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Assignment. No. 3.1 CSMP

Plane Stress
$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2\epsilon_{xy} \end{bmatrix} \quad \ldots\ldots \text{Eq (A)}$$

Plane Strain
$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2\epsilon_{xy} \end{bmatrix} \quad \ldots \text{Eq (B)}$$

Part-a

Replace $E \to E^*$ & $\nu \to \nu^*$ in plan stress constitutive matrix.

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{E^*}{1-\nu^{*2}} & \frac{\nu^* E^*}{1-\nu^{*2}} & 0 \\ \frac{\nu^* E^*}{1-\nu^{*2}} & \frac{E^*}{1-\nu^{*2}} & 0 \\ 0 & 0 & \frac{E^*}{2(1+\nu^*)} \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{bmatrix} \quad \ldots \text{Eq (C)}$$

To form the plan strain matrix, each entity of constitutive matrix must be equal to each entity (corresponding) of Eq (B).

So, By setting Equal A_{11} of (C) with A_{11} of (B)

$$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)} = \frac{E^*}{1-\nu^{*2}} \Rightarrow E^* = \frac{E(1-\nu)(1-\nu^*)(1+\nu^*)}{(1+\nu)(1-2\nu)}$$

By setting equal A_{33} of (C) with A_{33} of (B)

$$\frac{E}{2(1+\nu)} = \frac{E^*}{2(1+\nu^*)} \Rightarrow E^* = \frac{E(1+\nu^*)}{(1+\nu)}$$

By putting E^* value in above equation.

$$\frac{E(1+\nu^*)}{(1+\nu)} = \frac{E(1-\nu)(1-\nu^*)(1+\nu^*)}{(1+\nu)(1-2\nu)} \Rightarrow \boxed{\nu^* = \frac{\nu}{1-\nu}}$$

and $$E^* = \frac{E\left[1+\left(\frac{-\nu}{1-\nu}\right)\right]}{1+\nu} \Rightarrow \boxed{E^* = \frac{E}{1-\nu^2}} \quad (2)$$

Part_b

Replace $E = E^*$ & $\nu = \nu^*$ in plane Strain Constitutive Matrix

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{E^*(1-\nu^*)}{(1+\nu^*)(1-2\nu^*)} & \frac{E^*\nu^*}{(1+\nu^*)(1-2\nu^*)} & 0 \\ \frac{E^*\nu^*}{(1+\nu^*)(1-2\nu^*)} & \frac{E^*(1-\nu^*)}{(1+\nu^*)(1-2\nu^*)} & 0 \\ 0 & 0 & \frac{E^*}{2(1+\nu^*)} \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{bmatrix} \quad \text{--- (D)}$$

To produce the plane Stress matrix, each entity of Constitutive matrix must be equal to each corresponding entity of (A)

So, By equating A_{11} of (D) with A_{11} of (A) & A_{33} of (D) with A_{33} of (A)

$$\frac{E^*(1-\nu^*)}{(1+\nu^*)(1-2\nu^*)} = \frac{E}{1-\nu^2} \quad \text{—— (1)}$$

$$\frac{E^*}{2(1+\nu^*)} = \frac{E}{2(1+\nu)} \Rightarrow \nu^* = \frac{E^*(1+\nu) - E}{E} \quad \text{— (2)}$$

put (2) into (1)

$$\frac{E^*\left(\frac{E - E^*(1+\nu) + E}{E}\right)}{\left(\frac{E + E^*(1+\nu) - E}{E}\right)\left(\frac{E - 2E^*(1+\nu) + 2E}{E}\right)} = \frac{E}{1-\nu^2}$$

$$E^*(1+\nu)\left(3E - 2E^*(1+\nu)\right) = \left(E^*(2E - E^*(1+\nu))\right)\left((1-\nu^*)(1+\nu)\right)$$

$$E^*(1+\nu).3E - E^*(1+\nu).2E^*(1+\nu) = \left[(E^*.2E - E^*.E^*(1+\nu)\right](1-\nu)(1+\nu)$$

