Computational Structural Mechanics and Dynamics

Master's Degree in Numerical Methods in Engineering

Assignment 6: Beams

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Abstract

1 Part A

For the first part of the assignment, it is required to implement the program in Matlab the Timoshennko 2 Nodes Beam with reduce integration for the shear stiffness matrix. The only change produced respect to the Full Integration is the shear stiffness matrix shown in figure 1.

```
%IMPLEMENTATION OF TIMOSHENKO REDUCED INTEGRATION MATRIX:

K_s = [   1    ,    len/2 ,    -1    ,    len/2 ;
        len/2 , len^2/4 , -len/2 , len^2/4 ;
         -1    ,  -len/2 ,     1    ,  -len/2 ;
        len/2 , len^2/4 , -len/2 , len^2/4 ];
```

Figure 1: Implemenentation of Timoshenko reduced integration matrix.

It is needed two Gauss points for the exact integration of the shear stiffness matrix Ks.

2 Part B

For the second part, it is required to solve the following problem with 64 element mesh for different dimension "a" shown in figure 2 with the:

- 2 nodes Timoshenko Reduce Integration element.
- 2 nodes Timoshenko Full Integrate element.
- 2 nodes Euler-Bernoulli element.

Finally, it is compared the maximum displacements, moments and shear for the 3 elements against the a/L relationship.

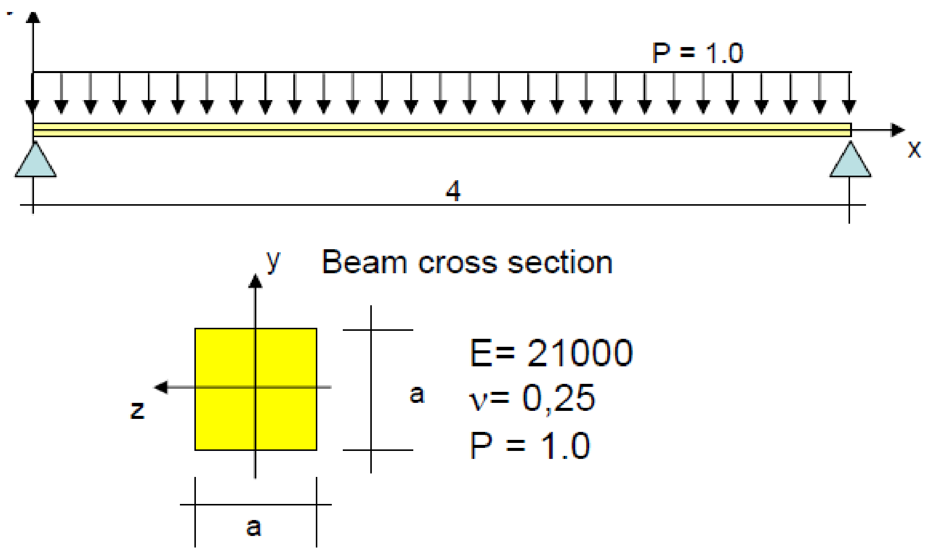

Figure 2: Problem statement.

The first step, it has been calculated the area and momentum of inertia for each length "a". It has been created the geometry of the beam with its conditions given in the problem with the software GiD. Then, it has been created an input file for the program given by MatFem for each of the methods in Matlab in order to analyse the beam and finally, it has been obtained the results.

a	(a/L)	Area	Moment of Inertia
0.001	0.00025	0.000001	8.33333E-14
0.005	0.00125	0.000025	5.20833E-11
0.01	0.0025	0.0001	8.33333E-10
0.02	0.005	0.0004	1.33333E-08
0.05	0.0125	0.0025	5.20833E-07
0.1	0.025	0.01	8.33333E-06
0.2	0.05	0.04	0.000133333
0.4	0.1	0.16	0.002133333

Figure 3: Areas and Moments of inertia.

Next, it is presented the tables with the results for each one of the methods.

