Universitat Politècnica de Catalunya

Master of Science in Computational Mechanics

Computational Structural Mechanics and Dynamics

Assignment 6
Bending of Beams

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1 Bending of simple-supported Beam

A beam, supported on both ends, is under a uniform distributed force of 1 N/m along all of it's length. Figure 1.1 depicts the beam, it's cross-section and material properties.

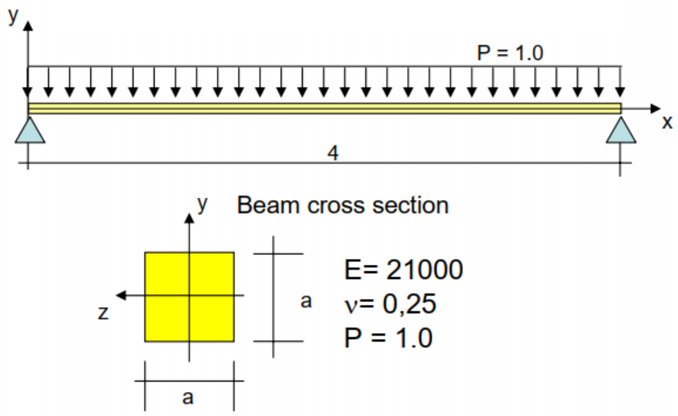

Figure 1.1: Problem

Several different beams with different squared cross-sections of side a were evaluated using the Euler-Bernoulli, the Timoshenko and the reduced Timoshenko theories. Table 1.1 presents the test cases. The idea is to check the behaviour of each method with a growing thickness of the beam.

Table 1.1: Test cases

a							
0,001	0,005	0,010	0,020	0,050	0,100	0,200	0,400

1.1 Reduced Timoshenko

The provided code for solving the state of the beam with 64 nodes was still lacking the reduced Timoshenko method. To add the missing method, a modification of the original Timoshenko theory was made, by reducing the influence of the transverse shear stiffness.

Two Gauss points are needed for the exact integration of the shear stiffness matrix $\boldsymbol{K}_s$. By using just one Gauss point, the influence of the $\boldsymbol{K}_s$ matrix is reduced, allowing the method to also be applied on cases where shear is not relevant.

The reduced shear stiffness matrix has a very straight-forward implementation and is presented on Equation 1.1, where $\hat{D}_s$ is the product of the shear modulus and the reduced cross-sectional area and $l^{(e)}$ is the size of the element.

$$\boldsymbol{K}_s = \left(\frac{\hat{D}_s}{l}\right)^{(e)} \begin{bmatrix} 1 & \frac{l^{(e)}}{2} & -1 & \frac{l^{(e)}}{2} \\ & \frac{(l^{(e)})^2}{4} & -\frac{l^{(e)}}{2} & \frac{(l^{(e)})^2}{4} \\ & & 1 & -\frac{l^{(e)}}{2} \\ Sym. & & & \frac{(l^{(e)})^2}{4} \end{bmatrix} \tag{1.1}$$

1.2 Evaluation

The maximum displacements for all test cases and methods are plotted on Figure 1.2. As expected, the Euler-Bernoulli and the Timoshenko theory behave similarly for the most of the test cases, but diverge for slender beams (low a/L). The Timoshenko method overrepresents the shear stiffness, making the beam "stiffer" than the Euler-Bernoulli prediction. Hence the lower maximum displacement values for small cross-sections. The reduced Timoshenko is able to "fix" the problem, reaching almost the same values as the Euler-Bernoulli (within a margin of 1%) just by performing a one-point instead of a two-point Gauss integration.