$$E^*.3E - E^*2E(1-\nu) = E^*.2E^*(1+\nu) - E^*E^*(1-\nu)(1+\nu)$$

$$E^*E(3-2-2\nu) = E^*.E^*(1+\nu)\left[2-1+\nu\right]$$

$$E^{*}E(1-2\nu) = E^{*}E^{*}(1+\nu)(1+\nu)$$ ③

$$\boxed{E^{*} = \frac{E(1-2\nu)}{(1+\nu)^2}}$$

put E^* in Eq ②

$$\frac{E(1-2\nu)}{(1+\nu)(1+\nu)} = \frac{E(1+\nu^*)}{(1+\nu)} \Rightarrow \boxed{\nu^* = \frac{-3\nu}{1+\nu}}$$

Assignment. No. 3.2

As we have

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \quad \text{— (I)}, \qquad \mu = \frac{E}{2(1+\nu)} \quad \text{— (II)}$$

Part-a From (I) & (II),

$$E = \frac{\lambda(1+\nu)(1-2\nu)}{\nu} \quad \text{— (III)} \quad \& \quad E = \mu\, 2(1+\nu) \quad \text{— (IV)}$$

By Comparing (III) & (IV)

$$\frac{\lambda(1+\nu)(1-2\nu)}{\nu} = 2\mu(1+\nu)$$

$$\boxed{\nu = \frac{\lambda}{2(\mu+\lambda)}} \quad \& \quad \boxed{E = \frac{\mu(2\mu+3\lambda)}{(\mu+\lambda)}}$$

Part. b Elastic Matrix for plane Stress & plane Strain

$$\frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad \& \quad \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

for plane Stress

$$\frac{\dfrac{\mu(2\mu+3\lambda)}{(\mu+\lambda)}}{1-\left(\dfrac{\lambda}{2(\mu+\lambda)}\right)^2}\begin{bmatrix} 1 & \dfrac{\lambda}{2(\mu+\lambda)} & 0 \\ \dfrac{\lambda}{2(\mu+\lambda)} & 1 & 0 \\ 0 & 0 & \dfrac{1-\left(\dfrac{\lambda}{2(\mu+\lambda)}\right)}{2} \end{bmatrix}$$

$$\frac{\dfrac{\mu(2\mu+3\lambda)}{(\mu+\lambda)}}{\dfrac{4(\mu+\lambda)^2-\lambda^2}{4(\mu+\lambda)(\mu+\lambda)}}\begin{bmatrix} 1 & \dfrac{\lambda}{2(\mu+\lambda)} & 0 \\ \dfrac{\lambda}{2(\mu+\lambda)} & 1 & 0 \\ 0 & 0 & \dfrac{2\mu+\lambda}{4(\mu+\lambda)} \end{bmatrix}$$

$$\frac{\mu(2\mu+3\lambda)(4(\mu+\lambda))}{(2\mu+2\lambda-\lambda)(2\mu+2\lambda+\lambda)}\begin{bmatrix} 1 & \dfrac{\lambda}{2(\mu+\lambda)} & 0 \\ \dfrac{\lambda}{2(\mu+\lambda)} & 1 & 0 \\ 0 & 0 & \dfrac{2\mu+\lambda}{2(\mu+\lambda)} \end{bmatrix}$$

$$\boxed{\frac{4\mu(\mu+\lambda)}{(2\mu+\lambda)}\begin{bmatrix} 1 & \dfrac{\lambda}{2(\mu+\lambda)} & 0 \\ \dfrac{\lambda}{2(\mu+\lambda)} & 1 & 0 \\ 0 & 0 & \dfrac{2\mu+\lambda}{2(\mu+\lambda)} \end{bmatrix}}$$

For plane Strain

$$\frac{\dfrac{\mu(2\mu+3\lambda)}{(\mu+\lambda)}}{\left[1+\dfrac{\lambda}{2(\mu+\lambda)}\right]\left[1-2\left(\dfrac{\lambda}{2(\mu+\lambda)}\right)\right]}\begin{bmatrix} 1-\left(\dfrac{\lambda}{2(\mu+\lambda)}\right) & \dfrac{\lambda}{2(\mu+\lambda)} & 0 \\ \dfrac{\lambda}{2(\mu+\lambda)} & 1-\left(\dfrac{\lambda}{2(\mu+\lambda)}\right) & 0 \\ 0 & 0 & \dfrac{1-2\left(\dfrac{\lambda}{2(\mu+\lambda)}\right)}{2} \end{bmatrix}$$