2 nodes Timoshenko Reduce Integration element				
a	(a/L)	Max displ	Max Mz	Max Shear
0.001	0.00025	1.90E+09	1.999	1.9687
0.005	0.00125	3.05E+06	1.999	1.9687
0.01	0.0025	1.90E+05	1.999	1.9687
0.02	0.005	1.19E+04	1.999	1.9687
0.05	0.0125	3.05E+02	1.999	1.9687
0.1	0.025	1.91E+01	1.999	1.9687
0.2	0.05	1.197	1.999	1.9687
0.4	0.1	0.076	1.999	1.9687

Figure 4: Results for 2 nodes Timoshenko Reduce Integration element.

2 nodes Timoshenko Full Integration element				
a	(a/L)	Max displ	Max Mz	Max Shear
0.001	0.00025	1.46E+06	0.001534	1.9687
0.005	0.00125	5.74E+04	0.037658	1.9687
0.01	0.0025	1.36E+04	0.14345	1.9687
0.02	0.005	2.80E+03	0.469783	1.9687
0.05	0.0125	2.00E+02	1.314426	1.9687
0.1	0.025	1.69E+01	1.768721	1.9687
0.2	0.05	1.1596	1.936003	1.9687
0.4	0.1	0.0755	1.982887	1.9687

Figure 5: Results for 2 nodes Timoshenko Full Integrate element.

2 nodes Euler-Bernoulli element				
a	(a/L)	Max displ	Max Mz	Max Shear
0.001	0.00025	1.90E+09	1.9991	2
0.005	0.00125	3.05E+06	1.9991	2
0.01	0.0025	1.90E+05	1.9991	2
0.02	0.005	1.19E+04	1.9991	2
0.05	0.0125	3.05E+02	1.9991	2
0.1	0.025	19.047	1.9991	2
0.2	0.05	1.1905	1.9991	2
0.4	0.1	0.0744	1.9991	2

Figure 6: 2 nodes Euler-Bernoulli element.

Finally, it is presented the graphs in order to study the behaviour of each parameter with the method used.

