SUMMARY

Assignment Beams

Programming of Timoshenko 2-nodes beam element with reduced integration for the shear stiffness matrix

In order to code the reduced integration, MAT-FEM program is used as a reference and some modifications are carried out to the code.

The shear stiffness matrix of the reduced integration for every element results as follows

$$K_s^{(e)} = \left(\frac{GA^*}{l}\right)^{(e)} \begin{bmatrix} 1 & \frac{l^{(e)}}{2} & -1 & \frac{l^{(e)}}{2} \\ \cdots & \frac{(l^{(e)})^2}{4} & -\frac{l^{(e)}}{2} & \frac{(l^{(e)})^2}{4} \\ & \cdots & 1 & -\frac{l^{(e)}}{2} \\ symm. & & .. & \frac{(l^{(e)})^2}{4} \end{bmatrix}$$

After obtaining the displacements, shear is evaluated at only one gauss point in order to be consistent with the integration of the stiffness matrix.

For both Timoshenko beams the bending moment and the shear force are evaluated at the mid-point of the element, which is optimal for the evaluation of stresses.

Comparative study of Timoshenko full/reduced and Euler-Bernoulli beams solving a simple problem

Description of the problem

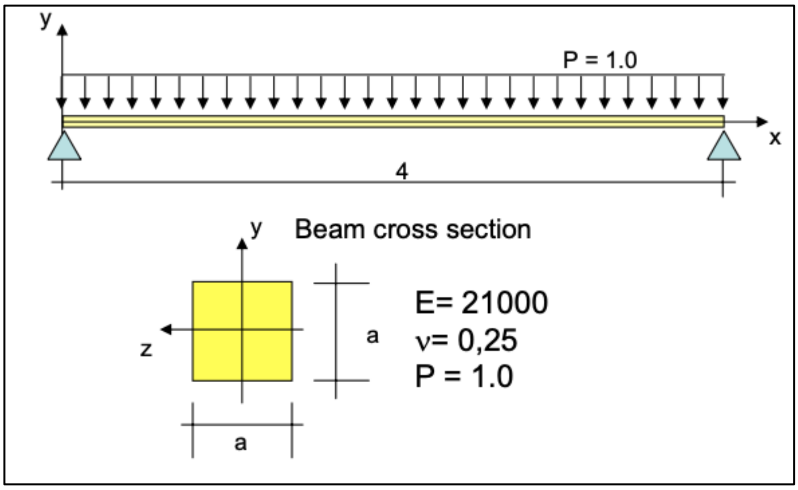

Figure 1: Description of the 1D beam problem

A 64-element mesh is applied to discretize the structure. The parameter of the cross-section $\boldsymbol{a}$ takes different values in order to asses the evolution of the results with respect to the ratio $\boldsymbol{a/l}$, being $\boldsymbol{l}$ a constant.

Here is the table with all the values that the parameter takes

a =	0.001	0.005	0.010	0.020	0.050	0.100	0.200	0.400

Then, the results are computed and presented below

Table 2: Results of the maximum values with FTI model

Full Timoshenko beam			
Maximum values			-
Displacement	Bending moment	Shear	*a/l*
1,199E+06	0,001	1,969	0,00025
4,723E+04	0,031	1,969	0,00125
1,128E+04	0,118	1,969	0,0025
2,395E+03	0,402	1,969	0,005
186,40	1,223	1,969	0,0125
16,45	1,725	1,969	0,025
1,15	1,923	1,969	0,05
0,08	1,979	1,969	0,1

Table 1: Results of the maximum values with RTI model

Reduced Timoshenko beam			
Maximum values			-
Displacement	Bending moment	Shear	*a/l*
1,904E+09	1,999	1,969	0,00025
3,046E+06	1,999	1,969	0,00125
1,904E+05	1,999	1,969	0,0025
1,190E+04	1,999	1,969	0,005
304,74	1,999	1,969	0,0125
19,06	1,999	1,969	0,025
1,20	1,999	1,969	0,05
0,08	1,999	1,969	0,1

Table 3: Results of the maximum values with EUB model

Euler - Bernoulli beam			
Maximum values			-
Displacement	Bending moment	Shear	*a/l*
1,905E+09	2,000	Not applicable	0,00025
3,048E+06	2,000	Not applicable	0,00125
1,905E+05	2,000	Not applicable	0,0025
1,190E+04	2,000	Not applicable	0,005
304,76	2,000	Not applicable	0,0125
19,05	2,000	Not applicable	0,025
1,19	2,000	Not applicable	0,05
0,07	2,000	Not applicable	0,1

To see more clearly the differences between the three types of beam element some graphs are plotted.

