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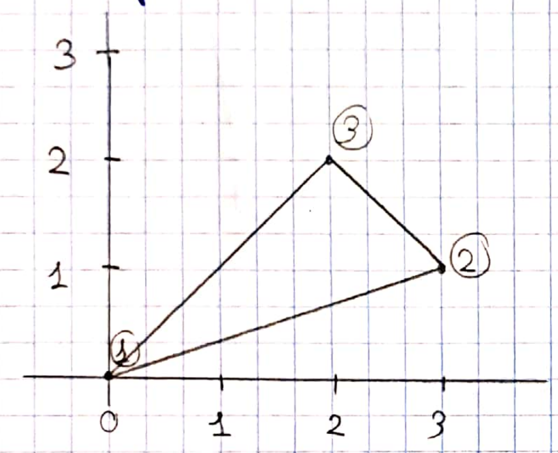
Assignment 3.1

1. Compute $K^{(e)}$



$x_1 = y_1 = 0$

$x_2 = 3 ; y_2 = 1$

$x_3 = 2 ; y_3 = 2$

$$E = \begin{bmatrix} 100 & 25 & 0 \\ 25 & 100 & 0 \\ 0 & 0 & 50 \end{bmatrix} ; \quad h = 1$$

\# Area of the triangle:

$$A = \frac{1}{2} \cdot \begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 0 & 3 & 2 \\ 0 & 1 & 2 \end{vmatrix} = 2$$

\# The element stiffness matrix:

$$K^{(e)} = \int_{\Omega^e} h \cdot B^T E B \, d\Omega$$

where B and E are constant over the triangle area.

The element strain matrix :

$$B = \frac{1}{2A}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

$$B = \frac{1}{4}\begin{bmatrix} -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & -2 & 0 & 3 \\ -1 & -1 & -2 & 2 & 3 & -1 \end{bmatrix}$$

$$K_{(e)} = \frac{h}{4A} \cdot B^T \cdot E \cdot B =$$

$$K_{e)} = \begin{bmatrix} 18.75 & 9.375 & -12.5 & -6.25 & -6.25 & -3.125 \\ & 18.75 & 6.25 & 12.5 & -15.625 & -31.25 \\ & & 75 & -37.5 & -62.5 & 31.25 \\ & \text{SYM} & & 75 & 43.75 & -87.5 \\ & & & & 68.75 & -28.125 \\ & & & & & 118.75 \end{bmatrix}$$

2. Show the sum of the rows and columns 1, 3, 5 and sum of 2, 4, 6 must vanish and why.

* Sum of rows 1, 3 and 5; 2, 4 and 6

$$\begin{array}{cccccc}
[18.75 & 9.375 & -12.5 & -6.25 & -6.25 & -3.125] \\
[-12.75 & 6.25 & 75 & -37.5 & -6.25 & 31.25] \\
[-6.25 & -15.625 & -62.5 & 43.75 & 68.75 & -28.125] \\
\hline
0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

$$\begin{array}{cccccc}
[9.375 & 18.75 & 6.25 & 12.5 & -15.625 & -31.25] \\
[-6.25 & 12.5 & -37.5 & 75 & 48.75 & -87.5] \\
[-3.125 & -31.25 & 31.25 & -87.5 & -28.125 & 116.75] \\
\hline
0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

* Sum of columns 1, 3, 5 and 2, 4, 6.
They are the same values of the rows and they result to 0.

* Rows and columns 1, 3, 5 represents the stiffness corresponding to axis X and 2, 4, 6 to axis Y.
The elemental stiffness is singular and the element needs a constraint to be able to hold an external load and have the internal equilibrium.
There will be infinite different possible desplacements for a given set of external forces if there are not constraint imposed. The sum of these rows / columns are 0 because rigid body motions are allowed with no constraint applied.

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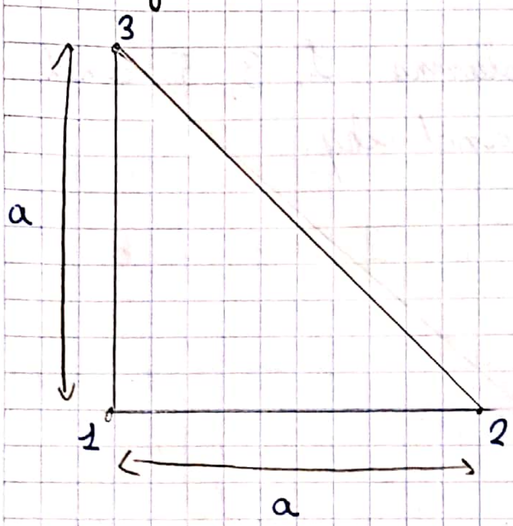
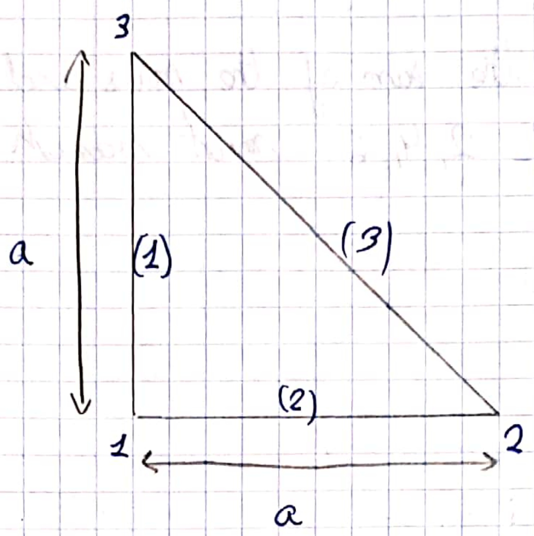
Assignment 3.2





