Computational Structural Mechanics and Dynamics
Assignment 6 - Beams

Federico Parisi
federicoparisi95@gmail.com

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Assignment 6.1:

a) Program In Mat Lab the Timoshenko 2 Nodes Beam element with reduce integration for the shear stiffness matrix.

Answer

Bending is an important behaviour of any mechanical system to be analyzed. The theory behind the study of beams behaviours is based on two main theories, the Euler-Bernoulli and the Timoshenko one. The first one is for thin beam and the second one for thick beams.

To solve the assignment, the Timoshenko method has to be implemented. They are based on the same principles, differing from the deflection plane; the Timoshenko theory doesn't accept that the plane originally ortogonal to the beam axis, remains ortogonal after deformation. That leads to a ϕ term to be computed apart from the displacement. This term is a measure of the shear strain.

The Timoshenko's method leads so to a different element stiffness matrix, split in a bending stiffness matrix and a shear one.

$$\begin{aligned} K^e a^e - f^e &= q^e \\ K^e &= K_b^e + K_s^e \end{aligned} \tag{1}$$

The problem comes from the ratio a/L when it becomes too small, so the beam can not be considered thick and it is called the shear locking effect.

In order to ensure that the shear stiffness matrix is singular, it is used a reduced integration method using only one integration point instead of two. This matrix will be obtained:

$$K_s^e = \left(\frac{GA^*}{l}\right)^{(e)} \begin{bmatrix} 1 & \frac{l}{2} & -1 & \frac{l}{2} \\ & \frac{l^2}{4} & -\frac{l}{2} & \frac{l^2}{4} \\ & & 1 & -\frac{l}{2} \\ sym & & & \frac{l^2}{4} \end{bmatrix} \tag{2}$$

In which $A^* = \alpha A$ with α is the sharping parameter of the section, for a rectangular one is 5/6.

So to implement the code is necessary adding the shear stiffness matrix defined below:

```
K_s = [ 1 , len/2 , -1 , len/2 ;
        len/2 , len^2/4 , -len/2 , len^2/4 ;
        -1 , -len/2 , 1 , -len/2 ;
        len/2 , len^2/4 , -len/2 , len^2/4 ];
```

Assignment 6.2

b) Solve the following problem with a 64 element mesh with the
2 nodes Euler Bernulli element
2 nodes Timoshenko Full Integrate element
2 nodes Timoshenko Reduce Integration element.
Compare maximum displacements, moments and shear for the 3 elements against the a/L relationship.

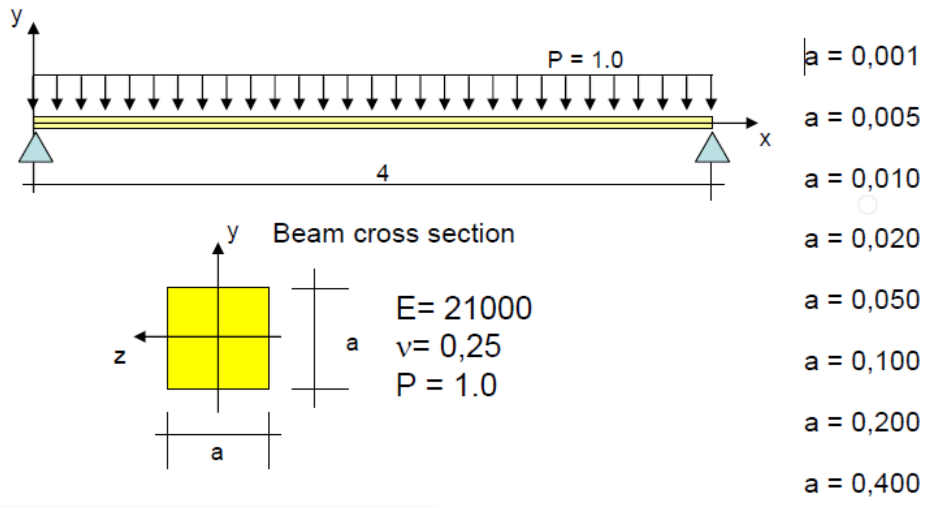

Figure 1: Problem data

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Answer

It is asked to analyze the behaviour of this beam, considering the different methods and the changing of the a dimension. What will change will be the Area and the Inertia, introduced respectively in computing the K_b and K_s matrices as following:

$$\begin{aligned} K_b &= \int_{l^e} B_b^T(E\mathbf{I})B_b dx \\ K_s &= \int_{l^e} B_s^T(G\alpha\mathbf{A})B_s dx \end{aligned} \tag{3}$$

remembering that $\alpha = 5/6$.

In the following plots are shown the behaviours of the different methods for changing the section of the beam, increasing the thickness.

On the x-axis is shown the ratio a/L while in the y-axis takes place the logarithmic scale respectively of the absolute values of displacements (fig. 2), momentum (fig. 3) and shear (fig. 4).

From the graph in figure 2 can be appreciate the shear locking effect introduced before. As can be noticed, with the Timoshenko's full integrated element, the displacements are not so accurate for slender beams and simulates smaller values. On the other hand, while using the reduced element integration, the Timoshenko's method has almost the same results as the Euler's, well-suited for slender beams. As expected and anticipated in the theory, the Timoshenko with the full integrated element works well for thick beams, so when the a/L ratio is large enough the values are almost the same.

The effectiveness of the reduced integration can be appreciated especially in the momentum plot (figure 3). When the beam is thin, the Timoshenko with full integration is far away from the accurate results, while the reduced one is clearly overcoming the problem of the shear locking effect.

Evaluating the shear values, with the Euler method is not possible to do it directly, while with the Timoshenko's ones yes and they give very similar results as expected.

