Assignment 6, Computational Structural Mechanics and Dynamics

Simen Lieng, Universitat Politecnica de Catalunya 23/03/2020

Problem a

MATLAB scripts and functions are available on the CIMNE Virtual Centre, which makes this exercise easier to solve. In the function `Beam.Timoshenko.m` (I renamed it) the solution is used for a regular Timoshenko Beam Element. This means a fully integrated Timoshenko Beam element. To adjust this code for a Reduced Timoshenko Beam, the $\mathbf{K}_s$-matrix need to be changed. This results in:

```
K_s = [ 1 , len/2 , -1 , len/2 ; len/2 , len^2/4 , -len/2 , len^2/4 ;
-1 , -len/2 , 1 , -len/2 ; len/2 , len^2/4 , -len/2 , len^2/4 ];
```

The results is evaluated and compared with the Euler Bernoulli and Timoshenko beam in the next exercise.

Problem b

The MatLab scripts and functions that were provided needed some adjusting of variables to fit this exact problem. Further this resulted in obtaining the max displacement, moment and shear values for the three different beam elements stated in the assignment (Euler Bernoulli, Fully integrated Timoshenko and Reduced integrated Timoshenko). These values were found for a given **a**, which is the length of the sides in the square cross-section. There were eight different **a**-values used, to compare the results of the different elements to the given slenderness of the beam. The results is both given in a table and by plots:

Timoshenko Full integration	a dimension	Max displacement	Max moment	Max shear force
	0.001	14614550	0.0015	1.9688
	0.005	5.7400e+04	0.0377	1.9687
	0.010	1.3581e+04	0.1426	1.9686
	0.020	2.7962e+03	0.4696	1.9680
	0.050	199.9260	1.3111	1.9638
	0.100	16.7064	1.7510	1.9491
	0.200	1.1133	1.8586	1.8900
	0.400	0.00635	**1.6656**	1.6538

Figure 1: Fully integrated Timoshenko Beam

Timoshenko Reduced integration	a dimension	Max displacement	Max moment	Max shear force
	0.001	1.9040e+09	1.9990	1.9688
	0.005	3046360	1.9990	1.9687
	0.010	190386	1.9988	1.9686
	0.020	1.1896e+04	1.9982	1.9680
	0.050	303.9950	1.9940	1.9638
	0.100	18.8781	1.9790	1.9491
	0.200	1.1493	1.9191	1.8900
	0.400	0.0640	1.6792	1.6538

Figure 2: Reduced integrated Timoshenko Beam

Euler-Bernoulli	a dimension	Max displacement	Max moment	Max shear force
	0.001	1.9048e+09	2.0003	2.0000
	0.005	3047540	2.0003	1.9999
	0.010	190457	2.0001	1.9998
	0.020	11900	1.9995	1.9992
	0.050	304	1,9953	1.9950
	0.100	18.8571	1.9803	1.9800
	0.200	1.1429	1.9203	1.9200
	0.400	0.0625	1.6803	1.6800

