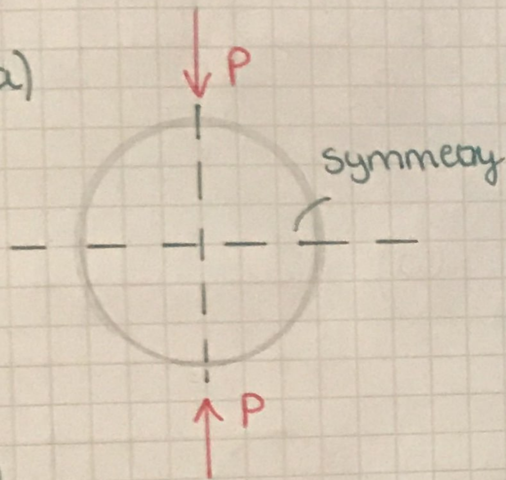
Assignment 2 - Computational Structural Mechanics and Dynamics

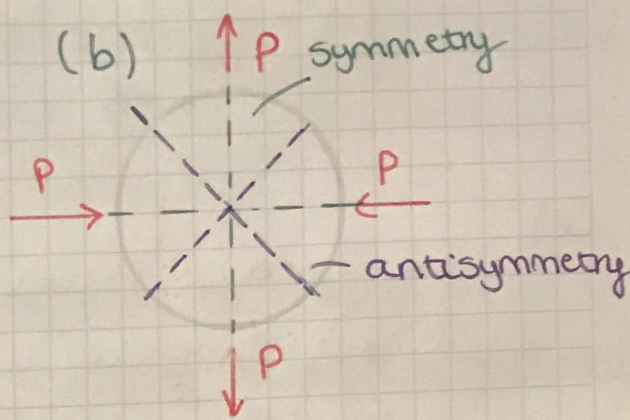
Assignment 2.1

1)

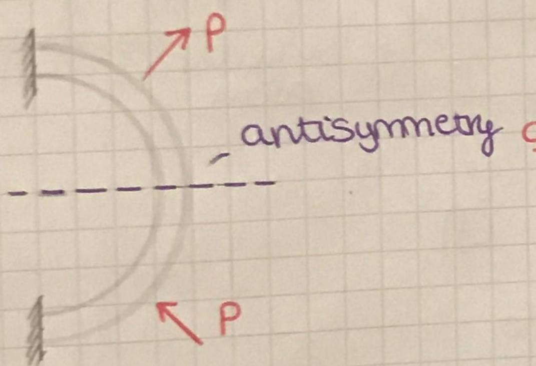
(a)



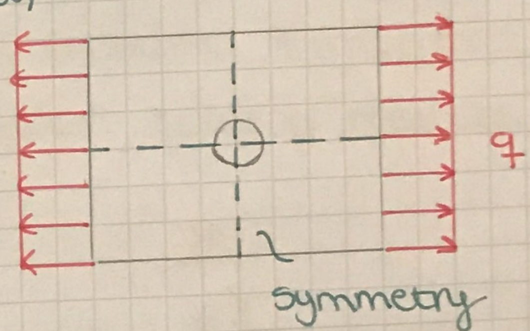
(b)



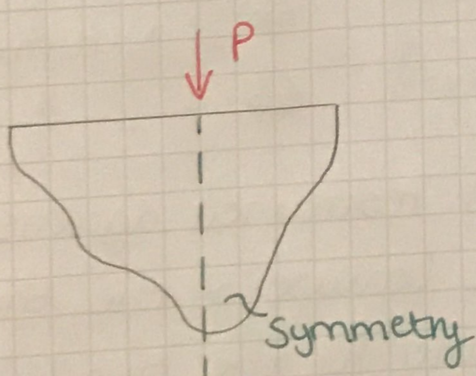
(c)