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$$\frac{\dfrac{\mu(2\mu+3\lambda)}{(\mu+\lambda)}}{\left[\dfrac{(2\mu+3\lambda)}{2(\mu+\lambda)}\right]\left[\dfrac{\mu}{(\mu+\lambda)}\right]} \begin{bmatrix} \dfrac{2\mu+\lambda}{2(\mu+\lambda)} & \dfrac{\lambda}{2(\mu+\lambda)} & 0 \\ \dfrac{\lambda}{2(\mu+\lambda)} & \dfrac{2\mu+\lambda}{2(\mu+\lambda)} & 0 \\ 0 & 0 & \dfrac{\mu}{2(\mu+\lambda)} \end{bmatrix}$$

$$\frac{1}{2(\mu+\lambda)} \begin{bmatrix} \dfrac{2\mu+\lambda}{2(\mu+\lambda)} & \dfrac{\lambda}{2(\mu+\lambda)} & 0 \\ \dfrac{\lambda}{2(\mu+\lambda)} & \dfrac{2\mu+\lambda}{2(\mu+\lambda)} & 0 \\ 0 & 0 & \dfrac{\mu}{2(\mu+\lambda)} \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 2\mu+\lambda & \lambda & 0 \\ \lambda & 2\mu+\lambda & 0 \\ 0 & 0 & \mu \end{bmatrix}}$$

Part-3

$$E = E_\lambda + E_\mu$$

Stress-Strain Matrix E of plane Strain

$$E = \begin{bmatrix} 2\mu+\lambda & \lambda & 0 \\ \lambda & 2\mu+\lambda & 0 \\ 0 & 0 & \mu \end{bmatrix}$$

$$E_\lambda + E_\mu = \begin{bmatrix} \lambda & \lambda & 0 \\ \lambda & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 2\mu & 0 & 0 \\ 0 & 2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix}$$

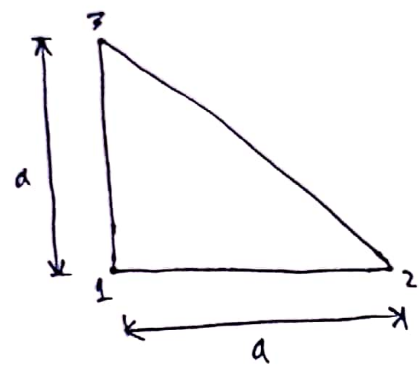
Part-d

$$E_\lambda = \lambda \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow E_\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$E_\mu = \mu \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow E_\mu = \frac{E}{2(1+\nu)} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Assignment. No. 3.3

Part-a



A plane Linear Turner Triangle

$a = a = h = 1$

h = thickness $= 1$ (Constant)

$\nu = 0$

For Turner Triangle, let's have Nodal Coordinates

Node	X	Y
1	0.0	0.0
2	1.0	0.0
3	0.0	1.0

Area $\Rightarrow$ $$2A = \det \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}$$

$$2A = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \Rightarrow \boxed{A = \frac{1}{2}}$$

we know stress-strain matrix E for plane stress

$$E = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \Rightarrow E = \begin{bmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E/2 \end{bmatrix}$$

As: Stiffness Matrix for Turner Triangle

$$K^e = \int_{\Omega^e} h B^T E B \, d\Omega = B^T E B \int_{\Omega^e} h \, d\Omega \quad , \text{for constant 'h'}$$

$$K^e = \frac{h}{4A} B^T E B = \frac{h}{4A} \begin{bmatrix} Y_{23} & 0 & x_{32} \\ 0 & x_{32} & Y_{23} \\ Y_{31} & 0 & x_{13} \\ 0 & x_{13} & Y_{31} \\ Y_{12} & 0 & x_{21} \\ 0 & x_{21} & Y_{12} \end{bmatrix} \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{12} & E_{22} & E_{23} \\ E_{13} & E_{23} & E_{33} \end{bmatrix} \begin{bmatrix} Y_{23} & 0 & Y_{31} & 0 & Y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & Y_{23} & x_{13} & Y_{31} & x_{21} & Y_{12} \end{bmatrix}$$

