UPC - BARCELONA TECH
MSc Computational Mechanics
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Computational Solid Mechanics & Dynamics

Assignment 6 Beams

Due 19/03/2018

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a) Program in MatLab the Timoshenko 2 Nodes Beam element with reduce integration for the shear stiffness matrix

In this assignment related with Euler-Bernoulli and Timoshenko beam theories, we are asked to solve a simple supported beam considering a uniformly distributed load acting on it with a mesh of 64 elements. We are given the fully implemented codes for the Euler-Bernoulli and Timoshenko cases, and it just remains to include the reduced integration Timoshenko algorithm, which is usually implemented as a way to overcome the so-called *shear locking* effect. This reduced integration technique computes the matrix $\boldsymbol{K}_s^{(e)}$ using a quadrature of one order less than is needed for exact integration. The matrix is

$$\boldsymbol{K}_s^{(e)} = \left(\frac{GA^*}{L}\right)^{(e)} \begin{bmatrix} 1 & \frac{l^{(e)}}{2} & -1 & \frac{l^{(e)}}{2} \\ \ddots & \frac{(l^{(e)})^2}{4} & -\frac{l^{(e)}}{2} & \frac{(l^{(e)})^2}{4} \\ & \ddots & 1 & -\frac{l^{(e)}}{2} \\ Symm & & \ddots & \frac{(l^{(e)})^2}{4} \end{bmatrix}$$

Then, we can just change the code provided for the Timoshenko case,

```
%     K_shear = [   1    ,    len/2 ,    -1    ,    len/2 ;
%                 len/2 , len^2/3 , -len/2 , len^2/6 ;
%                  -1    ,  -len/2 ,     1    ,  -len/2 ;
%                 len/2 , len^2/6 , -len/2 ,  len^2/3 ];

    K_shear = [   1    ,    len/2 ,    -1    ,    len/2 ;
                len/2 , len^2/4 , -len/2 , len^2/4 ;
                 -1    ,  -len/2 ,     1    ,  -len/2 ;
                len/2 , len^2/4 , -len/2 ,  len^2/4 ];
```

Figure 0.1: Definition of the $\boldsymbol{K}_s^{(e)}$ matrix for the case of reduced integration in the Timoshenko code.

As we can see, we have just included the definition of the matrix for the reduced integration case and comment the matrix of the full integration case, so that a user can choose if he/she wants to use one type of analysis or the other. Also, for stress evaluation we have set the parameters *gaus1* and *gaus2* equal to zero in order to have a consistent formulation with the reduced integration technique.

b) Solve the following problem with a 64 element mesh with the 2 nodes Euler-Bernoulli element, 2 nodes Timoshenko Full Integrate element and 2 nodes Timoshenko Reduce Integration element. Compare maximum displacements, moments and shear for the 3 elements against the a/L relationship.

First of all, let us recall the applications discussed in class for the Euler-Bernoulli and Timoshenko beam theories. The Euler-Bernoulli is the classical approach to study the bending of slender plane beams. Its basic assumption is that a transverse cross section remains plane and orthogonal to the beam axis after the deformation. It required the use of C^1 elements, and when using a cubic interpolation it is able to exactly reproduce the displacement field on the nodes. On the other hand, Timoshenko beam theory accounts for the effect of transverse shear deformation since it considers that cross sections of the beam do not neccesarily remain orthogonal to the axis. Therefore, this theory applies for "thick" beams $\left(\lambda = \frac{L}{h} < 10\right)$ where the transverse shear deformation plays a role and also for slender beams ($\lambda > 100$) where shear deformation is irrelevant.

In the next three figures, a comparison among the maximum values of vertical displacements, bending moments and shear forces is presented. In order to make these plots to be more explanatory and easy comparable, the logarithm of these variables is plotted.

Figure 0.2 shows a comparison of the results obtained for the maximum displacements of the beam under consideration. As it can be seen in the plot, the solution for the Timoshenko full integrate element differs from the one obtained for both Euler-Bernoulli and Reduced Timoshenko approaches when the ratio a/L is relatively low. These values of the ratio correspond to slender beams, i.e. those beams which have a slenderness ratio of $\lambda > 100$.

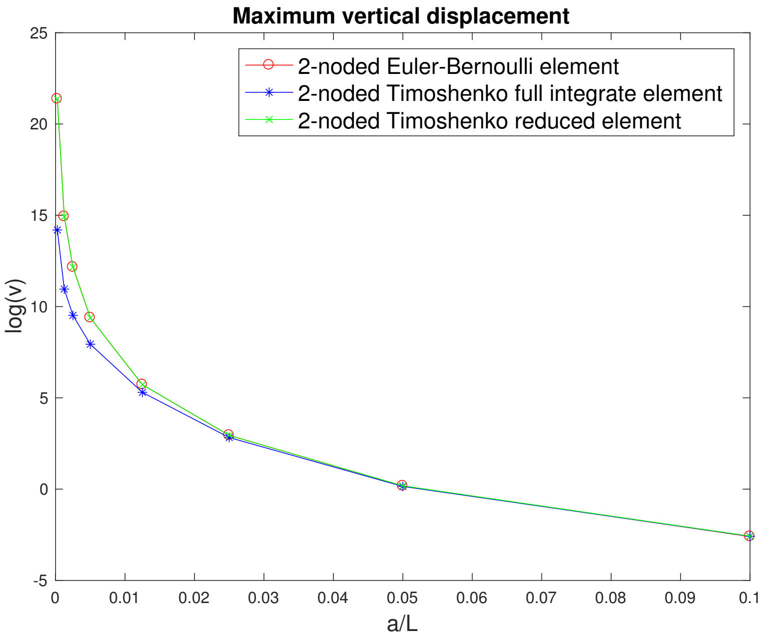

Figure 0.2: Representation of the natural logarithm of the maximum displacement for the different cases considered versus the ratio a/L where a is the width of the square section of the beam and L is the total length of the beam.

