Master of Science in Computational Mechanics

Computational Structural Mechanics and Dynamics

Assignment 6: Beam elements

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Implementation of Timoshenko beam element with reduced integration

Several numerical studies have pointed out that Timoshenko beam elements are unable to reproduce the conventional solution as the beam slenderness increases, producing solutions that are progressively stiffer than the exact one. This phenomenon, known as *shear locking*, can be mitigated by reducing the influence of transverse shear stiffness by under-integrating the terms $\mathbf{K_s}^{(e)}$ with only one Gauss point. Thus, for a homogeneous material, $\mathbf{K_s}^{(e)}$ becomes:

$$\mathbf{K_s}^{(e)} = \left(\frac{GA^*}{l}\right)^{(e)} \begin{bmatrix} 1 & \frac{l^{(e)}}{2} & -1 & \frac{l^{(e)}}{2} \\ \frac{l^{(e)}}{2} & \frac{(l^{(e)})^2}{4} & -\frac{l^{(e)}}{2} & \frac{(l^{(e)})^2}{4} \\ -1 & -\frac{l^{(e)}}{2} & 1 & -\frac{l^{(e)}}{2} \\ \frac{l^{(e)}}{2} & \frac{(l^{(e)})^2}{4} & -\frac{l^{(e)}}{2} & \frac{(l^{(e)})^2}{4} \end{bmatrix} \tag{1}$$

To implement this approach, MATFEM was used as based computing program. Thus, for the reduced Timoshenko beam element a new function was created replacing the definition of $\mathbf{K_s}^{(e)}$ by the reduced one in equation (1). Please note that the terms in $\mathbf{K_b}^{(e)}$ are still integrated exactly.

Moreover, for stress evaluation, the integration of the matrix $\mathbf{B}$ is performed imposing the Gaussian points to be equal to 0.

Solution of sample problem

For the comparison of the Euler, Timoshenko and reduced Timoshenko beam elements a sample problem will be solved. The boundary conditions and material properties are depicted in Figure (1)

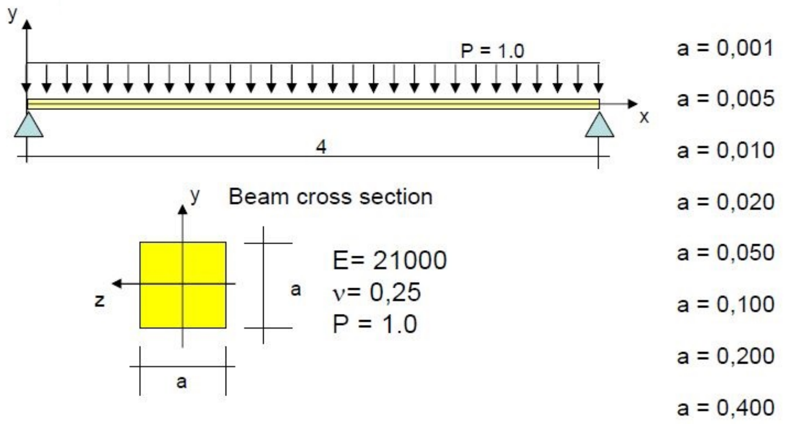

Fig. 1 – Boundary conditions and material properties

As seen in Figure (1), the value a is varied to study the influence of slenderness on the results obtained with the different beam elements.

Table (1) shows the results of numerical computations using the three different elements with different slenderness ratios. The results were computed using a 64-element mesh.

Table 1 – Comparison of maximum vertical displacement, bending moment and shear forces for beams with different slenderness ratio using Euler-Bernoulli and Timoshenko elements

	Euler-Bernoulli Element			**Timoshenko Element**			**Timoshenko Element (reduced integration)**		
Slenderness Ratio λ	**max** w [m]	**max** M [N·m]	**max** Q [N]	**max** w [m]	**max** M [N·m]	**max** Q [N]	**max** w [m]	**max** M [N·m]	**max** Q [N]
4000	1.905E+09	1.99991	2.000	1.461E+06	0.0015	1.9688	1.904E+09	1.9990	1.9688
800	3.048E+06	1.99990	2.000	5.740E+04	0.0377	1.9687	3.046E+06	1.9990	1.9688
400	1.905E+05	1.99990	2.000	1.358E+04	0.1426	1.9688	1.904E+05	1.9990	1.9687
200	1.191E+04	1.99990	2.000	2.797E+03	0.4698	1.9688	1.190E+04	1.9990	1.9687
80	3.048E+02	1.99990	2.000	2.004E+02	1.3144	1.9688	3.048E+02	1.9990	1.9687
40	1.905E+01	1.99990	2.000	1.688E+01	1.7687	1.9687	1.907E+01	1.9990	1.9688
20	1.191E+00	1.99990	2.000	1.160E+00	1.9360	1.9688	1.197E+00	1.9990	1.9688
10	7.440E-02	1.99990	2.000	7.560E-02	1.9829	1.9688	7.620E-02	1.9990	1.9687

Figure (2) shows a comparison of how the vertical displacements changes with respect to the slenderness ratio and how it is predicted by the different theories. Since the effect of transverse shear stresses becomes negligible as the slenderness ratio decreases, the Timoshenko solution coincide with conventional Euler-Bernoulli theory. It can be seen that for slender beams, i.e. large values of λ, Timoshenko solution predicts an over-stiff beam and the vertical displacement are smaller in comparison with reduced Timoshenko and Euler-Bernoulli approaches.

