Assignment 3

1)

1) $1+\nu = \frac{E}{2\mu}$

$$\nu = \frac{E}{2\mu} - 1$$

$$\lambda = \frac{\left(\frac{E}{2\mu}-1\right)E}{\left(\cancel{1}+\frac{E}{2\mu}\cancel{-1}\right)\left(1-2\left[\frac{E}{2\mu}-1\right]\right.}$$

$$\lambda = \frac{\cancel{\frac{E}{2\mu}}\left(E-2\mu\right)}{\cancel{\frac{E}{2\mu}}\left(1-2\left[\frac{E}{2\mu}-1\right]\right)}$$

$$\lambda = \frac{E-2\mu}{1-\frac{2E}{2\mu}+2} = \frac{(E-2\mu)2\mu}{2\mu-2E+4\mu} = \frac{\mu(E-2\mu)}{3\mu-E}$$

$$3\mu\lambda - \lambda E = \mu E - 2\mu^2$$

$$E(\lambda+\mu = \mu(3\lambda+2\mu)$$

$$E = \frac{\mu(3\lambda+2\mu)}{\lambda+\mu}$$

$$\nu = \frac{\cancel{\mu}(3\lambda+2\mu)}{2\cancel{\mu}(\lambda+\mu)} - 1$$

$$\nu = \frac{3\lambda+2\mu-2\lambda-2\mu}{2(\lambda+\mu)}$$

$$\nu = \frac{\lambda}{2(\mu+\lambda)}$$

2) Plane stress Matrix

$$\mathbb{C} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

Substatuting E and ν, with λ and μ we get :

$$\mathbb{C}_{\lambda\mu} = 2\mu\begin{bmatrix} \frac{4(\mu+\lambda)}{2\mu+\lambda} & \frac{2\lambda}{2\mu+\lambda} & 0 \\ \frac{2\lambda}{2\mu+\lambda} & \frac{4(\mu+\lambda)}{2\mu+\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3) Plane ~~stress~~ strain Matrix

$$\mathbb{C} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{(1-2\nu)}{2} \end{bmatrix}$$

substatuting E and ν

$$\mathbb{C}_{\lambda\mu} = \begin{bmatrix} 2\mu+\lambda & \lambda & 0 \\ \lambda & 2\mu+\lambda & 0 \\ 0 & 0 & \mu \end{bmatrix}$$

$$\mathbb{C}_{\lambda\mu} = \mathbb{C}_\lambda + \mathbb{C}_\mu$$

$$\mathbb{C}_\lambda = \begin{bmatrix} \lambda & \lambda & 0 \\ \lambda & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \mathbb{C}_\mu = \begin{bmatrix} 2\mu & 0 & 0 \\ 0 & 2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix}$$

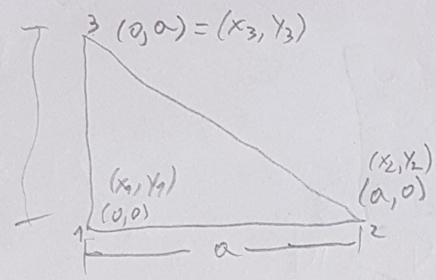
4) Express $\mathcal{E}_{\lambda}$ and $\mathcal{E}_{\mu}$ in terms of E and ν

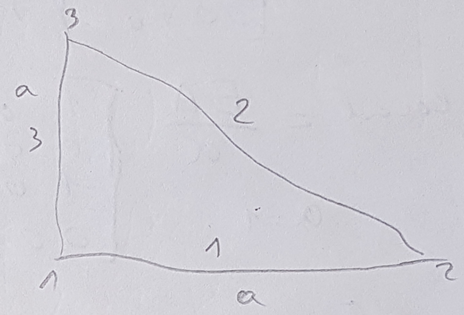
$$\mathcal{E}_{\lambda}(E,\nu) = \frac{E\nu}{(1+\nu)(1-2\nu)}\begin{bmatrix}1&1&0\\1&1&0\\0&0&0\end{bmatrix}$$

$$\mathcal{E}_{\mu}(E,\nu) = \frac{E}{(1+\nu)}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1/2\end{bmatrix}$$

$\boxed{3-2}$

$a=1,\ h=1,\ \nu=0$





$$B = DN = \frac{1}{2A_e}\begin{bmatrix}y_{23}&0&y_{31}&0&y_{12}&0\\0&x_{32}&0&x_{13}&0&x_{21}\\x_{32}&y_{32}&x_{13}&y_{31}&x_{21}&y_{12}\end{bmatrix} \quad \begin{cases}y_{ij} = y_i - y_j\\ x_{ij} = x_i - x_j\end{cases}$$

$$A_e = \frac{1}{2}\det\begin{bmatrix}1&1&1\\x_1&x_2&x_3\\y_1&y_2&y_3\end{bmatrix}$$