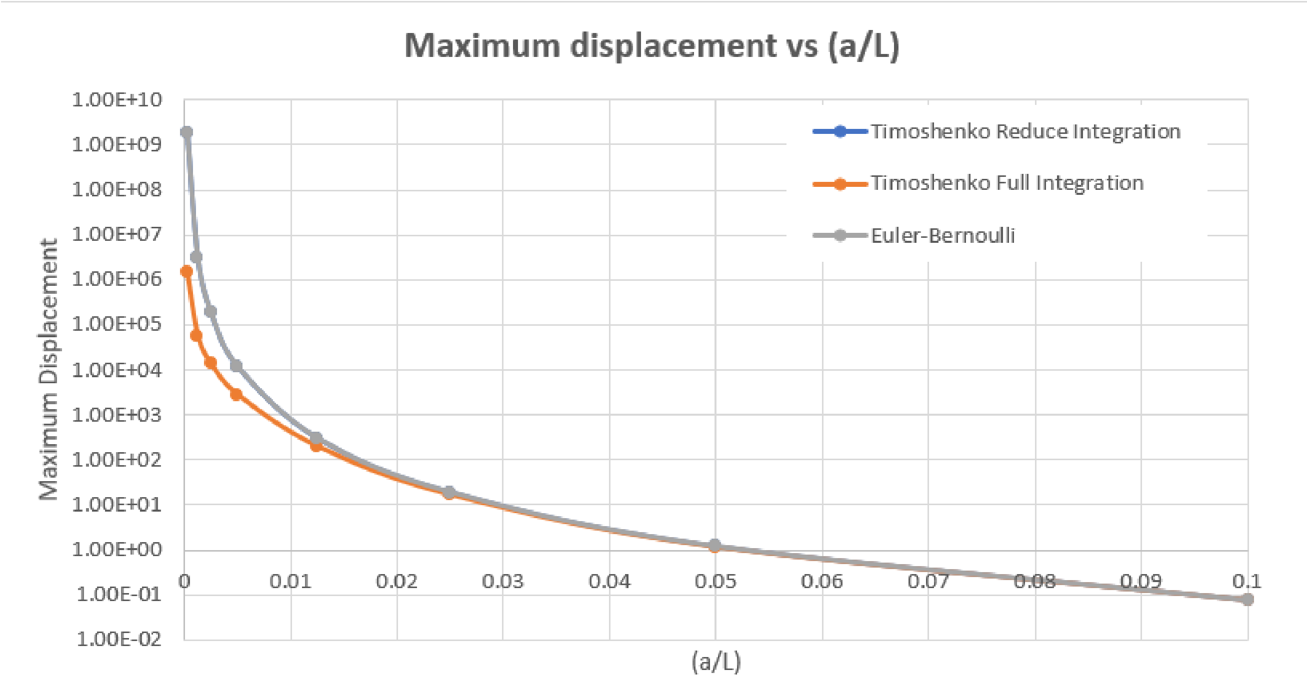

Figure 7: Maximum displacement.

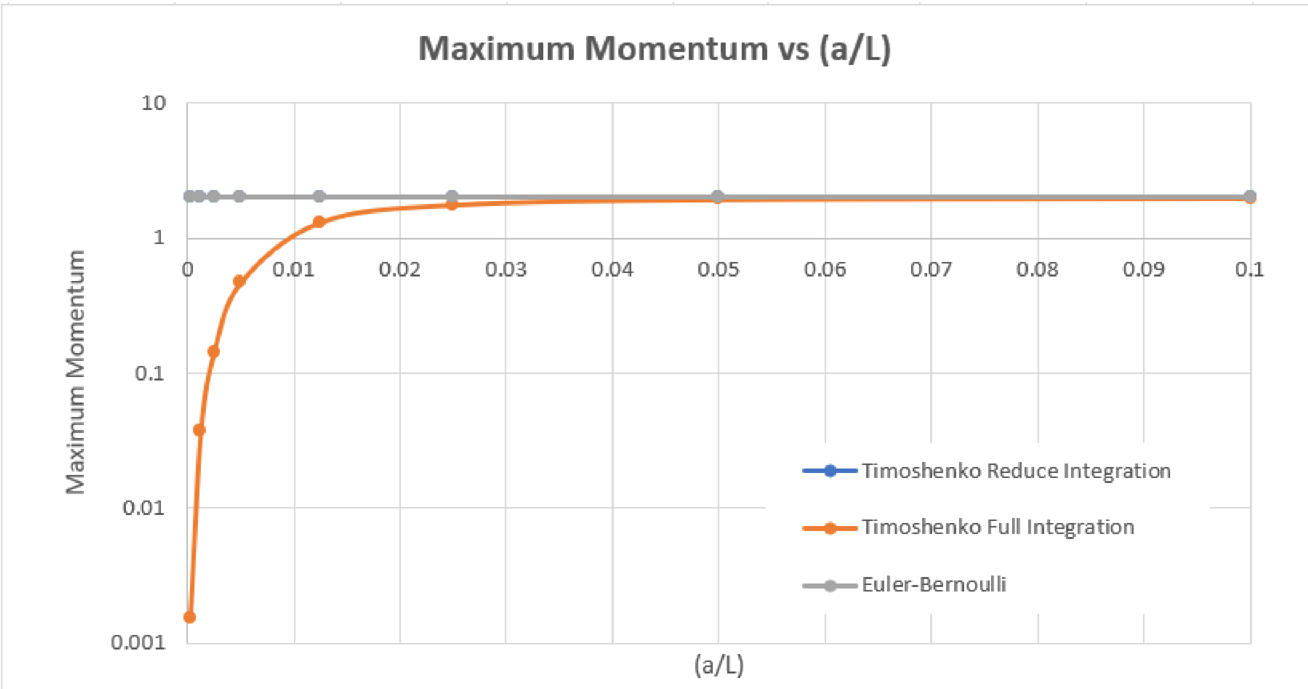

Figure 8: Maximum moment.

