Universitat Politècnica de Catalunya

Computational Solid Mechanics and Dynamics

Master's Degree in Numerical Methods in Engineering

Beam analysis using Matlab

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Academic Year 2019-2020

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1 Part A

As we are given the shear stifness matrix already, it is only necessary to change the variable $\mathbf{K_{shear}}$ with the following one.

$$
\mathbf{K_{shear}} =
\begin{array}{c}
\\
1 \\
2 \\
3 \\
4
\end{array}
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
\left[\begin{array}{cccc}
1 & L/2 & -1 & L/2 \\
 & L^2/4 & -L/2 & L^2/4 \\
 & & -1 & -L/2 \\
Symm. & & & L^2/4
\end{array}\right]
\end{array}
$$

2 Part B

To elaborate this part, three different Matlab programs have to be used. These are:

- `Beam_EulerBernoulli_v1_2.m`

- `Beam_Timoshenko.m`

- `Beam_EulerBernoulli_v1_2.m`

These programs are designed so as to be used with an input file that collects the main variables of the problem so that the finite element code can return the computed variables. In doing so, the characteristics of this input file are summarized in the following box.

> 1   **input_file**
>
> **Brief description:** Contains the values of the variables that the problem needs to be executed.
> **Input variables:** Young's modulus (`young` = 210000), Poisson's ratio (`poiss` = 0.25), density (`denss` = 0), coordinates of the nodes (`coordinates`), connectivity matrix (`elements`) and boundary conditions (`fixdesp`, `pointload`.
> **Output variables:** [].

Now, it is very easy to generate the connectivity matrix and the coordinates vector for this case as it is a single beam with 64 nodes. The connectivity matrix will be a matrix with as many rows as nodes and only two columns with the ordered nodes of each element. The coordinates vector is simply a row with the coordinate of each node. The point loads in this are modified so that each nodal element receives half the proportional load of the element. The two extreme nodes have their displacement fixed. Furthermore, those parameters that depend on the area of the section side-length a, such as the area or the inertia, are defined inside each subroutine of calculation. Therefore, the three Matlab programs are defined as a function with the only input parameter being a, which is generated in an iterative loop that increases the value of a
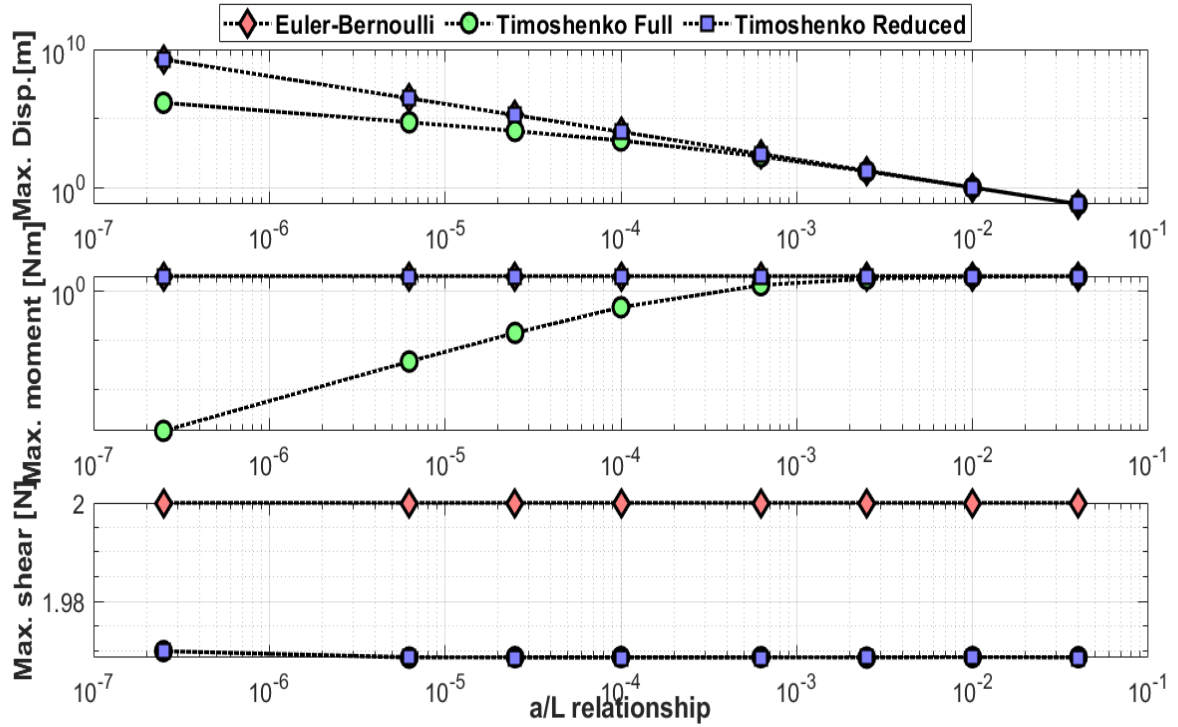
Figure 1: Evolution of the maximum values of moment, stress and displacement for different values of the a/L ratio.

to the values proposed. In turn, each of these functions return the maximum value (absolute) of the stresses, moments and displacements. The results can be seen in Fig. 1.

The discussion of the results lies on mentioning that the Reduced Timoshenko and the Euler-Bernoulli programs give the same results (even if plotted logarithmic) as both graphics are superposed for the displacements and moments. In the case of the shear stress, the values are different but its difference is residual and both graphs maintain the same pattern of values (an straight line) throughout all the values of the ratio. The values of the shear stress could have been expected by knowing that given a uniform load of value unity, for a beam of length $4m$, the shear stresses will be maximum at the extreme points, which will have to compensate for a symmetrically equal load of $2N$ at each support. As for the displacements, the value which is whown on the graphs clearly corresponds to the one at the center of the beam, since the problem is symmetric and the slides show the displacement distribution over the beam.

On the other hand, the Timoshenko full integration program gives results that are far from the other two methods for low values of a, (i.e. low values of the ratio of interest). It is however observable that these results tend to converge to the values of the other methods when high values of a are reached. It means that the order of convergence is different for this method. The reason for that divergence can be found on the calculation of the stiffness matrix for the local element, $\mathbf{K_{element}}$, which is calculated as a product between $\mathbf{K_{shear}}$ and a constant. This constant, which depends on the model, adds some extra stiffness to the Timoshenko Full

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Integration which is noticeable for low values of the a/L ratio.