The effect of transverse shear deformation is negligible for a slender beam (i.e. for a sufficiently large value of λ). Hence, Timoshenko solution should coincide for this case with that of conventional Euler-Bernoulli theory as the assumptions for the computations will reduce to the same ones. What happens in reality is that as the parameter λ increases, the numerial solution is progressively stiffer than the exact one (the matrix $\boldsymbol{K}_s$ becomes larger). This basically means that the Timoshenko full integrated element is in fact unable to reproduce the behaviour of slender beams. This effect is the so-called *shear locking* and it is the explanation for the results obtained in the graph. As we consider less stiffer beams, i.e. higher values of a/L, the Timoshenko full element solution approaches more and more to the conventional "exact" Euler-Bernoulli solution.

One of the most popular techniques to overcome the problem of *shear locking* is the reduced integration strategy, which basically consist of under-integrating the terms in the $\boldsymbol{K}_s^{(e)}$ matrix using a quadrature of one order less than needed for exact results. This approach enables to reduce the influence of the transverse shear stiffness and the 2-noded Timoshenko reduced element yields to be valid for both thick and slender beams. This is why the solution basically matches the one from Euler-Bernoulli beam theory.

Figure 0.3 shows a comparison for the maximum values of the bending moment in the beam obtained with the three approaches.

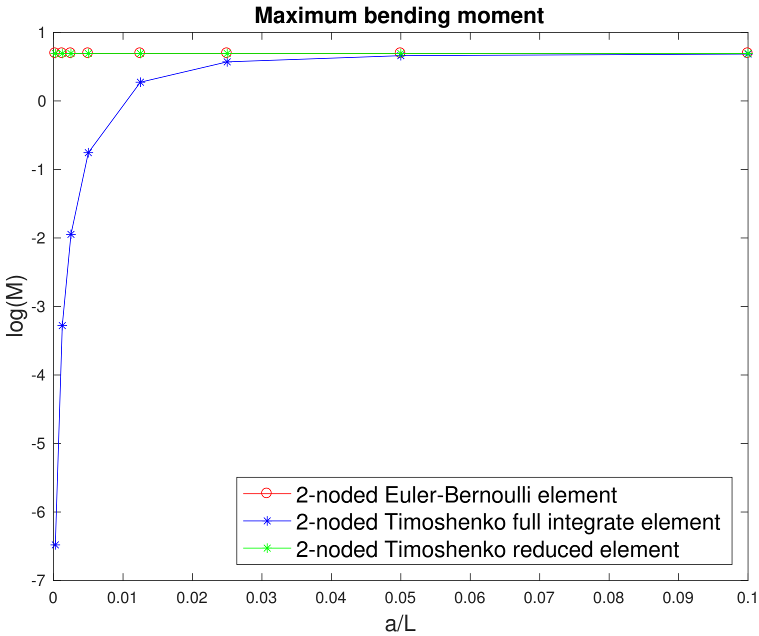

Figure 0.3: Representation of the natural logarithm of the maximum bending moment for the different cases considered versus the ratio a/L.

As before, for really slender beam (low values of the ratio a/L), the Timoshenko full integrated element gives non-desirable results as it does not properly accounts for the behaviour of slender beams. On the other hand, the Timoshenko reduced integration element mainly matches de convencional Euler-Bernoulli approach. Both basically give the same result (nearly constant moment) for all the cases considered, as we can see in the plot.

Finally, figure 0.4 shows the results for the shear force. In this case, we just have the results for both cases of the Timoshenko theory since the Euler-Bernoulli approach does not consider the effect of transverse shear deformation. Again, one can note the the 2-noded Timoshenko full integrated element solution is corrupted by spurious oscillations for slender beams. This is due to the fact of the *shear locking*, already discussed previously in detail. The solution for the reduced approach, is mainly constant and acceptable.

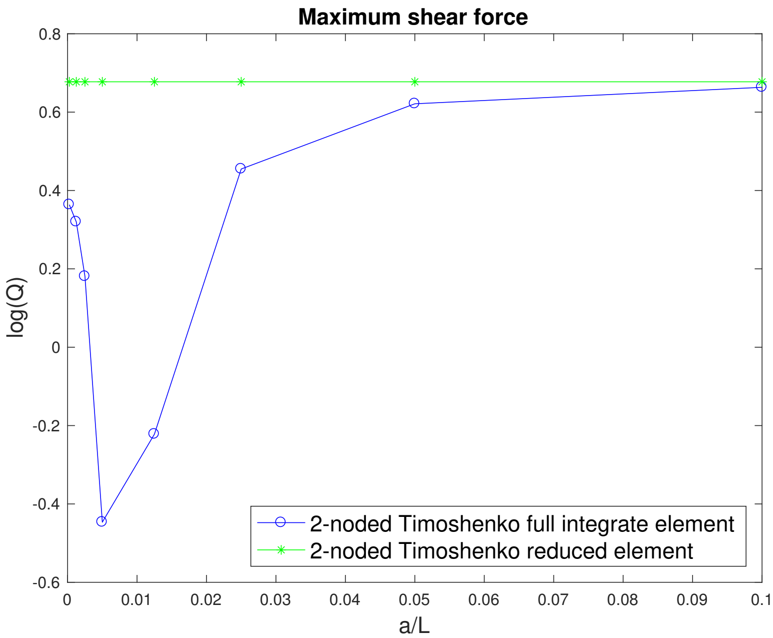

Figure 0.4: Representation of the natural logarithm of the maximum shear force for the different cases considered versus the ratio a/L.