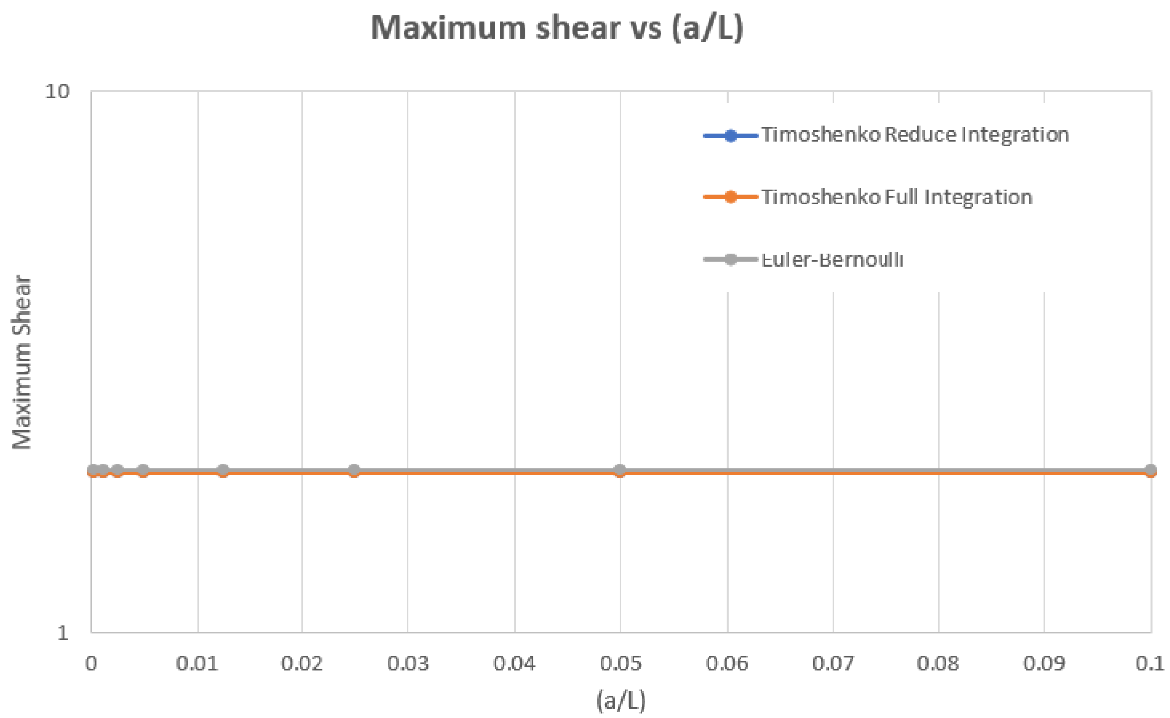

Figure 9: Maximum shear.

All the results shown are for beams with a ratio (a/L) smaller than 0.1, therefore, they are all slender beams. From the theory, the Euler-Bernoulli method is a good method for slender beams.

For the study of displacement, it is noticed that there are big variations for the maximum value of the displacement for each ratio (a/L). On the one hand, Euler-Bernoulli and the reduced integration Timoshenko's model have the same displacements, They work well with a set of slender ratio values. On the other hand, for the full integration of Timoshenko's model, there are smaller values of the displacements for the smaller values of the ratios. This is due to the fact the shear locking effect which produces an increase of the stiffness of the beam and smaller strains.

For the study of bending moment and shear, the models of Timoshenko Reduce Integration and Euler-Bernoulli show the same values for bending moment and shear. However, the full integrated Timoshenko has an increasing behaviour for the maximum moment values because the cross-section and stiffness that produces a variation in the integration of the global stiffness for the bending problem. The shear locking affects to the rotational degrees of freedom. It is necessary to say that exists a little difference for the shear values because of the hypothesis in the Timoshenko model.

It is shown that the Euler-Bernoulli's method describes this problem properly and gives accurate result for the bending problem. The Thimoshenko's method is better for the shear problem, but it is not required for a slender beam. Therefore, to conclude the assignment, it is preferable to avoid the full integration of Timoshenko's model because it has problems with the bending calculation.