Màster en Mètodes Numèrics

Computational Structural Mechanics and Dynamics

Assignment 6: Beams

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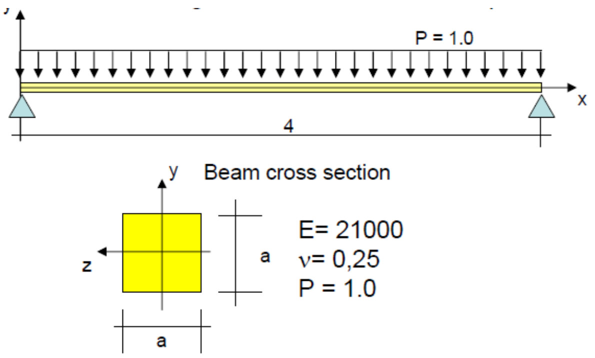
a) Program in Matlab the Timoshenko 2 Nodes Beam element with reduce integration for the shear stiffness matrix.

So as to implement the Timoshenko 2 nodes beam element with reduce integration, the elemental shear stiffness matrix is coded as shown:

```
K_s = [ 1 , len/2 , -1 , len/2 ;
    len/2 , len^2/4 , -len/2 , len^2/4 ;
    -1 , -len/2 , 1 , -len/2 ;
    len/2 , len^2/4 , -len/2 , len^2/4 ];
```

b) Solve the following problem with a 64 element mesh with the:
   a. 2 nodes Euler-Bernoulli element.
   b. 2 nodes Timoshenko Full Integrate element.
   c. 2 nodes Timoshenko Reduce Integration element.

Compare maximum displacements, moments and shear for the 3 elements against the a/L relationship.



Eight different configurations for a/L are considered for every beam element.

a dimension [m]	Area [m^2]	Inertia [m^4]
0.001	1.0000e-06	8.3333e-14
0.005	2.5000e-05	5.2083e-11
0.01	1.0000e-04	8.3333e-10
0.02	4.0000e-04	1.3333e-08
0.05	0.0025	5.2083e-07
0.1	0.01	8.3333e-06
0.2	0.04	1.3333e-04
0.4	0.16	0.0021

Here below there are plotted the figures representing maximum deflection, maximum bending moment and maximum shear forces of the beam for different configurations of slenderness ratio a/L for 3 different 2-noded beam elements with a 64 element mesh.

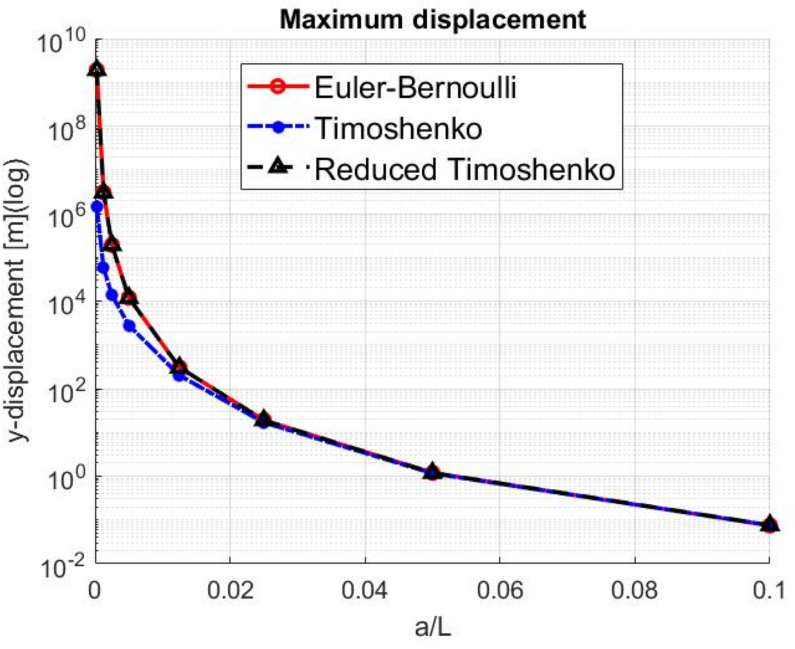


Figure 1. Maximum y-displacement.