$a = h = 1$ $\quad$ $A_1 = A_2$, A_3

E , $\nu = 0$

a) $K^{(e)}$ for both models

* Stiffness matrix:

$$K^{(e)} = h \cdot \underline{B}^T \cdot \underline{E}\, \underline{B}$$

* For plane stress problem, where $\nu = 0$

$$\underline{E} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} = E \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$$

$$B = \frac{1}{2A}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

+ Coordinates of the nodes:

$x_1 = y_1 = 0$

$x_2 = a = 1 \quad y_2 = 0$

$x_3 = 0 \quad y_3 = a = 1$

+ Area of the triangles:

$$A = \frac{1}{2} \cdot \begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix} = \frac{a^2}{2} \overset{a=1}{=} \frac{1}{2}$$

$y_{12} = 0 \;;\quad y_{23} = -a \;;\quad y_{31} = a$

$x_{32} = -a \;;\quad x_{13} = 0 \;;\quad x_{21} = a$

$$B = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

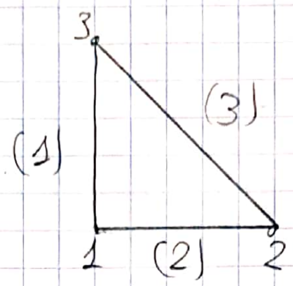
+ So, the stiffness matrix for Turner triangle

$$K^{(e)}_{triangle} = E \begin{bmatrix} 0.75 & 0.25 & -0.5 & -0.25 & -0.25 & 0 \\ & 0.75 & 0 & -0.25 & -0.25 & -0.5 \\ & & 0.5 & 0 & 0 & 0 \\ & & & 0.25 & 0.25 & 0 \\ & SYM & & & 0.25 & 0 \\ & & & & & 0.5 \end{bmatrix}$$

b) Set of values for $A_1 = A_2 = A$ and $A_3 = A'$ to make $K_{bar} = K_{triangle}$

$L_1 = L_2 = a = 1$

$L_3 = \sqrt{2}\, a = \sqrt{2}$



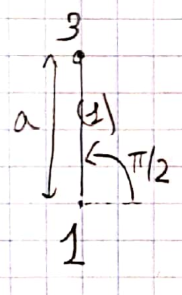
* Stiffness matrix of the bar.

$$K^{(e)} = \frac{(EA)^e}{L^e} \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & c^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -c^2 & sc & s^2 \end{bmatrix}$$

$s = \sin\phi$

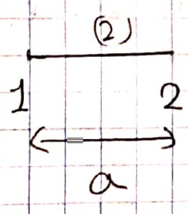
$c = \cos\phi$

* Element 1. $s = 1$ $c = 0$



$$K^1 = EA \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

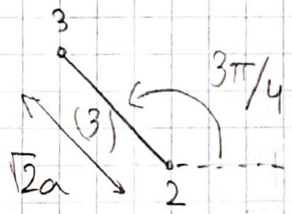
* Element 2: $s = 0$ $c = 1$



$$K^2 = EA \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

* Element 3 $\quad s = \frac{\sqrt{2}}{2} \quad c = -\frac{\sqrt{2}}{2}$

$$K^{(3)} = \frac{EA'}{\sqrt{2}} \begin{bmatrix} 1/2 & -1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & -1/2 & 1/2 \end{bmatrix}$$



* Global stiffness matrix:

$$K_{bar} = E \begin{bmatrix} A & 0 & -A & 0 & 0 & 0 \\ & A & 0 & 0 & 0 & -A \\ & & A + \frac{A'}{2\sqrt{2}} & -\frac{A'}{2\sqrt{2}} & -\frac{A'}{2\sqrt{2}} & \frac{A'}{2\sqrt{2}} \\ & & & \frac{A'}{2\sqrt{2}} & \frac{A'}{2\sqrt{2}} & -\frac{A'}{2\sqrt{2}} \\ & SYM & & & \frac{A'}{2\sqrt{2}} & -\frac{A'}{2\sqrt{2}} \\ & & & & & A + \frac{A'}{2\sqrt{2}} \end{bmatrix}$$

It is not possible to make Kbar = Ktriangle
First of all! there are 0 entries impossible to make equal!

In order to make them similar

\# $K_{bar,11} = k_{tri,11}$

$$\left[A = 0.75\right] \Rightarrow K_{11} = K_{22} \quad ; \quad K_{13\,bar} = -0.75$$

$K_{bar,55} = K_{tri,55}$

\# $\frac{A'}{2\sqrt{2}} = 0.25$; $\left[A' = \frac{1}{\sqrt{2}}\right]$

\# There are other components impossible to make them equal.

c) Why they are not equivalent? Physical explanation.

The two matrices represents two different structural problems, so it is not possible to make them equivalent. On one hand, in the case of bar elements, it is only considered axial displacements/forces in the bars. There is always a node that is not connected with a bar (for instance, node 1 with bar 3), so it is expected to have 0 in the stiffness matrix
On the other hand, in the case of the triangle, perimeter of the material is distributed throughout the whole the triangle, whereas, in the case of the bars, the material is only considered where there is a bar.

d) Solve a) considering $\nu \neq 0$.

So, now it is the same procedure considering:

$$\underline{E} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1-\nu/2 \end{bmatrix}$$

The K triangle will change, whereas there is no impact on the K bar.

With that effect, it is possible to conclude and highlight the difference between both problems. The problem of the bars behaves like a truss and not like a cross-section material.