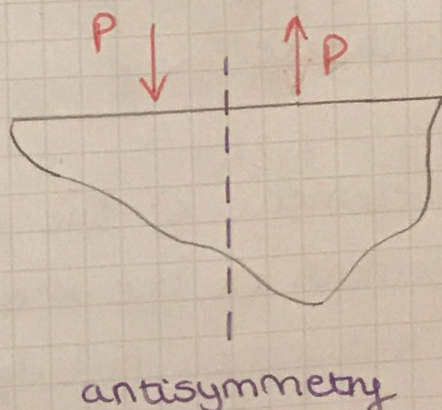
(d)



(e)

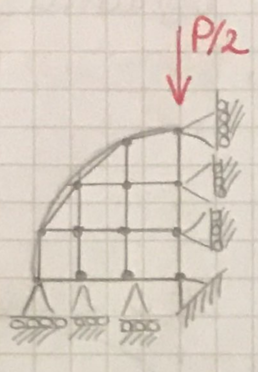


(f)

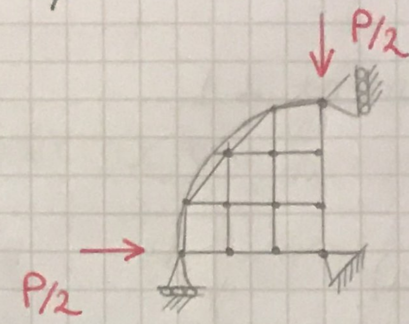


2)

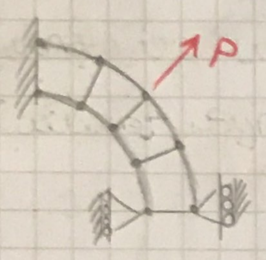
(a)



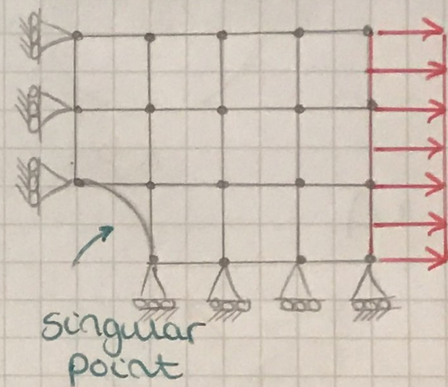
(b)



(c)



(d)



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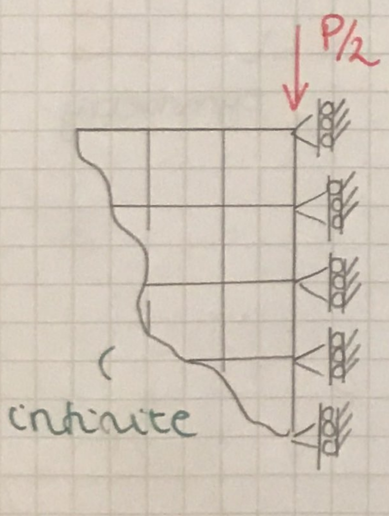

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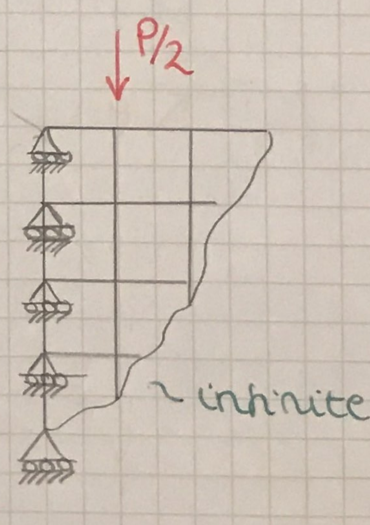


For element c) the part is restrained against movement in the x-direction because of the the forces in x-direction nullify eachother for the two halves.

Same concept for element (f), but in y-direction.

