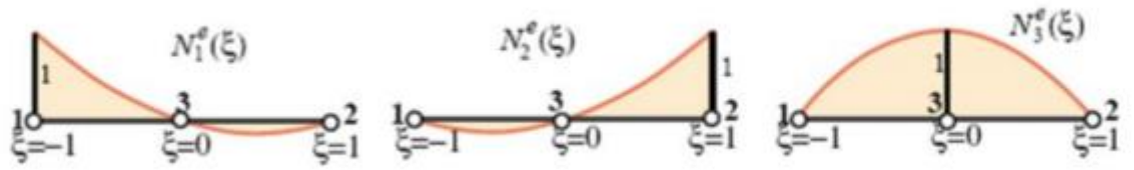
Computational Structural Mechanics and Dynamics

Assignment 5 – Trond Jørgen Opheim

Problem 5.2



a. Determine the coefficient for the shape functions

$N_1(-1) = 1$ $\quad$ $a_0 = 0$

$N_1(0) = 0$ $\quad \rightarrow \quad$ $a_1 = -\frac{1}{2}$ $\quad \rightarrow \quad$ $N_1 = -\frac{1}{2}\xi + \frac{1}{2}\xi^2$

$N_1(1) = 0$ $\quad$ $a_2 = \frac{1}{2}$

$N_2(-1) = 0$ $\quad$ $b_0 = 0$

$N_2(0) = 0$ $\quad \rightarrow \quad$ $b_1 = \frac{1}{2}$ $\quad \rightarrow \quad$ $N_2 = \frac{1}{2}\xi + \frac{1}{2}\xi^2$

$N_2(1) = 1$ $\quad$ $b_2 = \frac{1}{2}$

$N_3(-1) = 0$ $\quad$ $c_0 = 1$

$N_3(0) = 1$ $\quad \rightarrow \quad$ $c_1 = 0$ $\quad \rightarrow \quad$ $N_3 = 1 - \xi^2$

$N_3(1) = 0$ $\quad$ $c_2 = -1$

b. Sum of the shape functions

$$N = N_1 + N_2 + N_3 = \left(-\frac{1}{2}\xi + \frac{1}{2}\xi^2\right) + \left(\frac{1}{2}\xi + \frac{1}{2}\xi^2\right) + (1 - \xi^2) = 1$$

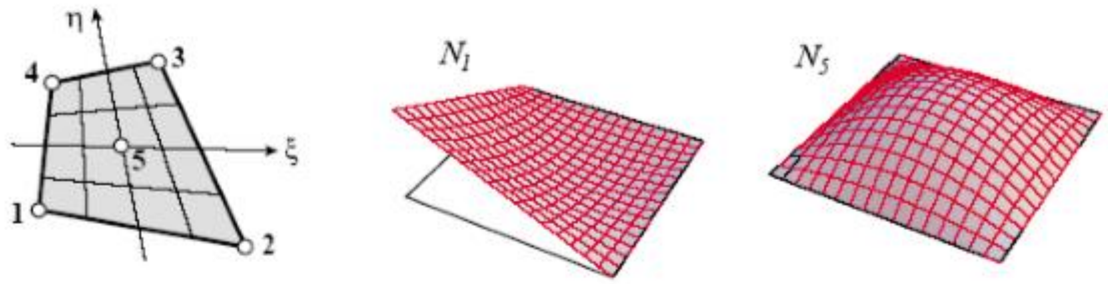
c. Derivatives of the shape functions

$$N_1 = -\frac{1}{2}\xi + \frac{1}{2}\xi^2 \qquad \frac{dN_1}{d\xi} = -\frac{1}{2} + \xi$$

$$N_2 = \frac{1}{2}\xi + \frac{1}{2}\xi^2 \qquad \frac{dN_1}{d\xi} = \frac{1}{2} + \xi$$

$$N_3 = 1 - \xi^2 \qquad \frac{dN_1}{d\xi} = 2\xi$$

Problem 5.2 Shape functions for the 5-noded quadrilateral element



To solve this problem, I use the hint given in the assignment. That is to first compute the shape function N_5 for the fifth node, then use that $N_i = \underline{N_i} + \propto N_5$, where $\underline{N_i}$ is the i-th shape function for the 4-noded quadrilateral element.

To find $N_5(\xi,\eta)$ I use the line product method:

$$N_5(\xi,\eta) = c * (1-\xi)(1-\eta)(1+\xi)$$

$$N_5(0,0) = 1 \ \rightarrow c = 1$$

Shape functions $\underline{N_i}$ for the 4-noded quadrilateral element:

$$\underline{N_1}(\xi,\eta) = \frac{1}{4}(1-\xi)(1-\eta)$$

$$\underline{N_2}(\xi,\eta) = \frac{1}{4}(1+\xi)(1-\eta)$$

$$\underline{N_3}(\xi,\eta) = \frac{1}{4}(1+\xi)(1+\eta)$$

$$\underline{N_4}(\xi,\eta) = \frac{1}{4}(1-\xi)(1+\eta)$$

Define α by $N_i = \underline{N_i} + \propto N_5$:

$$N_1(\xi,\eta) = \frac{1}{4}(1-\xi)(1-\eta) + \propto (1-\xi)(1-\eta)(1+\xi)$$

$$N_1(0,0) = 0 \ \rightarrow \ \frac{1}{4} + \propto = 0 \ \rightarrow \ \alpha = -\frac{1}{4}$$

Shape functions for the 5-noded quadrilateral element:

$$N_1(\xi,\eta) = \frac{1}{4}(1-\xi)(1-\eta) - \frac{1}{4}(1-\xi)(1-\eta)(1+\xi)$$

$$N_2(\xi,\eta) = \frac{1}{4}(1+\xi)(1-\eta) - \frac{1}{4}(1-\xi)(1-\eta)(1+\xi)$$

$$N_3(\xi,\eta) = \frac{1}{4}(1+\xi)(1+\eta) - \frac{1}{4}(1-\xi)(1-\eta)(1+\xi)$$

$$N_4(\xi,\eta) = \frac{1}{4}(1-\xi)(1+\eta) - \frac{1}{4}(1-\xi)(1-\eta)(1+\xi)$$

$$N_5(\xi,\eta) = (1-\xi)(1-\eta)(1+\xi)$$

Which are the five shape functions for the 5-noded quadrilateral element that satisfy compatibility and sum equal to 1.

Problem 5.3

n: number of nodes
n_F: element degrees of freedom
n_R: number of independent rigid body modes
n_G: number of Gauss points
n_E: order of strain-stress matrix(in this case 3 because of the assumption of plane stress)

Table 1 – Rank sufficient Gauss rules for hexahedron element in plane stress

Element	n	n_F	$n_F - n_R$	Min. n_G	Integration rule
8-node hexahedron	8	24	21	7	3x3
20-node hexahedron	20	60	57	19	5x5
27-node hexahedron	27	81	78	26	6x6
64-node hexahedron	64	192	189	63	8x8