$$E = \frac{E}{(1-\nu)^2}\begin{bmatrix}1&\nu&0\\\nu&1&0\\0&0&\frac{1-\nu}{2}\end{bmatrix}$$

$$K^e_{tri} = \int_{\Omega} h\,B^T E B\, d\Omega = h\,B^T E B\int_0^{A_e} d\Omega = h\,B^T E B A_e$$

$$K^e_{tri} = h\,B^T E B A_e$$

$$k_{tri} = \begin{bmatrix} \frac{3E}{4} & \frac{E}{4} & -\frac{E}{2} & -\frac{E}{4} & -\frac{E}{4} & 0 \\ \frac{E}{4} & \frac{3e}{4} & 0 & -\frac{E}{4} & -\frac{e}{4} & -\frac{E}{2} \\ -\frac{E}{2} & 0 & \frac{E}{2} & 0 & 0 & 0 \\ -\frac{E}{4} & -\frac{E}{4} & 0 & \frac{E}{4} & \frac{E}{4} & 0 \\ -\frac{E}{4} & -\frac{E}{4} & 0 & \frac{e}{4} & \frac{E}{4} & 0 \\ 0 & -\frac{E}{2} & 0 & 0 & 0 & \frac{E}{2} \end{bmatrix} \leftarrow \begin{cases} a = 1 \\ h = 1 \\ \nu = 0 \end{cases}$$

$$K_1 local = \frac{EA}{a} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad K_2\, local = \frac{\sqrt{2}\, EA}{2a} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K_3\, local = \frac{EA}{a} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$a = 1$

$$K_1 = T_1^T\, k_1 local\, T_1 = EA \begin{array}{c} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \end{array}$$

$$K_2 = T_2^T\, k_2\, local\, T_2 = \frac{\sqrt{2}\, EA}{4} \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{array}{l} u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \end{array}$$

$$K_3 = T_3^T\, k_3\, local\, T_3 = EA \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{array}{l} u_{x_3} \\ u_{y_3} \\ u_{x_1} \\ u_{y_1} \end{array}$$

$$K_1G = EA \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \end{array}$$

$$K_2G = \frac{EA\sqrt{2}}{4} \begin{array}{c} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & -1 & 1 & 1 & -1 \\ 0 & 0 & -1 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix} \end{array} \begin{array}{l} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \end{array}$$

$$K_3G = EA \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$K bar = K_{1G} + K_{2G} + K_{3G} = K_{iG} \in \{1,2,3\}$$

$$K bar = \begin{bmatrix} EA & 0 & -EA & 0 & 0 & 0 \\ 0 & EA & 0 & 0 & 0 & -EA \\ -EA & 0 & \frac{EA(\sqrt{2}+4)}{4} & -\frac{EA\sqrt{2}}{4} & -\frac{\sqrt{2}EA}{4} & \frac{\sqrt{2}EA}{4} \\ 0 & 0 & -\frac{\sqrt{2}EA}{4} & \frac{\sqrt{2}EA}{4} & \frac{\sqrt{2}EA}{4} & -\frac{\sqrt{2}EA}{4} \\ 0 & 0 & -\frac{\sqrt{2}EA}{4} & \frac{\sqrt{2}EA}{4} & \frac{\sqrt{2}EA}{4} & -\frac{\sqrt{2}EA}{4} \\ 0 & -EA & \frac{\sqrt{2}EA}{4} & -\frac{\sqrt{2}EA}{4} & -\frac{\sqrt{2}EA}{4} & \frac{EA(\sqrt{2}+4)}{4} \end{bmatrix}$$

2) No; It $A_1 = A_3 = \frac{3}{4}$ $A_2 = \frac{\sqrt{2}}{2}$

3) The triangular matrix has more non-zero terms, meaning that the DOF are more intricate than the truss bar global matrix.
In the triangular stiffness matrix almost all elements depends on of first line contributes.

ii) $\nu \neq 0$

The bar, truss is not affected by this proposition.

The increase of ν to 1 make several position of ~~the~~ matrix $\to \infty$.

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \end{bmatrix}$$

$$\sigma_{xx} = \frac{E}{1-\nu^2} \left[\varepsilon_{xx} + \nu \varepsilon_{yy} \right] \to \nu = 0 \to \sigma_{xx} = E\,\varepsilon_{xx}$$

But $\nu \neq 0$ the longtitudial component (ε_{xx}) play a bigger role in composition to $\nu = 0$ and the transverse component starts to get important. Let's say $\nu = 0.2$

$$\sigma_{xx_2} = \frac{E}{1-0{,}04} \left[\varepsilon_{xx} + 0.2\, \varepsilon_{yy} \right]$$

$$\sigma_{xx_2} = E \left[1{,}042\, \varepsilon_{xx} + 0{,}208\, \varepsilon_{yy} \right]$$

For the same strains, stresses with higher poisson ratio are also higher.

For same stresses the strains are smaller.

If $\nu > 1$ several terms of the matrix diagonal becomes negative.