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Assignment 6 Beams (MATLAB + GiD)

a) Program In MatLab the Timoshenko 2 Nodes Beam element with reduce integration for the shear stiffness matrix:

$$\begin{split} \mathbf{K}_{b}^{(e)} &= \left(\frac{\mathbf{E}_{l}^{\prime}}{\mathbf{I}}\right)^{(e)} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \text{ (The point interpolation is exact for } \mathbf{K}_{b}^{(e)}\text{)} \\ \mathbf{K}_{s}^{(e)} &= \left(\frac{\mathbf{G}\mathbf{A}^{*}}{l}\right)^{(e)} \begin{bmatrix} 1 & \frac{l^{(e)}}{2} & -1 & \frac{l^{(e)}}{2} \\ \cdots & \frac{l^{(e)}}{2} & -\frac{l^{(e)}}{2} & \frac{l^{(e)}}{2} \\ \cdots & 1 & -\frac{l^{(e)}}{2} \\ \cdots & 1 & -\frac{l^{(e)}}{2} \\ \text{Simetr.} & \cdots & \frac{l^{(e)}}{4} \end{bmatrix} \text{ (Reduced integration)} \end{split}$$

For implementation of the Timoshenko 2 Nodes Beam element with reduce integration for the shear stiffness matrix following piece of code was modified. The results were obtained by eliminating gauss point. Then simulation was performed for various area of beam and compared with other types of element.

```
% One gauss point for stress evaluation
gaus0 = 0.0; % One Gauss point for stresses evaluation
bmat_b = [ 0, -1/len, 0, 1/len];
bmat_s1 = [-1/len,-(1-gaus0)/2, 1/len,-(1+gaus0)/2];
Str1_g0 = D_matb*(bmat_b *transpose(u_elem));
Str2_g0 = D_mats*(bmat_s1*transpose(u_elem));
Strnod(lnods(1),1) = Strnod(lnods(1),1) + Str1_g0;
Strnod(lnods(2),1) = Strnod(lnods(2),1) + Str1_g0;
Strnod(lnods(1),2) = Strnod(lnods(1),2) + Str2_g0;
Strnod(lnods(2),2) = Strnod(lnods(2),2) + Str2_g0;
Strnod(lnods(1),3) = Strnod(lnods(1),3) + 1;
Strnod(lnods(2),3) = Strnod(lnods(2),3) + 1;
end
```

b) Solve the following problem with a 64 element mesh with the:

- 2 nodes Euler-Bernoulli element
- 2 nodes Timoshenko Full Integrate element
- 2 nodes Timoshenko Reduce Integration element.

Compare maximum displacements, moments and shear for the 3 elements against the a/L relationship.



Results:

The problem was solved with the given data for all the 3 elements and following results were obtained:

1 Maximum Displacement:

As it can be seen in the figure 1 Timoshenko Full Integrate element fails catastrophically for thin beam while other two elements gives better results but not a. But as the problem comes under thick beam all three elements shows similar results.



Figure 1: Comparison of Displacement

2 Maximum Bending moment

Figure 2 shows the plot for bending moment observed in beam. The results are similar to the displacement for Timoshenko Full Integrate element. but for other two elements we get consistent results.



Figure 2: Comparison of Bending moment

3 Maximum Shear

The results are promising for all three elements with consistent results of which Euler-Bernoulli element performs the best of all 3 with accurate solution.