Maximum displacements

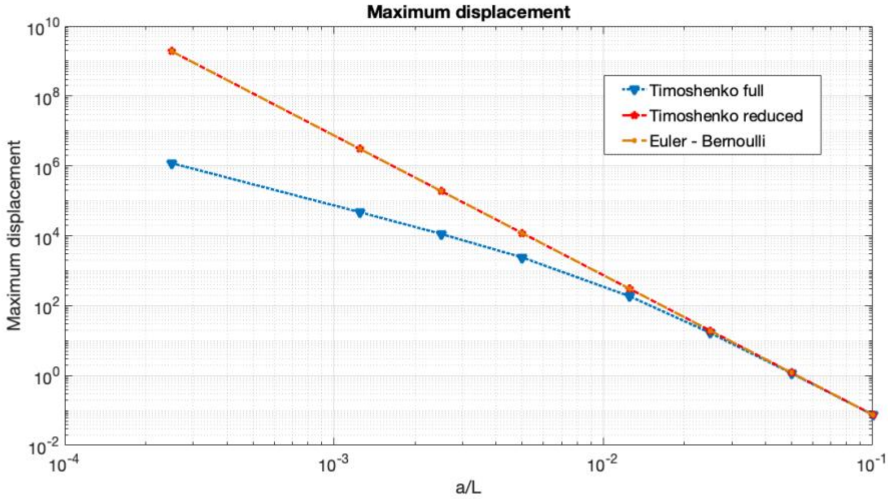

Figure 2: Maximum displacements vs slenderness ratio

Maximum bending moment

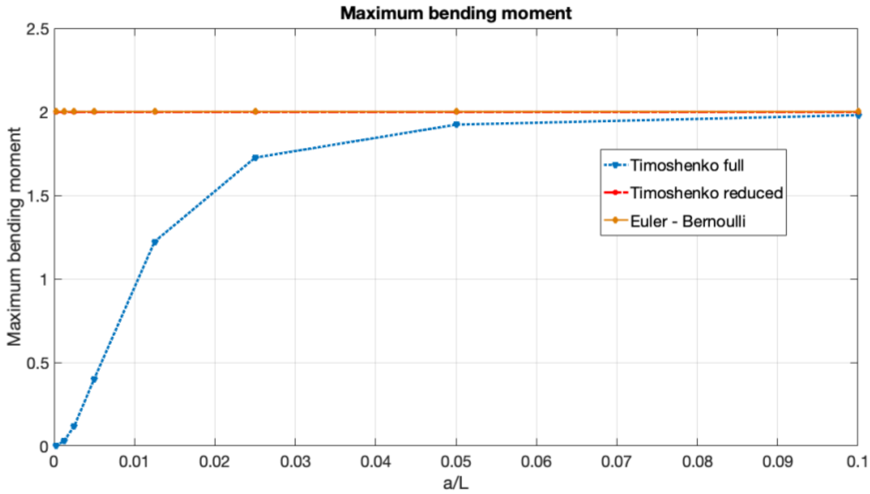

Figure 3: Maximum bending moment vs slenderness ratio

Maximum shear

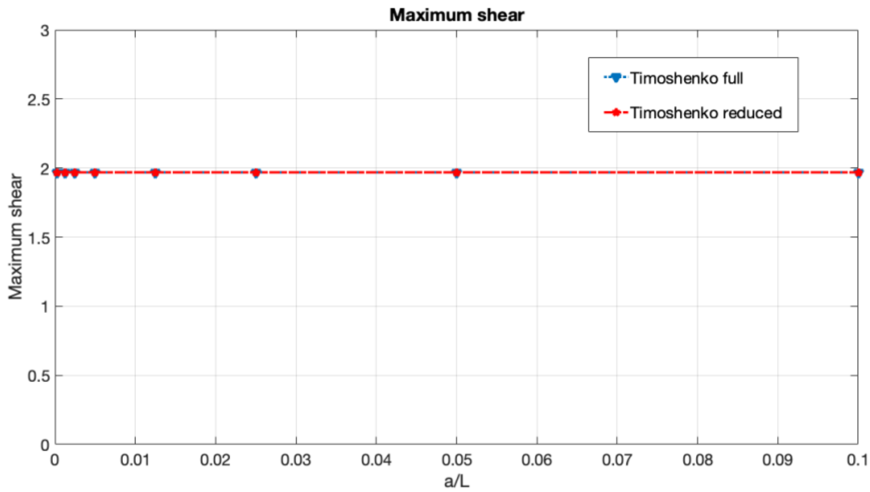

Figure 4: Maximum shear force vs slenderness ratio

Conclusions

Regarding the displacements, the Euler-Bernoulli model (EUB) is supposed to be more accurate for slender beams, in which the shear effects are negligible and its cubic approximation for the vertical displacement gives accurate results. Therefore, as can be seen in the figure 2 the Full-Timoshenko model (FTI) experiences a poor behaviour for slender beams, and its results are too stiff. However, the results of FTI model match with the ones of the other models for thicker beams with lower slenderness ratios ($L/a < 40$). As for the Reduced-Timoshenko model (RTI), its results regarding the maximum displacements coincides with the ones of the EUB. It is worth mentioning that all the maximum displacements are on the mid-point of the beam, as the analytical results predict.

As regards the maximum bending moment results (figure 3), the EUB and RTI models give highly accurate results respect to the analytical one (error <0.049% for RTI and <0.016% for EUB). However, the FTI model shows again problems with the slender beams and does not achieve acceptable results (error < 5%) up to $L/a < 20$.

Finally, although Euler-Bernoulli model does not take into account shear effects, the results of the maximum shear force for the other two models has been assessed (figure 4) and they have turned out to be pretty accurate (error < 1.56% in both cases). The conclusion is that the full integration does not lose accuracy regarding the shear force, since for higher slenderness ratio the shear stiffness matrix dominates the bending one and, furthermore, the polynomial approximation is sufficient to capture the linear evolution of this parameter.