Assignment 5

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Problem 1

a) ① $N_1^e(-1) = a_0 - a_1 + a_2 = 1$

$N_1^e(0) = a_0 = 0$

$N_1^e(1) = a_0 + a_1 + a_2 = 0$

$$\Rightarrow \begin{cases} a_0 = 0 \\ a_1 = -1/2 \\ a_2 = 1/2 \end{cases} \Rightarrow N_1^e = -\frac{1}{2}\cdot\xi + \frac{1}{2}\cdot\xi^2$$

② $N_2^e(-1) = b_0 - b_1 + b_2 = 0$

$N_2^e(0) = b_0 = 0$

$N_2^e(1) = b_0 + b_1 + b_2 = 1$

$$\Rightarrow \begin{cases} b_0 = 0 \\ b_1 = 1/2 \\ b_2 = 1/2 \end{cases} \Rightarrow N_2^e = \frac{1}{2}\xi + \frac{1}{2}\cdot\xi^2$$

③ $N_3^e(-1) = C_0 - C_1 + C_2 = 0$

$N_3^e(0) = C_0 = 1$

$N_3^e(1) = C_0 + C_1 + C_2 = 0$

$$\Rightarrow \begin{cases} C_0 = 1 \\ C_1 = 0 \\ C_2 = -1 \end{cases} \Rightarrow N_3^e = 1 - \xi^2$$

b) We can get:

$N_1^e + N_2^e + N_3^e = 1$

c). $\frac{\partial N_1^e}{\partial \xi} = \xi - \frac{1}{2}$; $\frac{\partial N_2^e}{\partial \xi} = \xi + \frac{1}{2}$; $\frac{\partial N_3^e}{\partial \xi} = -2\xi$

Problem 5.2

By using the line-product method:

$$N_5 = (1+\xi)(1-\xi)(1+\eta)(1-\eta) = (1-\xi^2)(1-\eta^2) \Rightarrow$$

① $N_1 = \frac{1}{4}(1-\xi)(1-\eta) + \alpha \cdot (1-\xi^2)(1-\eta^2)$

$N_1(0,0) = \frac{1}{4} + \alpha \Rightarrow \alpha = -\frac{1}{4} \Rightarrow \boxed{N_1 = \frac{1}{4}(1-\xi)(1-\eta) - \frac{1}{4}(1-\xi^2)(1-\eta^2)}$

② $N_2 = \frac{1}{4}(1+\xi)(1-\eta) + \alpha(1-\xi^2)(1-\eta^2)$

$N_2(0,0) = \frac{1}{4} + \alpha \Rightarrow \alpha = -\frac{1}{4} \Rightarrow \boxed{N_2 = \frac{1}{4}(1+\xi)(1-\eta) - \frac{1}{4}(1-\xi^2)(1-\eta^2)}$

③ $N_3 = \frac{1}{4}(1+\xi)(1+\eta) + \alpha(1-\xi^2)(1-\eta^2)$

$N_3(0,0) = \frac{1}{4} + \alpha \Rightarrow \alpha = -\frac{1}{4}$ $\boxed{N_3 = \frac{1}{4}(1+\xi)(1+\eta) - \frac{1}{4}(1-\xi^2)(1-\eta^2)}$

④ $N_4 = \frac{1}{4}(1-\xi)(1+\eta) + \alpha(1-\xi^2)(1-\eta^2)$

$N_4(0,0) = \frac{1}{4} + \alpha \Rightarrow \alpha = -\frac{1}{4}$ $\boxed{N_4 = \frac{1}{4}(1-\xi)(1+\eta) - \frac{1}{4}(1-\xi^2)(1-\eta^2)}$

$$N_1+N_2+N_3+N_4 = \frac{1}{4} \times \left[(1-\eta-\xi+\xi\eta) + (1-\eta+\xi-\xi\eta) + (1+\eta+\xi+\xi\eta) + (1+\eta-\xi-\xi\eta)\right] - \underbrace{(1-\xi^2)(1-\eta^2)}_{N_5}$$

$$= \frac{1}{4} \times 4 - N_5 = 1 - N_5$$

$$\Rightarrow N_1+N_2+N_3+N_4+N_5 = 1$$

Problem 5.3

For choosing the integration rules, we need to use following relation:

$n_E \cdot n_G \geq n_F - n_R$;

n_G - number of Gauss points

n_E - order of $\underset{\sim}{E}$ stress-strain matrix (for 3-D, $n_E = 6$)

n_F - number of element DoF.

n_R - number of independent rigid body modes (for 3-D, $n_R = 6$)

① 8-node hexahedron

$6 \times n_G \geq 3 \times 8 - 6 = 18 \Rightarrow n_G \geq 3 \Rightarrow$ choose $2 \times 2 \times 2$ rule.

② 20-node hexahedron

$6 \times n_G \geq 3 \times 20 - 6 = 54 \Rightarrow n_G \geq 9 \Rightarrow$ choose $3 \times 3 \times 3$ rule.

③ 27-node hexahedron

$6 \times n_G \geq 3 \times 27 - 6 = 75 \Rightarrow n_G \geq 13 \Rightarrow$ choose $3 \times 3 \times 3$ rule

④ 64-node hexahedron

$6 \times n_G \geq 3 \times 64 - 6 = 186 \Rightarrow n_G \geq 31 \Rightarrow$ choose $4 \times 4 \times 4$ rule.