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Assignment 6- Beam

Assignment 1: *Program Timoshenko 2-nodes beam with reduce integration for shear stiffness matrix*

If the length of the element is considerably larger than the thickness, we can approximate the element with a beam. The best known beam theories are Eulero Bernoulli's beam theory, used for slender beams, and Timoshenko's ray theory, used for squat beams.

By the theory, defining the beam slenderness ratio as $\lambda = \frac{l^2}{h}$, as the beam slenderness increases, the solution for Timoshenko element is progressivity stiffer than the exact one. To solve this problem, Timoshenko's reduced integration method can be used. which is able to reduce the influence of transverse stiffness using a Gauss quadrature of an order lower than that needed to exactly integrate the terms in K_s^e.

In order to implement Timoshenko's treatment in a MATLAB code, we need to modify the stiffness matrix K_s^e and:

```
K_s = [  1     ,  len/2   ,  -1     ,  len/2   ;
        len/2  ,  len^2/4 ,  -len/2 ,  len^2/4 ;
        -1     ,  -len/2  ,   1     ,  -len/2  ;
        len/2  ,  len^2/4 ,  -len/2 ,  len^2/4 ];
```

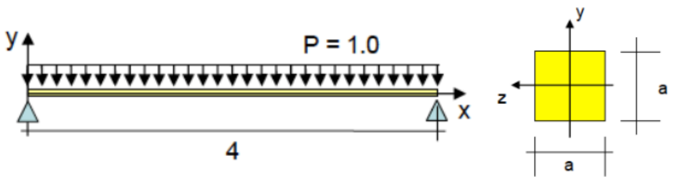
In the following exercise we can compare the results of the beam of Eulero Bernoulli and Timoshenko.

Assignment 2: *Solve the following problem with 64 elements mesh with*

- *2 nodes Euler Bernulli element*
- *2 nodes Timoshenko Full Integrate element*
- *2 nodes Timoshenko Reduce Integration element.*

Compare the maximum displacements, moments and shear for the 3 elements against the a/L relationship.

Beam data:
$$\begin{cases} E = 21000 \\ \nu = 0.25 \\ L = 4 \\ \textit{variable a and area} \\ \textit{inertia} = {a^4}/{12} \end{cases}$$

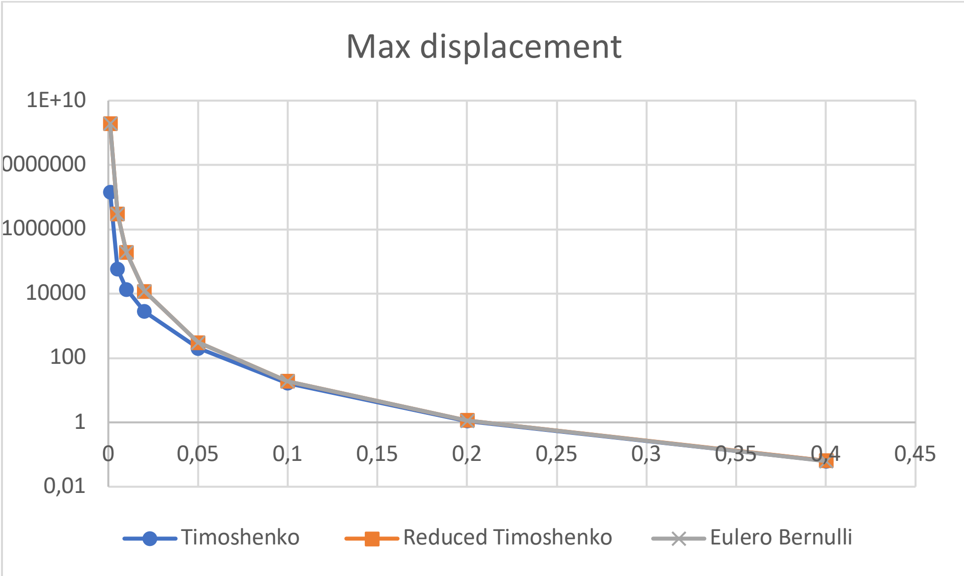


We will analyze the problem for each case mentioned (vary by a) and the results will be compared by means of a graph representing the displacement values (y direction), the maximum moment and cut for each type of element with respect to a / L.
The results were obtained using 8 different values of a to be able to compare different slenderness ratios.

