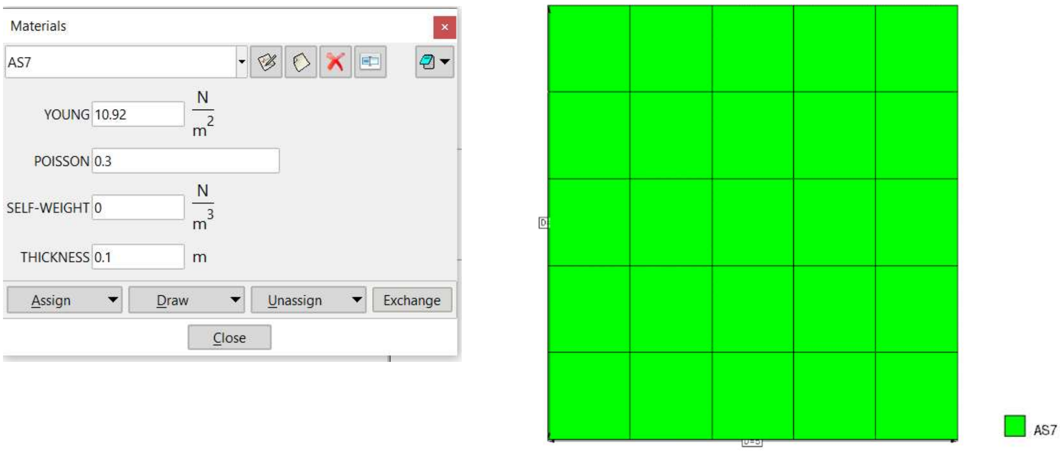


Figure 1.4 – Material

The value of the thickness of the plate depends on the case that we are running.

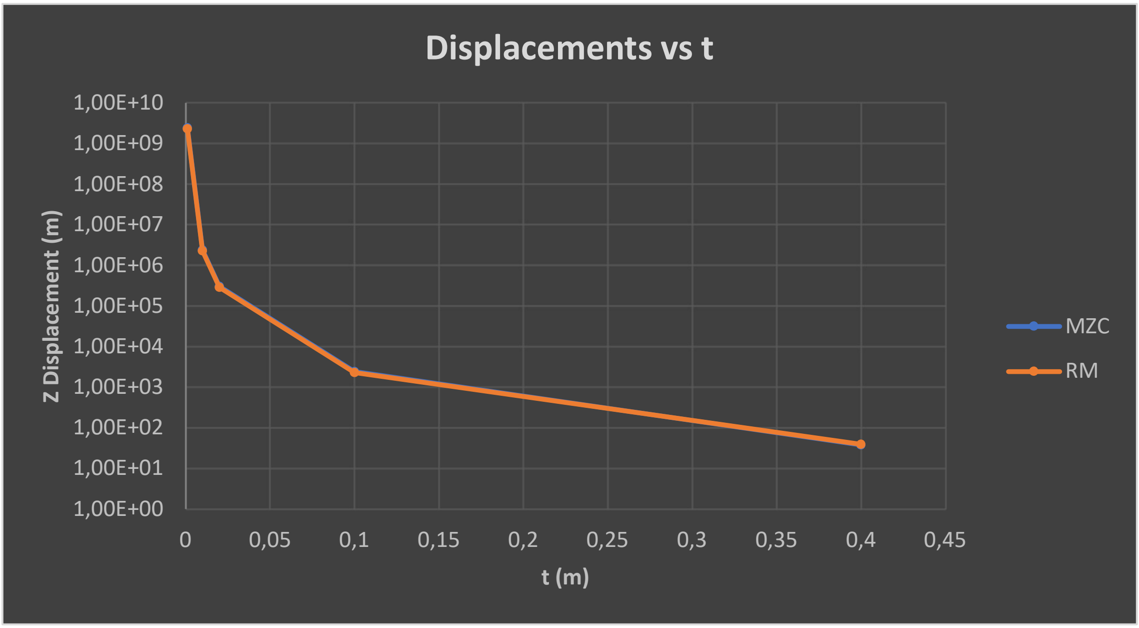
The results for both formulations are in the Table 1 and 2, and printed in the graphics:

MZC						
t (m)	Z Displacement (m)	Mxy (N*m)	Mx (N*m)	My (N*m)	Rotx (rad)	Roty (rad)
0,001	-2,43E+09	-0,86173	-1,1839	-1,1839	-1,68E+09	-1,68E+09
0,01	-2,41E+06	-0,86173	-1,1839	-1,1839	-1,68E+06	-1,68E+06
0,02	-3,02E+05	-0,86173	-1,1839	-1,1839	-2,10E+05	-2,10E+05
0,1	-2,41E+03	-0,86173	-1,1839	-1,1839	-1,68E+03	-1,68E+03
0,4	-3,77E+01	-0,86173	-1,1839	-1,1839	-2,63E+01	-2,63E+01

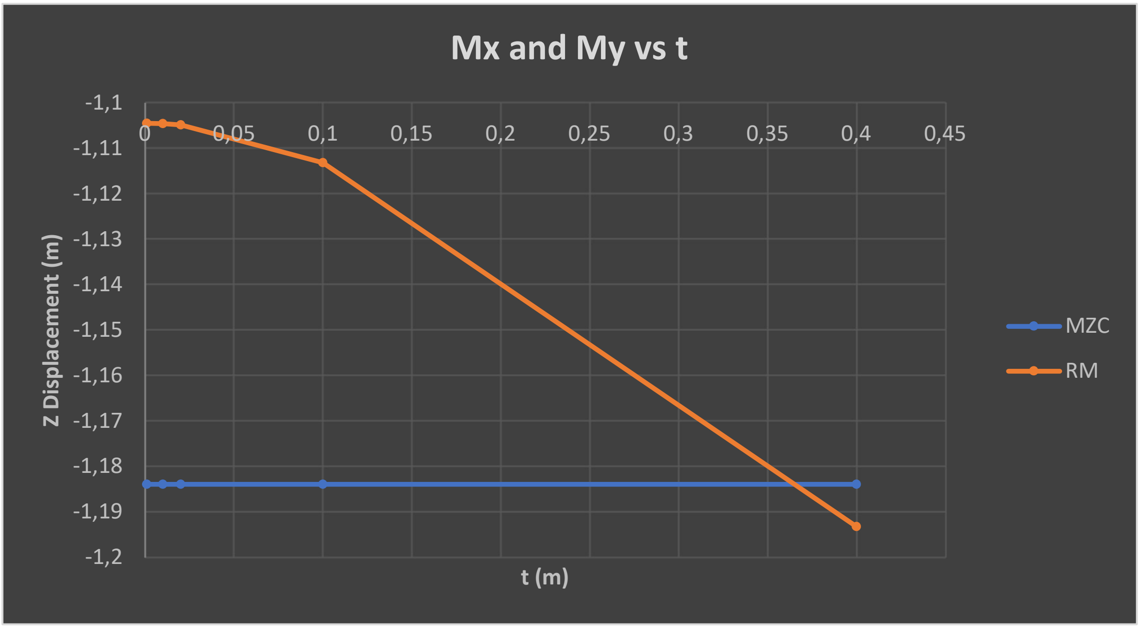
Table 1 – MZC formulation

RM						
t (m)	Z Displacement (m)	Mxy (N*m)	Mx (N*m)	My (N*m)	Rotx (rad)	Roty (rad)
0,001	-2,28E+09	-0,70899	-1,1045	-1,1045	-1,60E+09	-1,60E+09
0,01	-2,28E+06	-0,70878	-1,1046	-1,1046	1,60E+06	1,60E+06
0,02	-2,85E+05	-0,70814	-1,1049	-1,1049	-2,00E+05	-2,00E+05
0,1	-2,30E+03	-0,68873	-1,1132	-1,1132	-1,62E+03	-1,62E+03
0,4	-3,93E+01	-0,52615	-1,1932	-1,1932	-2,76E+01	-2,76E+01