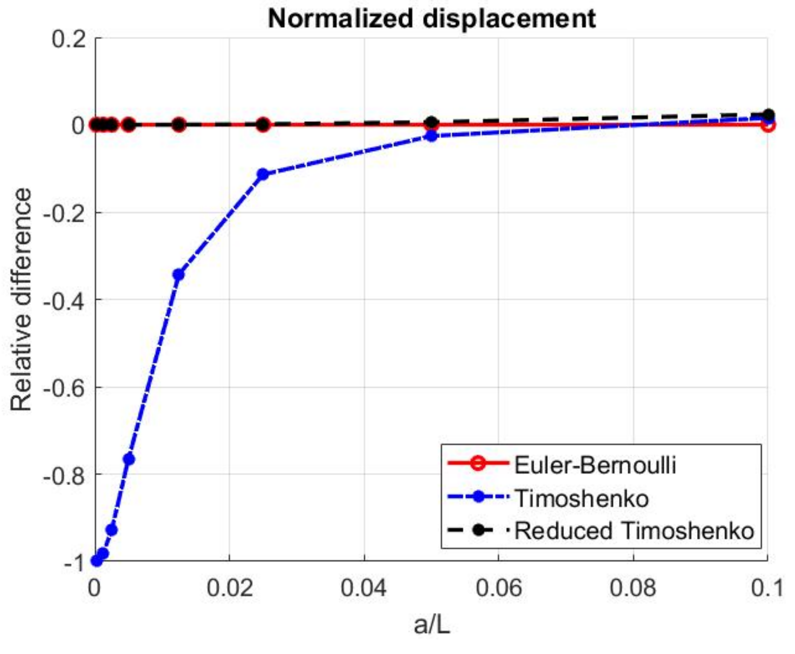


Figure 2. Normalized displacement.

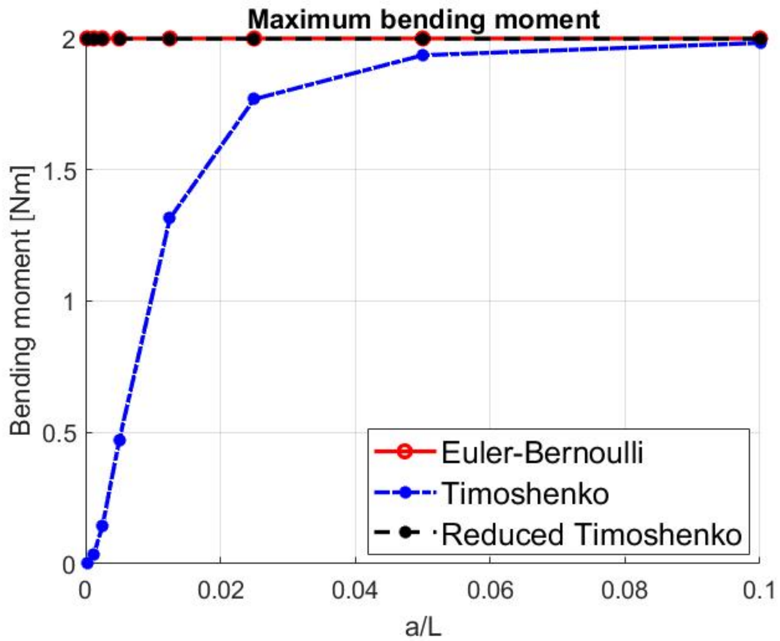


Figure 3. Maximum bending moment.