Similarly, the maximum bending moment (see Figure(3)) converges to a equal value as slenderness decreases, once again product of the tendency of the Timoshenko solution to predict over-stiff beam behavior.

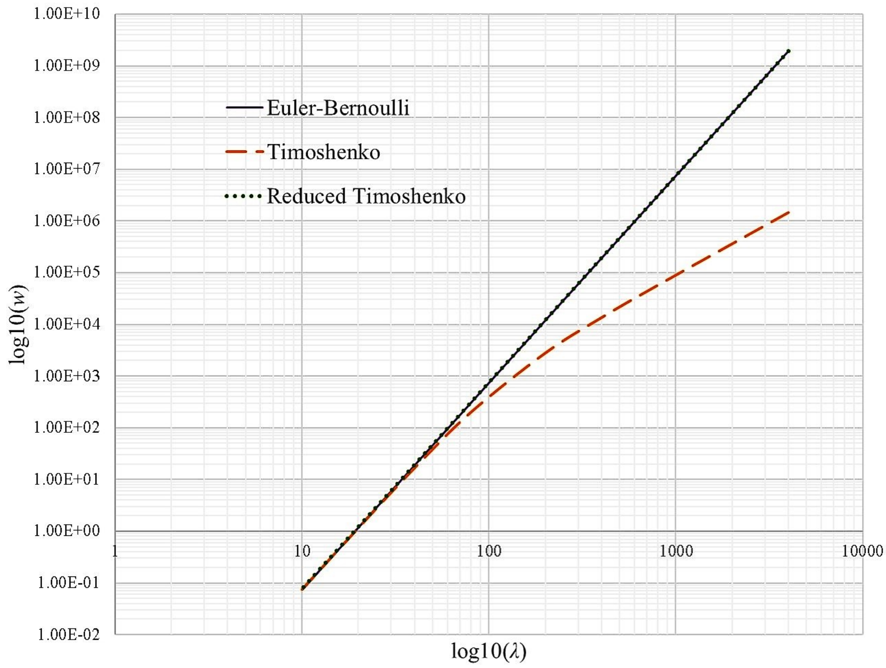

Fig. 2 – Maximum vertical displacement variation with respect to slenderness ratio. Solution using Euler-Bernoulli, Timoshenko and reduced-Timoshenko elements

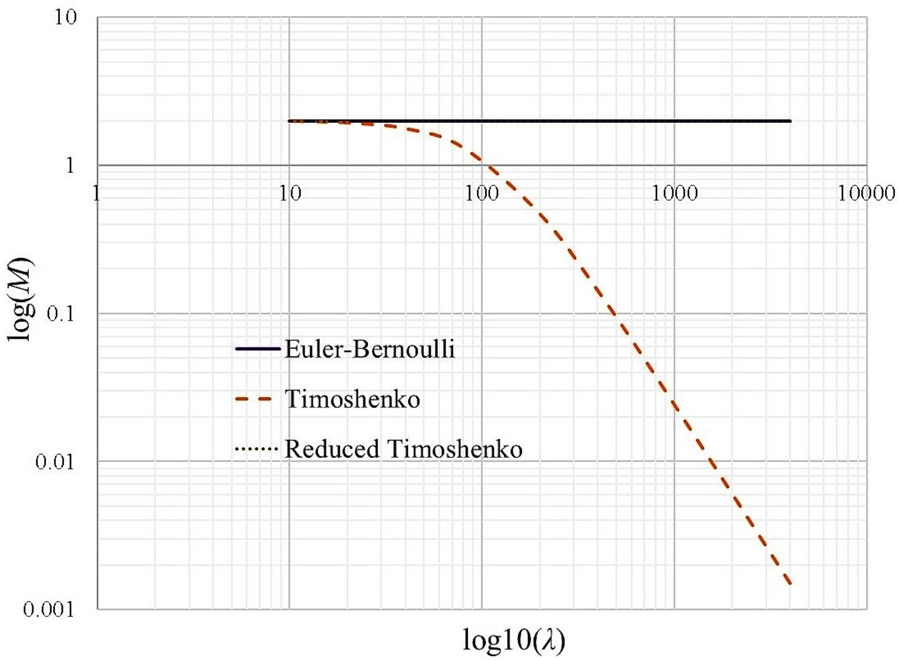

Fig. 3 – Maximum bending variation with respect to slenderness ratio. Solution using Euler-Bernoulli, Timoshenko and reduced-Timoshenko elements