As $b_i = Y_j - Y_k$ and $C_i = x_k - x_j$ where $i,j,k = 1,2,3$

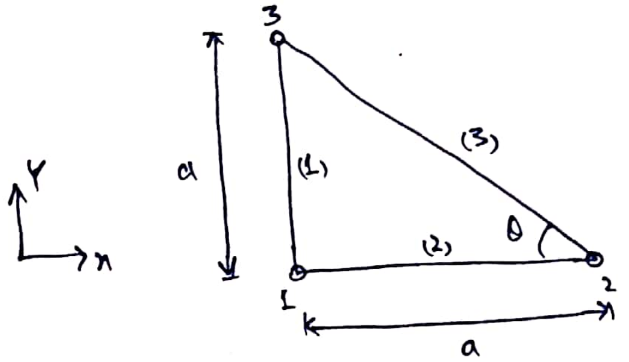
$b_1 = Y_2 - Y_3 \Rightarrow Y_{23} = -1$ | $C_1 = x_3 - x_2 \Rightarrow x_{32} = -1$

$b_2 = Y_3 - Y_1 \Rightarrow Y_{31} = 1$ | $C_2 = x_1 - x_3 \Rightarrow x_{13} = 0$

$b_3 = Y_1 - Y_2 \Rightarrow Y_{12} = 0$ | $C_3 = x_2 - x_1 \Rightarrow x_{21} = 1$

$$K^e = \frac{1}{4(\frac{1}{2})} \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E/2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$K_{Tri} = \frac{1}{2} \begin{bmatrix} \frac{3E}{2} & \frac{E}{2} & -E & -\frac{E}{2} & -\frac{E}{2} & 0 \\ \frac{E}{2} & \frac{3E}{2} & 0 & -\frac{E}{2} & -\frac{E}{2} & -E \\ -E & 0 & E & 0 & 0 & 0 \\ -\frac{E}{2} & -\frac{E}{2} & 0 & \frac{E}{2} & \frac{E}{2} & 0 \\ -\frac{E}{2} & -\frac{E}{2} & 0 & \frac{E}{2} & \frac{E}{2} & 0 \\ 0 & -E & 0 & 0 & 0 & E \end{bmatrix}$$



Three bar Elements
where $A_1 = A_2$ and A_3
$a = a = h = 1$, $\nu = 0$

Element stiffness matrix for a bar Element

$$\bar{K}^e = \frac{\bar{E}^e \bar{A}^e}{L^e} \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

where K^e = element stiffness matrix
E^e = Elastic constant , $A^e = A_1$, $L^e = a = 1 = L_1 = L_2$
$c = \cos\theta$ & $s = \sin\theta$ where $\theta = 45^\circ$ As $L_1 = L_2$

For Bar Element-1 , $\because \theta = 90^\circ$, $\because L_1 = 1$

$$K^1 = EA_1 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & EA_1 & 0 & -EA \\ 0 & 0 & 0 & 0 \\ 0 & -EA_1 & 0 & EA \end{bmatrix}$$

For Bar Element-2 $\because \theta = 0^\circ$, $\because L_2 = 1$

$$K^2 = EA_2 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} EA_2 & 0 & -EA_2 & 0 \\ 0 & 0 & 0 & 0 \\ -EA_2 & 0 & EA_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For Bar Element-3 $\because \theta = 45^\circ$, $\because L_3 = \sqrt{2}$