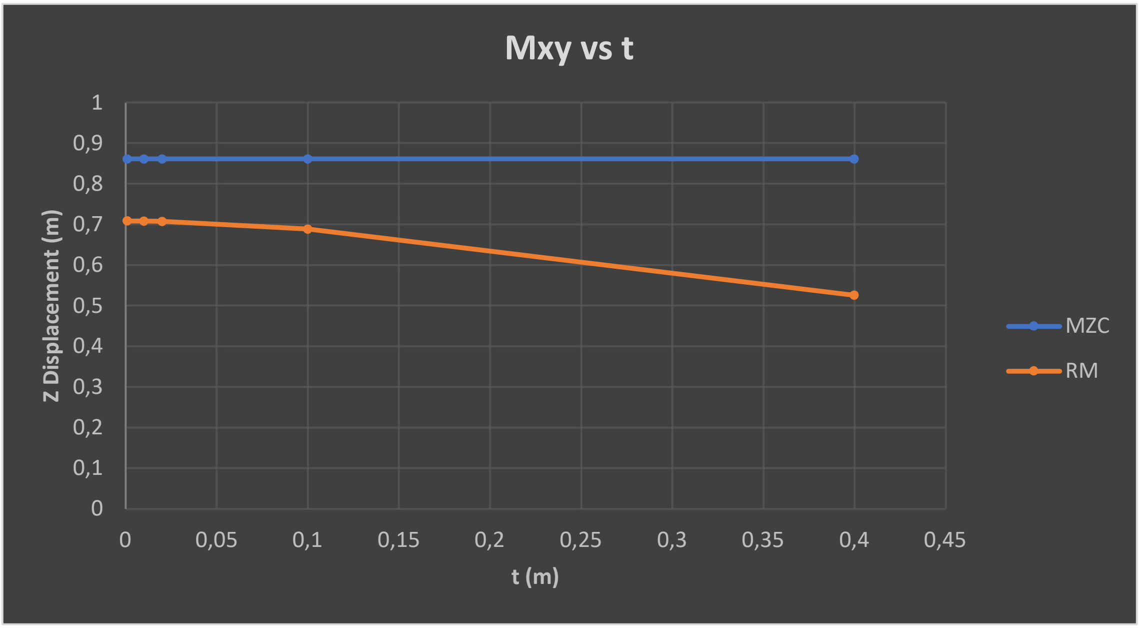
Table 2 – Reissner Mindlinl formulation



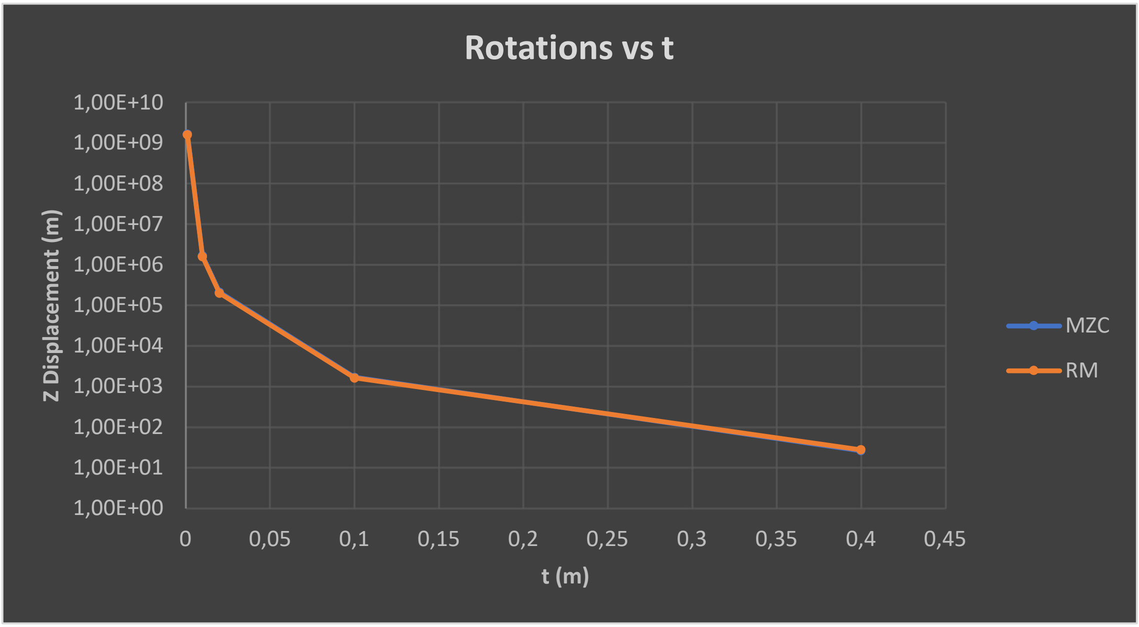
Graphic 1 – Z displacement vs Thickness



Graphic 2 – Moments Mx and My vs Thickness



Graphic 3 – Moments Mxy vs Thickness



Graphic 4 – Rotations ϑx and ϑy vs Thickness

<u>Conclusions</u>

We reach a series of conclusions provided by the previous graphics. We can highlight the following points

While the maximum moments obtained for the MZC model are constant for all plate thicknesses, the results for the RM vary slightly for each analysis, becoming closer to the results of the MZC model as the thickness increases, due to a reduction of shear locking effect.
However, Mxy moments present greater changes between methods, suffering a greater variation in percentage as the plate grows in thickness.

According to the values of the maximum displacement in Z direction does not change significantly from one formulation to the another, even with different values of thickness. The values of maximum displacement and rotations presented by the RM element formulation are smaller than the ones presented by the MZC element formulation when the thickness varies from 0.001 to 0.1 In such range, the plate is considered thin and the RM element formulation is not suitable
