Computational Structural Mechanics and Dynamics

As2 FEM Modelling:Introduction and Variational formulation

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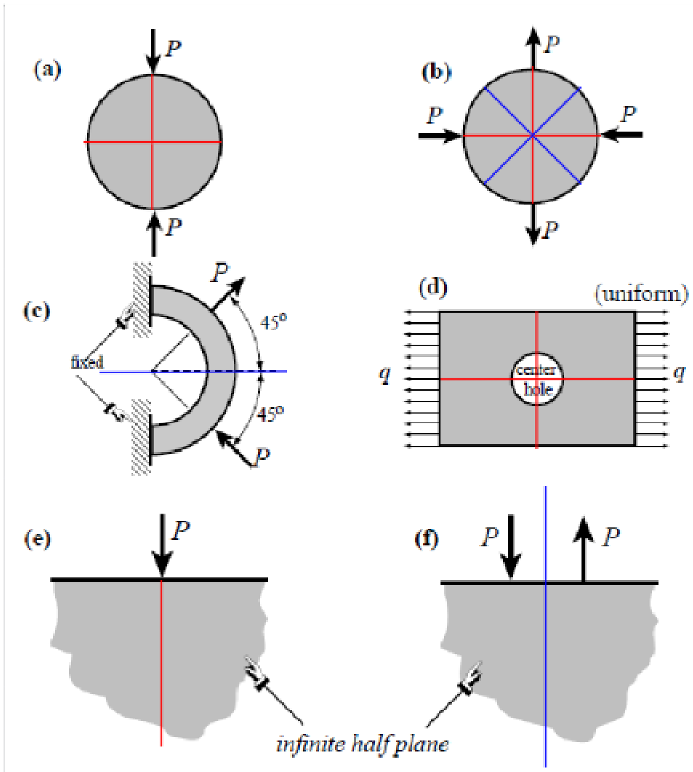
Assignment 2.1

1. **Identify the symmetry and antisymmetry line in the two-dimensional problems illustrated in the figure. They are:**
   (a) **A circular disk under two diametrically opposite point force (the famous "Brazilian test" for concrete)**
   (b) **The same disk under two diametrically opposite force pairs.**
   (c) **A clamped semiannulus under a force pair oriented as shown.**
   (d) **A stretched rectangular plate with a central circular hole.**
   (e) **And (f) are half-planes under concentrated loads.**

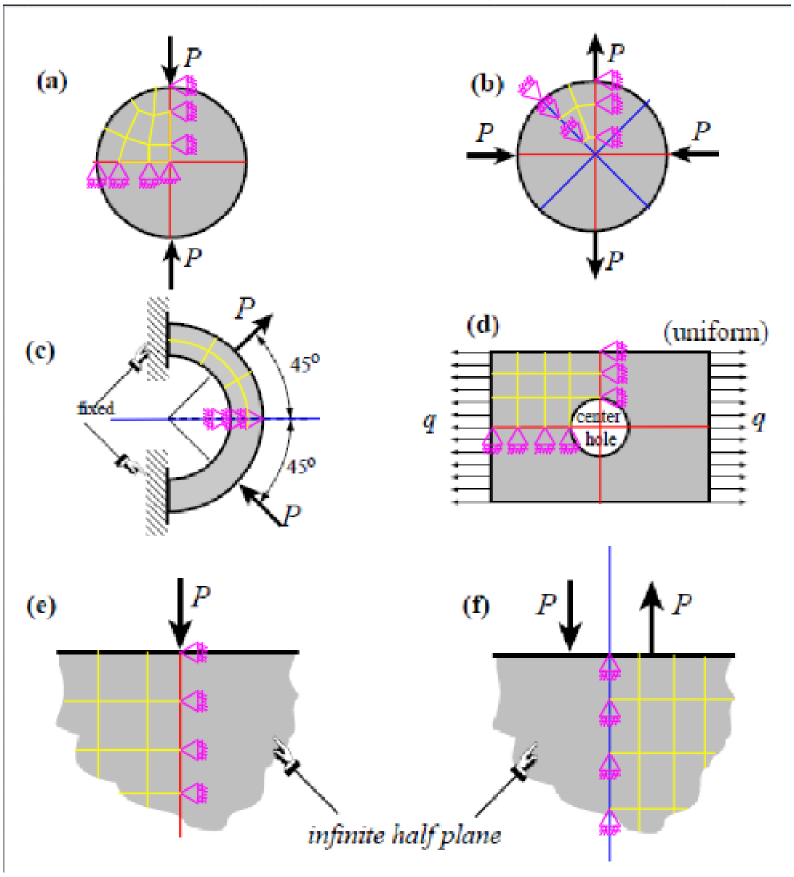
   [Answer]

   Symmetry lines are marked with red while antisymmetry are marked with blue.



2. **Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before lying out a finite element mesh. Then draw a coarse FE mesh indicating, with roller or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.**

   [Answer]



Assignment 2.2

Explain the difference between “verification” and “Validation” in the context of FEM-Modelling procedure.

[Answer]

Verification guarantees that the answer converged for the finite element (FE) solution of the model.

validation is needed to show that the model is truly representative of the problem at hand.

The FE solution must be verified and the model must be validated in order to guarantee a reliable analysis.

Assignment 2.3

A tapered bar element of length l and aeras A_i and A_j with A interpolated as $A = A_i(1-\xi) + A_j\xi$ and constant axis x along the rotating bar with origin at node i, the centrifugal axial force is $q(x) = \rho A \omega^2 x$ along the length in which x is the longitudinal coordinate $x = x^e$.

Find the consistent node force as function of ρ, A_i, A_j, ω and l. And specialize the result to the prismatic bar $A = A_i = A_j$

[Answer]

First, the ξ coordinate is expressed in terms of the element coordinate x:

$$\xi = \frac{x}{l}$$

Then the unknown variables are expressed in function of ξ

$$N = \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix}$$

The force vector is calculated as follows:

$$f = \int_0^1 N \cdot q \cdot J^{-1}\, d\xi = \int_0^1 \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} \cdot \rho(A_i(1-\xi) + A_j\xi)\omega^2 \xi l \cdot l\, d\xi$$

$$= \rho\omega^2 l^2 \int_0^1 A_i \begin{bmatrix} (1-\xi)^2\xi \\ \xi^2(1-\xi) \end{bmatrix} + A_j \begin{bmatrix} \xi^2(1-\xi) \\ \xi^3 \end{bmatrix} d\xi$$

$$= \rho\omega^2 l^2 (A_i \begin{bmatrix} \frac{1}{12} \\ \frac{1}{12} \end{bmatrix} + A_j \begin{bmatrix} \frac{1}{12} \\ \frac{1}{4} \end{bmatrix})$$

For the case of prismatic bar, $A = A_i = A_j$:

$$f = \rho\omega^2 l^2 A \begin{bmatrix} 1/6 \\ 1/3 \end{bmatrix}$$