$$K^3 = \frac{EA_3}{\sqrt{2}} \begin{bmatrix} 0.5 & -0.5 & -0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 & 0.5 \end{bmatrix} = \begin{bmatrix} \frac{0.5EA_3}{\sqrt{2}} & \frac{-0.5EA_3}{\sqrt{2}} & \frac{-0.5EA_3}{\sqrt{2}} \cdots \\ \frac{-0.5EA_3}{\sqrt{2}} & \vdots & \vdots \\ \frac{-0.5EA_3}{\sqrt{2}} & \vdots & \vdots \\ \frac{0.5EA_3}{\sqrt{2}} & \vdots & \vdots \end{bmatrix}$$

$$K^3 = \begin{bmatrix} \frac{0.5EA_3}{\sqrt{2}} & -\frac{0.5EA_3}{\sqrt{2}} & -\frac{0.5EA_3}{\sqrt{2}} & \frac{0.5EA_3}{\sqrt{2}} \\ -\frac{0.5EA_3}{\sqrt{2}} & \frac{0.5EA_3}{\sqrt{2}} & \frac{0.5EA_3}{\sqrt{2}} & -\frac{0.5EA_3}{\sqrt{2}} \\ -\frac{0.5EA_3}{\sqrt{2}} & \frac{0.5EA_3}{\sqrt{2}} & \frac{0.5EA_3}{\sqrt{2}} & -\frac{0.5EA_3}{\sqrt{2}} \\ \frac{0.5EA_3}{\sqrt{2}} & -\frac{0.5EA_3}{\sqrt{2}} & -\frac{0.5EA_3}{\sqrt{2}} & \frac{0.5EA_3}{\sqrt{2}} \end{bmatrix}$$

Assemble all 3 into Global Matrix (Stiffness)

$$K_{bar} = \begin{bmatrix} EA_2 & 0 & -EA_2 & 0 & 0 & 0 \\ 0 & EA_1 & 0 & 0 & 0 & -EA_1 \\ -EA_2 & 0 & EA_2+\frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} & \frac{EA_3}{2\sqrt{2}} \\ 0 & 0 & -\frac{EA_3}{2\sqrt{2}} & \frac{EA_3}{2\sqrt{2}} & \frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} \\ 0 & 0 & -\frac{EA_3}{2\sqrt{2}} & \frac{EA_3}{2\sqrt{2}} & \frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} \\ 0 & -EA_1 & \frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} & EA_1+\frac{EA_3}{2\sqrt{2}} \end{bmatrix}$$

Part - b

There are no exact values of cross-sectional areas A_1, A_2 & A_3 to make bar stiffness matrix equal to triangular stiffness matrix. if we set $A_1 = A_2 = \frac{3}{4}$ and $A_3 = \frac{1}{\sqrt{2}}$ we can make diagonal terms of both stiffness matrix equal (similar to each other). That's what best we can do in this case.

Part-C i) The Stiffness matrix for bar elements (K_{bar}) have more zero than K_{tri} matrix because, bar elements are less stiff than tri elements.

ii) Bar elements only capture the response of the boundary and nodes (connecting bars) while turmer triangler are 2D element, covers the whole triangular area.

iii) The variation of angle in Bar elements are more which make bar elements less stiff in comparison to turmer triangle.

Part-d

if $\nu \neq 0$

then Stiffness matrix will be $= \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$

then

$$K_{tri} = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$K_{tri} = \begin{bmatrix} \frac{3-\nu}{2} & \frac{\nu+1}{2} & -1 & \frac{\nu-1}{2} & \frac{\nu-1}{2} & -\nu \\ \frac{\nu+1}{2} & \frac{3-\nu}{2} & -\nu & \frac{\nu-1}{2} & \frac{\nu-1}{2} & -1 \\ -1 & -\nu & 1 & 0 & 0 & \nu \\ \frac{\nu-1}{2} & \frac{\nu-1}{2} & 0 & \frac{1-\nu}{2} & \frac{1-\nu}{2} & 0 \\ \frac{\nu-1}{2} & \frac{\nu-1}{2} & 0 & \frac{1-\nu}{2} & \frac{1-\nu}{2} & 0 \\ -\nu & -1 & \nu & 0 & 0 & 1 \end{bmatrix}$$

when $\nu \neq 0$ the Stiffness matrix have very few zero So, it behaves more Stiff as compared to the matrix when $\nu = 0$.