Figure 3: Euler Bernoulli Beam

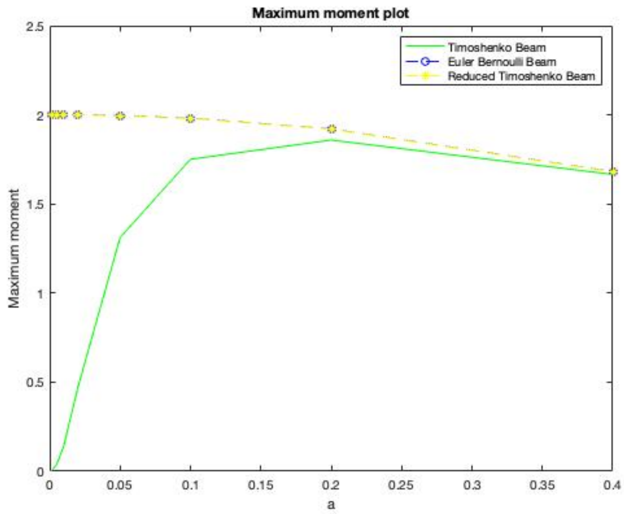

Figure 4: Maximum moment plot

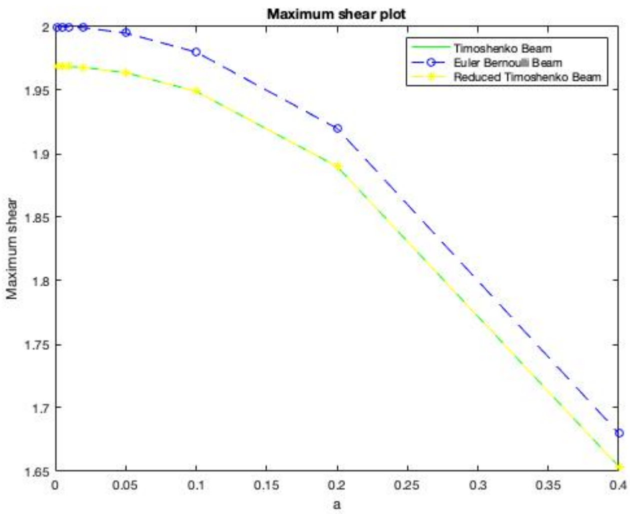

Figure 5: Maximum shear plot

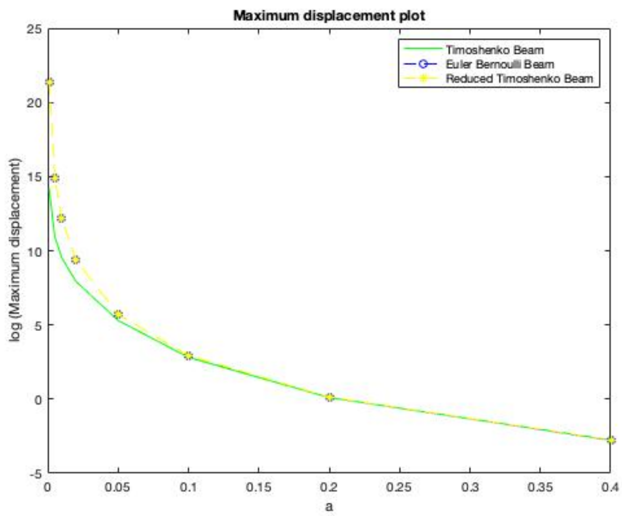

Figure 6: Maximum displacement plot

Discussion

To state the obvious, the beam will produce larger vertical displacement for a lower value of **a** (higher slenderness). This is easily seen in both the tables and the plots. The main difference between the element due to the displacement, is that fully integrated Timoshenko beam provides a stiffer solution. The solution of the other two elements is more or less the same, while the fully integrated Timoshenko beams results in a smaller displacement. This is not the case for all **a**-values, given that the result is more similar to the others for a less slender beam(the last value of displacement for fully integrated Timoshenko is wrong, correct answer is 0.0635).

The fully integrated Timoshenko beam also has problem with the accuracy when accessing the bending moment. Given that the exact value of the bending moment is found by $\frac{ql^2}{8}$. When $l = 4$ and $q = 1$ the exact value of the bending moment is equal to 2. The results improves for a larger cross-section, but is still quite inaccurate compared to the other two solutions. Given that the cross-section would be even bigger, the results could prove to be better than the other two cases.

When it comes to the maximum shear force, all three elements give roughly the same results. In general, the difference between the Timoshenko beam element, both full and reduced, and the Euler-Bernoulli element is that the Euler Bernoulli neglects the $\mathbf{K}_s$ term. For a beam with high slenderness this will not make the biggest difference, but for a larger cross-section, it will probably result in considerable difference. In fact, the Timoshenko Beam would probably be favourable for an analysis if the ratio between $\frac{a}{L}$ would we quite big.