Therefore, the RM element formulation presents a transverse shear locking effect. At such condition, the components of the shear stiffness matrix increase greatly, neglecting the components of the bending stiffness matrix, and the values of displacement and rotations decrease. If we apply a finer mesh to the geometry, we can verify such behaviour.

It can be concluded again that Reissner-Mindlin as well as Tymoshenko in the case of beams, will be a better estimator in the case of displacements, while MZC will work better with moments.

Assignment 7.2

The patch test is based in imposing at the boundary of a patch element a displacement field which can be exactly reproduced by the shape functions.

The displacements at the interior nodes of the patch should coincide with the exact values of the boundary and a constant strain field is obtained throughout the patch. This means that the patch should displace as a rigid body.

Using the previous meshing, we have to fix an unitary vertical displacement for the nodes at the boundary of the patch, while restraining the rotation around X and Y direction (Figure 2.1). The passing criteria is that the displacements of the nodes inside the patch (nodes 9, 15, 14 and 20) are identical to the one prescribed for the nodes at the boundary of the patch.

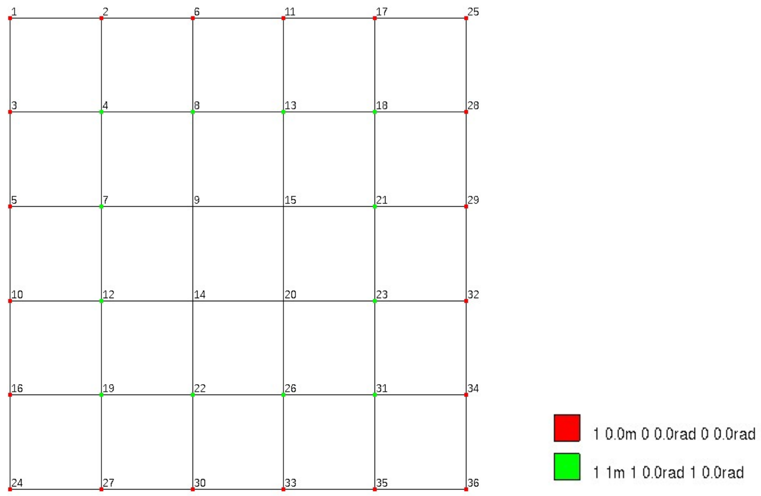


Figure 2.1 – Boundary Conditions

In the following Figures we can see the vertical displacements and the respective rotations of the plate:

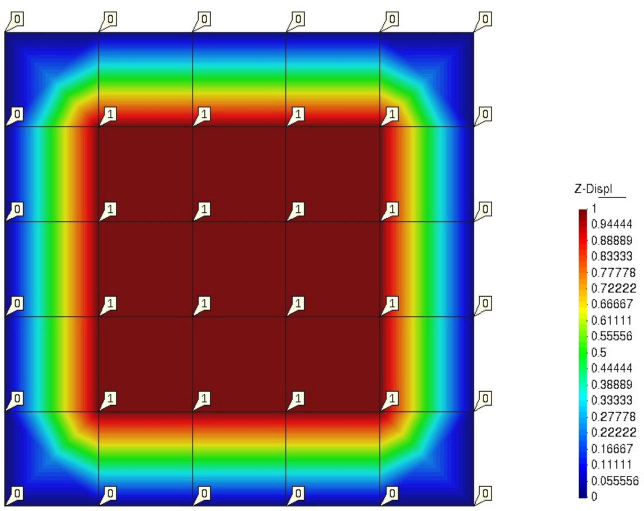


Figure 2.2 – Z Displacements

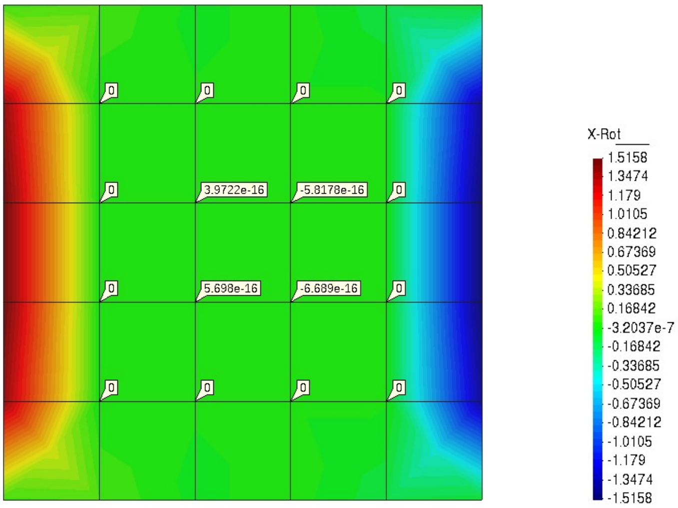


Figure 2.3 – ϑx

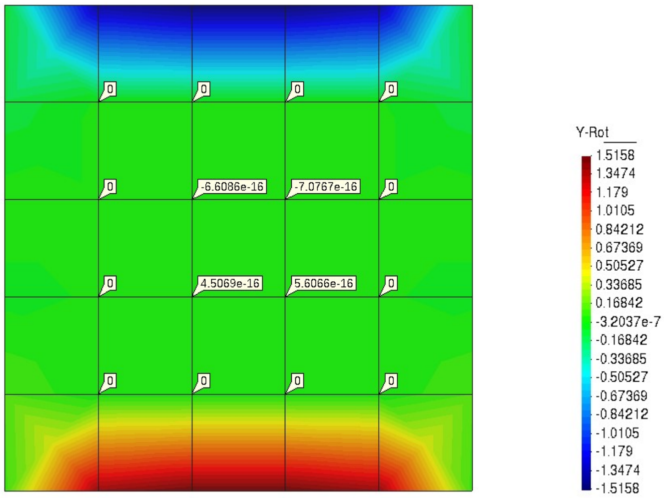


Figure 2.4 – ϑy

Node	Z Displacement (m)	ϑx	ϑy
4	1	0	0
8	1	0	0
13	1	0	0
18	1	0	0
21	1	0	0
23	1	0	0
31	1	0	0
26	1	0	0
22	1	0	0
19	1	0	0
12	1	0	0
7	1	0	0
9	1	3,97E-16	-6,61E-16
15	1	-5,82E-16	-7,08E-16
14	1	5,70E-16	4,51E-16
20	1	6,69E-16	5,61E-16

All vertical displacements are exactly equal to those at the boundary of the patch and rotations are practically equal to zero.