Assignment 2.2

Difference between "Verification" and "Validation":

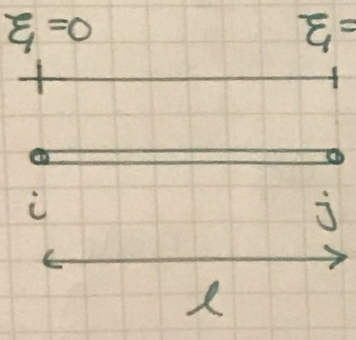
The verification is mostly related to the process of the Finite Element Analysis. This means to verify many of the steps in the analysis. The purpose is to provide a FEA that gives accurate solution for the given task.

The validitation is more related to making the FEA do what is supposed to do. To provide a realistic model that takes into account the physical features of the actual problem. A important part is to check the FEA against testing results.

Assignment 2.3

A tapered bar element of length ℓ and areas A_i and A_j interpolated as:

$$A = A_i(1-\xi) + A_j\xi$$

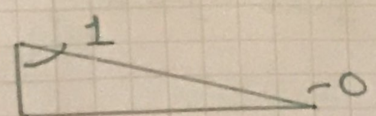


Consistent node force

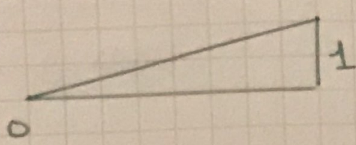
$$\Rightarrow f_{ext} = \int_0^{\ell} q(x) \cdot N(x)^T \, dx$$

Shape functions

Shape functions:



$\Rightarrow N_i = 1-\xi \qquad \Rightarrow N(\xi) = [1-\xi \quad \xi]$



$\Rightarrow N_j = \xi$

$$\xi_1 = \frac{x}{l} \quad (\xi_1 = 0 \Rightarrow x = 0 \ , \ \xi_1 = 1 \Rightarrow x = l)$$

$$\underline{x = \xi_1 \cdot l} \Rightarrow \frac{dx}{d\xi_1} = l \Rightarrow dx = d\xi_1 \cdot l$$

$$q(x) = A\rho\omega^2 x = \rho\omega^2 (A_i(1-\xi_1) + A_j \xi_1) \cdot \xi_1 \cdot l$$

$$q(\xi_1) = \rho\omega^2 (A_i(\xi_1 - \xi_1^2) + A_j \xi_1^2) \cdot l$$

$$\mathbb{f}_{ext} = \int_0^l q(x)\, \mathbb{N}^T(x)\, dx = \int_0^1 q(\xi_1) \cdot \mathbb{N}^T(\xi_1) \cdot d\xi_1 \cdot l$$

$$= \rho\omega^2 \cdot l^2 \int_0^1 (A_i(\xi_1 - \xi_1^2) + A_j \cdot \xi_1^2) \begin{bmatrix} 1-\xi_1 \\ \xi_1 \end{bmatrix} d\xi_1$$

$$= \rho\omega^2 \cdot l^2 \int_0^1 \begin{bmatrix} A_i(\xi_1 - \xi_1^2) + A_j \xi_1^2 - A_i(\xi_1^2 - \xi_1^3) - A_j \xi_1^3 \\ A_i(\xi_1^2 - \xi_1^3) + A_j \xi_1^3 \end{bmatrix}$$

$$= \rho\omega^2 \cdot l^2 \int_0^1 \begin{bmatrix} A_i(\xi_1^3 - 2\xi_1^2 + \xi_1) + A_j(\xi_1^2 + \xi_1^3) \\ A_i(\xi_1^2 - \xi_1^3) + A_j \xi_1^3 \end{bmatrix} d\xi_1$$

$$\underline{\underline{= \rho\omega^2 \cdot l^2 \begin{bmatrix} \frac{1}{12} A_i + \frac{1}{12} A_j \\ \frac{1}{12} A_i + \frac{1}{4} A_j \end{bmatrix}}} \Rightarrow A_i = A_j \Rightarrow \mathbb{f}_{ext} = \rho\omega^2 l^2 \begin{array}{l} \text{node } i \\ \begin{bmatrix} 1/6 \cdot A \\ 1/3 \cdot A \end{bmatrix} \\ \text{node } j \end{array}$$