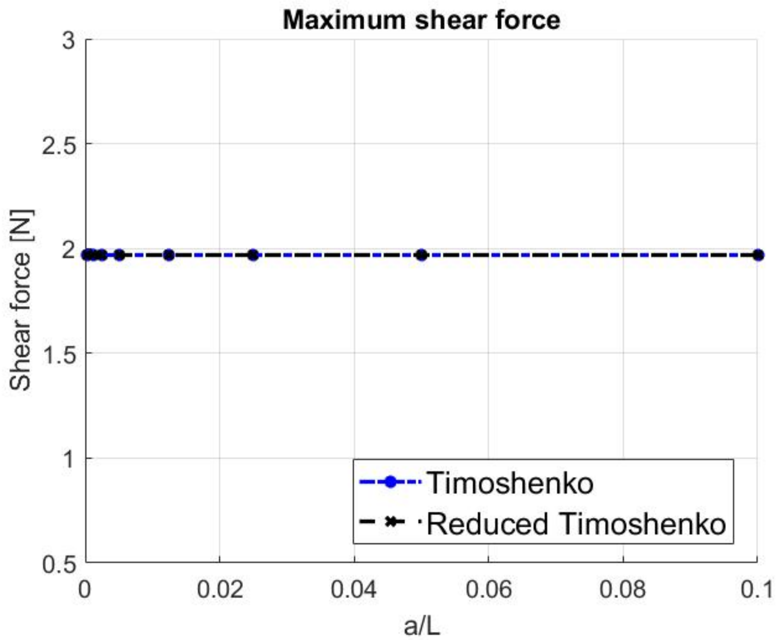


Figure 4. Maximum shear force.

The values holding these plots are resumed next:

Euler-Bernoulli model					
a	**L**	**a/L**	**Max. displacement**	**Max. Bending moment**	**Max. Shear force**
0.001	4	2.5000e-07	1.9048e+09	1.9999	2
0.005	4	6.2500e-06	3.0476e+06	1.9999	2
0.01	4	2.5000e-05	1.9048e+05	1.9999	2
0.02	4	1.0000e-04	1.1905e+04	1.9999	2
0.05	4	6.2500e-04	3.0476e+02	1.9999	2
0.1	4	0.0025	1.9048e+01	1.9999	2
0.2	4	0.01	1.1905	1.9999	2
0.4	4	0.04	7.4405e-02	1.9999	2

Full Timoshenko model					
a	**L**	**a/L**	**Max. displacement**	**Max. Bending moment**	**Max. Shear force**
0.001	4	2.5000e-07	1.4615e+06	1.5341e-03	1.9688
0.005	4	6.2500e-06	5.7401e+04	3.7658e-02	1.9688
0.01	4	2.5000e-05	1.3583e+04	1.4258e-01	1.9688
0.02	4	1.0000e-04	2.7973e+03	4.6978e-01	1.9688
0.05	4	6.2500e-04	2.0043e+02	1.3144	1.9688
0.1	4	0.0025	1.6875e+01	1.7687	1.9688
0.2	4	0.01	1.1596	1.9360	1.9688
0.4	4	0.04	7.5561e-02	1.9829	1.9688

Reduced Timoshenko model					
a	**L**	**a/L**	**Max. displacement**	**Max. Bending moment**	**Max. Shear force**
0.001	4	2.5000e-07	1.9040e+09	1.999	1.9688
0.005	4	6.2500e-06	3.0464e+06	1.999	1.9688
0.01	4	2.5000e-05	1.9041e+05	1.999	1.9688
0.02	4	1.0000e-04	1.1901e+04	1.999	1.9688
0.05	4	6.2500e-04	3.0476e+02	1.999	1.9688
0.1	4	0.0025	1.9069e+01	1.999	1.9688
0.2	4	0.01	1.1971	1.999	1.9688
0.4	4	0.04	7.6161e-02	1.999	1.9688

Results and conclusions

Firstly, it is clearly seen that as low values of a/L are used, Euler-Bernoulli and Reduced Timoshenko beam models show same results (due to hypothesis used in Timoshenko).

On the other hand, it can be appreciated that for the full integration Timoshenko model smaller values of vertical displacement occur. This mainly happens due to shear locking which makes the beam element stiffer. Due to this, bending moment gets underestimated.

As well, there is a rise on the shear effects as the ratio a/L is increased. In that case, Timoshenko models brings more accuracy to the calculation.

In *Figure2,* it might also be seen that both Timoshenko models converge to the same solution although Euler-Bernoulli reaches higher deflection values despite of not considering shear effects for that model.

Summing up, Euler-Bernoulli model is very accurate for the bending moment problem in the actual case-scenario (slender beam). Timoshenko's beam model performs better in a shear case-problem, however, full integrate Timoshenko might not be useful since it brings distortion in bending moment calculations (increment of stiffness matrix and decrease of strain due to shear locking effect).