Case number	a	a / L	$\text{Inertia} = \frac{1}{12} \cdot \alpha^4$
1	0.001	0.00025	8,3333E-14
2	0.005	0.00125	5,2083E-11
3	0.01	0.0025	8,3333E-10
4	0.02	0.005	1,3333E-08
5	0.05	0.0125	5,2083E-07
6	0.1	0.025	8,3333E-06
7	0.2	0.05	0,00013333
8	0.4	0.1	0,00213333

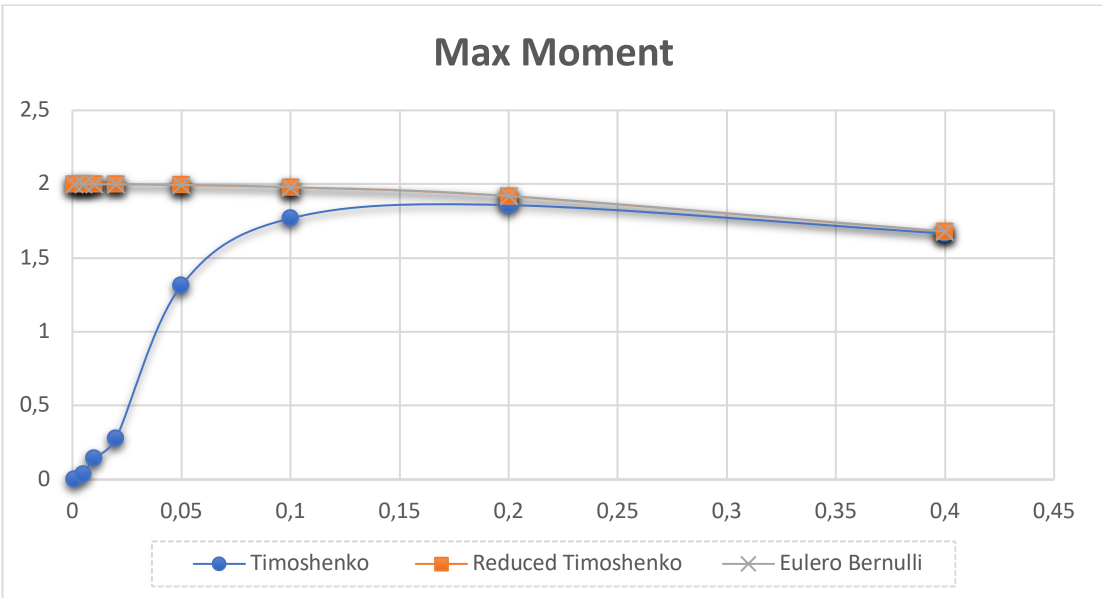
Max Displacement

Case numeber	*Timoshenko*	*Reduced Timoshenko*	*Eulero Bernulli*
1	$1.462 \cdot 10^6$	$1.904 \cdot 10^9$	$1.905 \cdot 10^9$
2	$5.7401 \cdot 10^4$	$3.046 \cdot 10^6$	$3.047 \cdot 10^6$
3	$1.3583 \cdot 10^4$	$1.904 \cdot 10^5$	$1.905 \cdot 10^5$
4	$2.7972 \cdot 10^3$	$1.1896 \cdot 10^4$	$1.1905 \cdot 10^4$
5	$1.999 \cdot 10^2$	$3.04 \cdot 10^2$	$3.048 \cdot 10^2$
6	$1.6707 \cdot 10^1$	$1.888 \cdot 10^1$	$1.885 \cdot 10^1$
7	1.1133	1.1493	1.1429
8	$6.352 \cdot 10^{-2}$	$6.4 \cdot 10^{-2}$	$6.252 \cdot 10^{-2}$