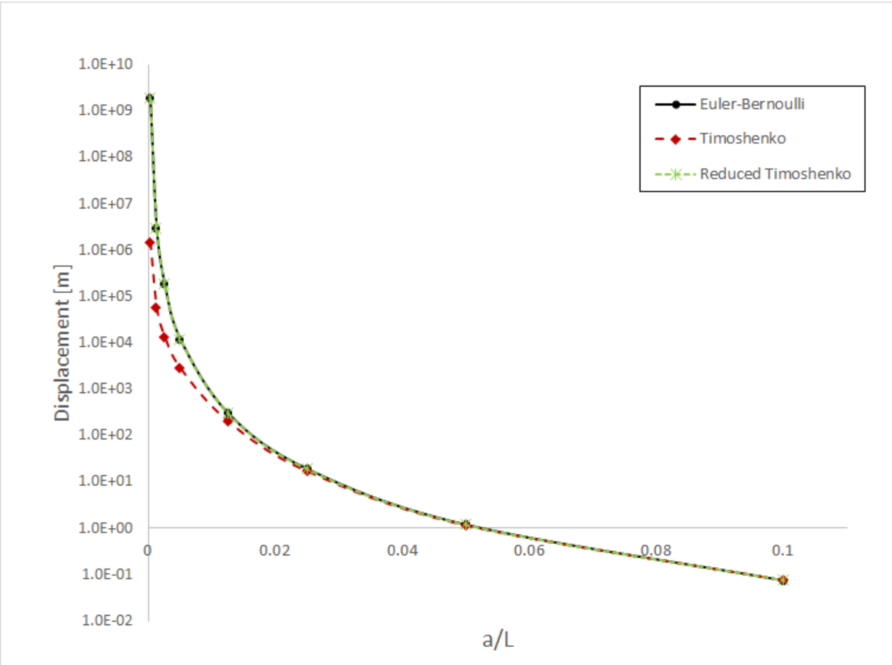

Figure 1.2: Maximum Displacement as a function of slenderness

Similarly, the Timoshenko theory fails to describe the momentum on very slender beams, while the reduced theory performs well as seen on Figure 1.3.

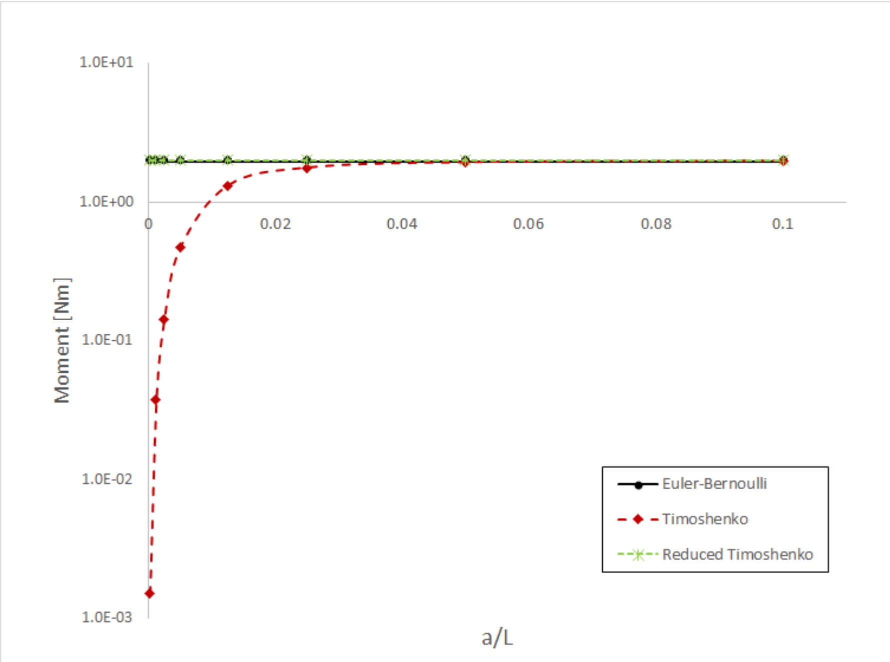

Figure 1.3: Maximum Momentum as a function of slenderness

As for the shear, all methods were able to capture the same maximum value. Evidently 2 N on the edges due to the reaction forces on the supports.

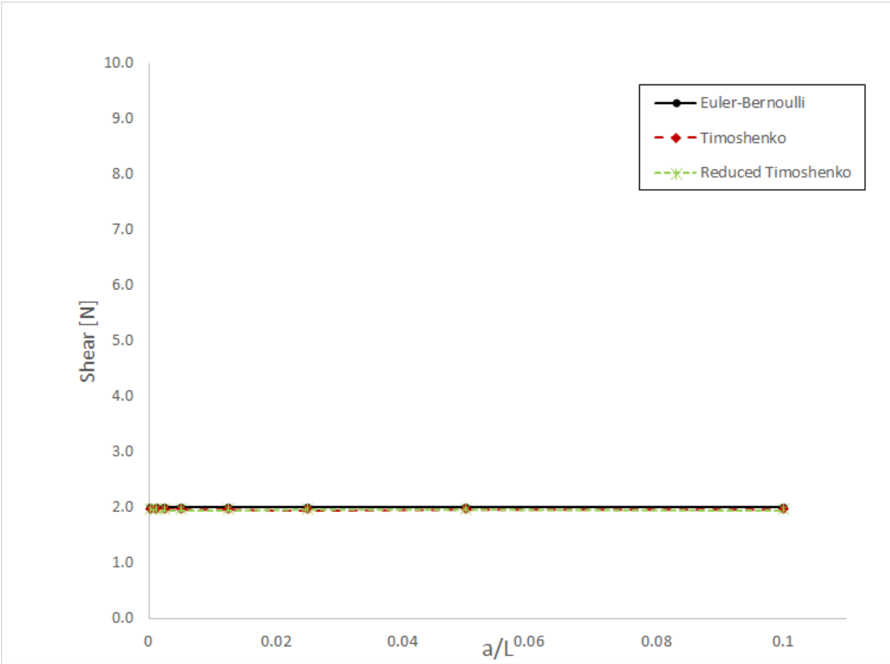

Figure 1.4: Maximum Shear as a function of slenderness