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2.3 Assignment

a)
$$K^e = \int_0^l \frac{EA}{l^2} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} l \, d\xi \quad E \text{ is constant}$$

$$K^e = E \int_0^1 \frac{[A_i(1-\xi) + A_j \xi]}{l^2} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} l \, d\xi$$

$$K^e = \frac{E}{l} \cdot \int_0^1 [A_i \ A_j] \cdot \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} d\xi$$

$$K^e = \frac{E}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \int_0^1 (A_i(1-\xi) + A_j \xi) \, d\xi =$$

$$= -\frac{A_i(1-\xi)^2}{2} + \frac{A_j \xi^2}{2} \Big|_0^1$$

$$K^e = \frac{E}{l}\left(\frac{A_i + A_j}{2}\right) \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad = \frac{A_j}{2} + \frac{A_i}{2} = \frac{A_i + A_j}{2}$$

$$K^e = \frac{E \cdot A'}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad \text{where } A' = \frac{A_i + A_j}{2}$$

then K^e is the same as if the cross section was constant with area $A' = \frac{A_i + A_j}{2}$

b)
$$f^e = \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} d\xi = \int_0^1 \rho g [A_i(1-\xi) + A_j \xi] \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} d\xi$$

$$= \rho g \int_0^1 \begin{bmatrix} A_i(1-\xi)^2 + A_j \xi(1-\xi) \\ A_i(1-\xi)\xi + A_j \xi^2 \end{bmatrix} d\xi$$

$$= \rho g \begin{bmatrix} -\frac{A_i(1-\xi)^3}{3} + A_j\left(\frac{\xi^2}{2} - \frac{\xi^3}{3}\right) \\ A_i\left(\frac{\xi^2}{2} - \frac{\xi^3}{3}\right) + \frac{A_j \xi^3}{3} \end{bmatrix} \Bigg|_0^1$$

$$f^{l} = \rho g \begin{bmatrix} A_j\left(\frac{1}{2}-\frac{1}{3}\right) + A_i \cdot \frac{1}{3} \\ A_i\left(\frac{1}{2}-\frac{1}{3}\right) + A_j \cdot \frac{1}{3} \end{bmatrix} =$$

$$f^{l} = \rho g \begin{bmatrix} \frac{A_i}{3} + \frac{A_j}{6} \\ \frac{A_i}{6} + \frac{A_j}{3} \end{bmatrix} = 3 \rho g \begin{bmatrix} A_i + \frac{A_j}{2} \\ \frac{A_i}{2} + A_j \end{bmatrix}$$

- If $A_i = A_j = A$

$$f^{l} = \rho g \begin{bmatrix} \frac{A}{2} \\ \frac{A}{2} \end{bmatrix} = \frac{1}{2} \rho g \begin{bmatrix} A \\ A \end{bmatrix}$$

- if $A_j = 0$

$$f^{l} = \rho g \begin{bmatrix} \frac{A_i}{3} \\ \frac{A_i}{6} \end{bmatrix} = 3 \rho g \begin{bmatrix} A_i \\ \frac{A_i}{2} \end{bmatrix}$$

c)