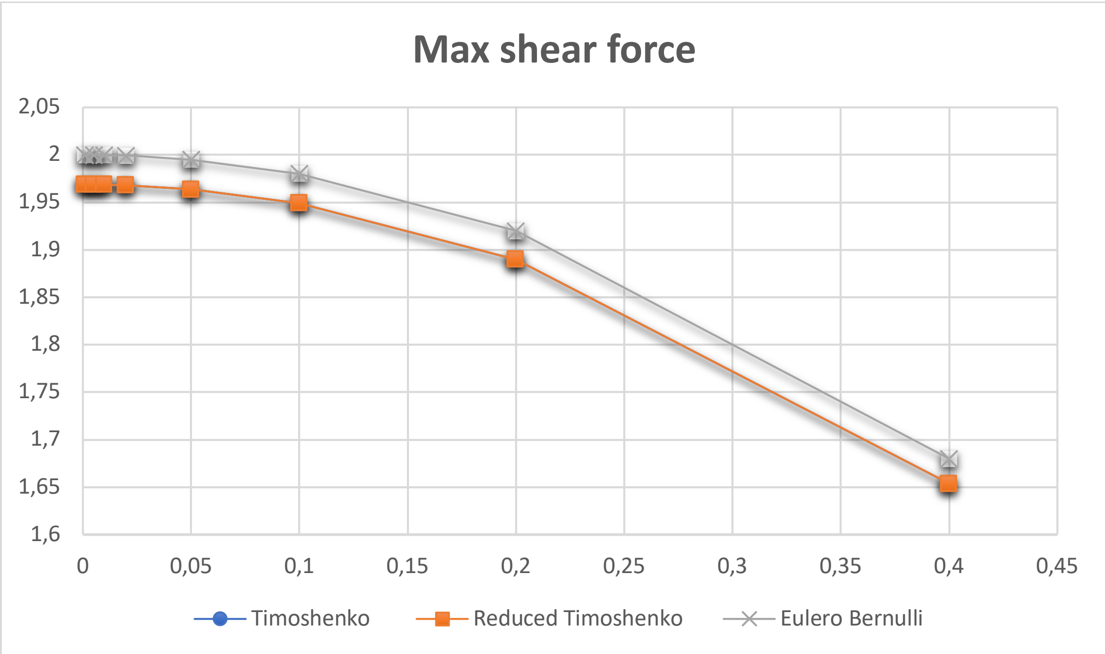
Max moment

Case numeber	*Timoshenko*	*Reduced Timoshenko*	*Eulero Bernulli*
1	$1.5341 \cdot 10^{-3}$	1.999	2.0003
2	$3.7658 \cdot 10^{-2}$	1.999	2.0003
3	$1.4258 \cdot 10^{-1}$	1.9988	2.0001
4	$2.7972 \cdot 10^{-1}$	1.9982	1.9995
5	1.3144	1.9940	1.9953
6	1.7687	1.9790	1.9803
7	1.9360	1.9191	1.9203
8	1.9829	1.6792	1.6803



Max shear force

Case numeber	*Timoshenko*	*Reduced Timoshenko*	*Eulero Bernulli*
1	1.9688	1.9688	2.000
2	1.9687	1.9687	1.9999
3	1.9686	1.9686	1.9998
4	1.9680	1.9680	1.9992
5	1.9638	1.9638	1.9950
6	1.9491	1.9491	1.980
7	1.8900	1.8900	1.920
8	1.6538	1.6538	1.680



We can see how the radius will produce a greater vertical displacement for a value less than a (greater slenderness).
If we look at the displacement graph, it is evident that the element represented with the integral Timoshenko method offers a much more rigid solution than the others, producing a smaller displacement. This is due to the fact that Timoshenko takes into account the shear deformation which is more realistic for thick beams. It can be observed when the a / L ratio is sufficiently small, the result using Timoshenko's theory is excessively rigid, which is called the shear blocking effect. Therefore in thin beams the effect of shear deformation is negligible. This problem can be overcome with a reduced integration into the Timoshenko method. Their results coincide with those obtained using the Eulero-Bernoulli theory.
The Euler-Bernoulli model is very precise for the bending moment problem in the case of a real scenario (slender beam).

The graphs show that the results obtained by Eulero Bernoulli's theory and Timoshenko's theory with reduced integration are similar. Therefore Timoshenko's theory with reduced integration is valid for thick and thin beams, as it provides good results for both cases, unlike, as mentioned before, of a theory of Timoshenko with full integration that fails for thin beams.
Since Euler Bernoulli's theory assumes that the effect of the transverse deformation of the cut is negligible for a thin radius, we cannot obtain the shear stress with this method. Shear forces are compared according to Timoshenko's theory with full and reduced integration.