As conclusion, it can be said that the two noded Timoshenko beam element with the reduced integration has excellent performance for slender and wide beams.

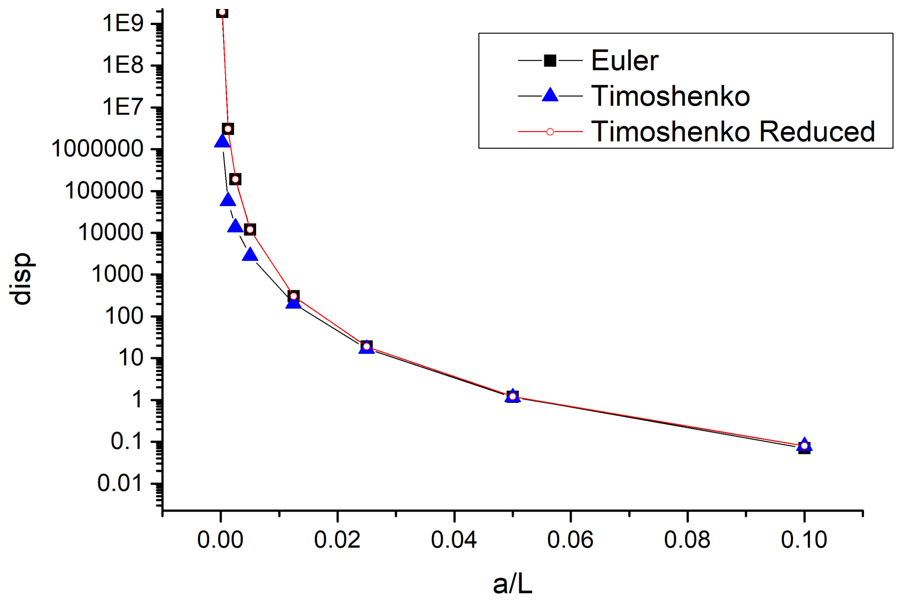

Figure 2: log(displacements)-a/L

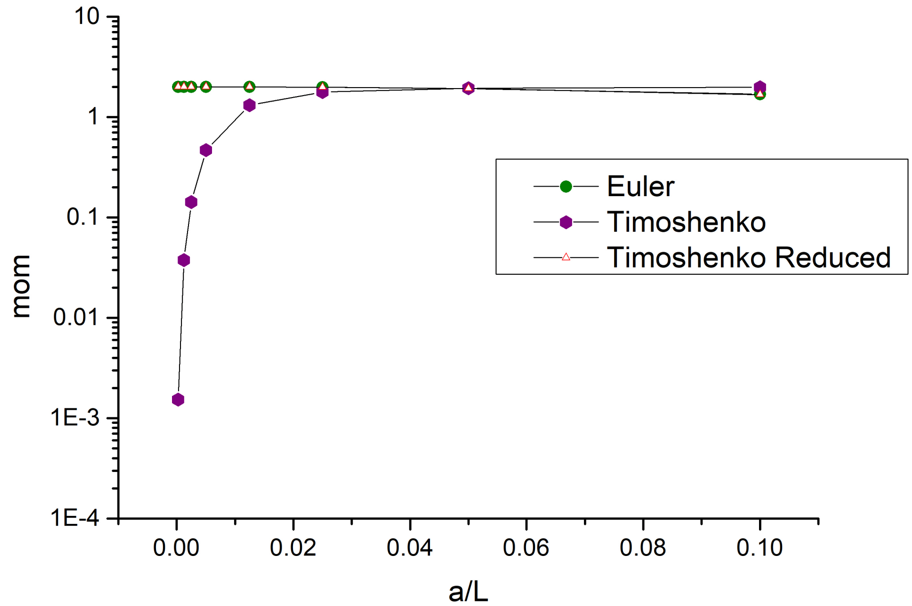

Figure 3: log(momentum)-a/L

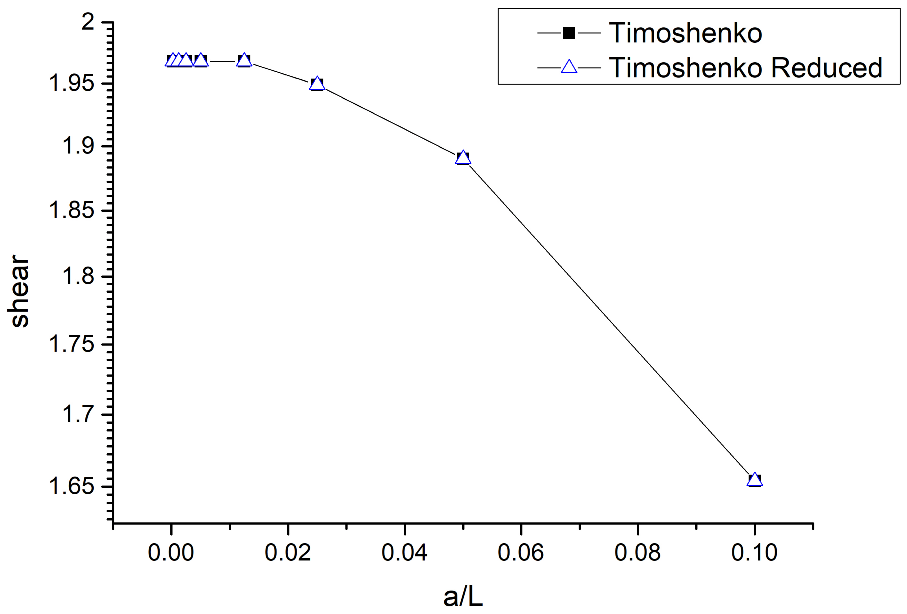

Figure 4: log(shear)-a/L