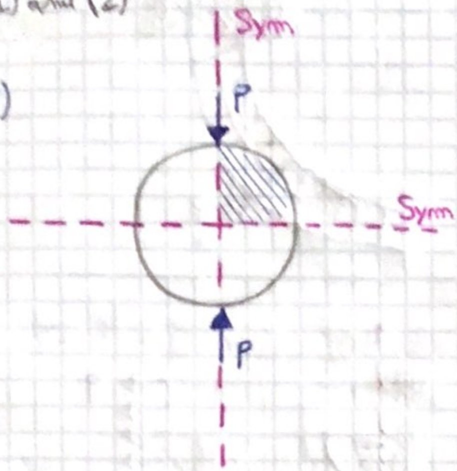
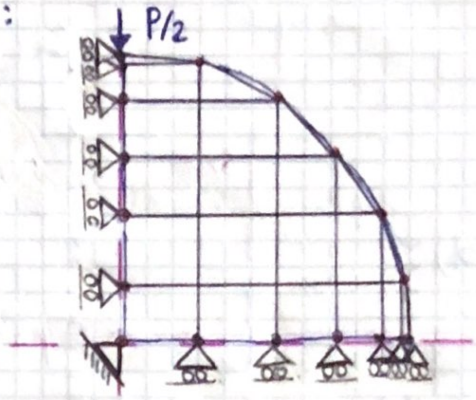
(1) and (2)

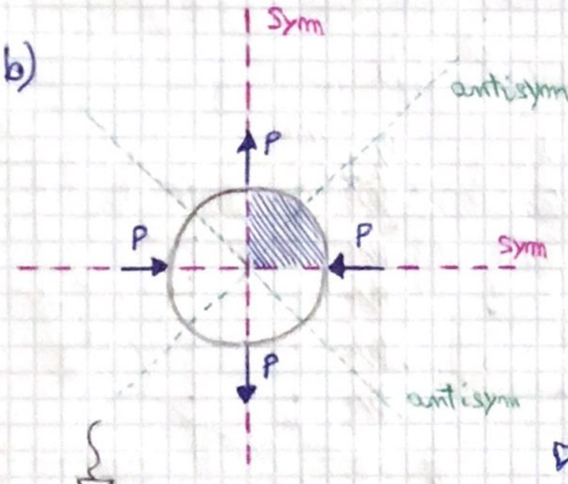
a)



It's possible to cut the structure to one quarter :

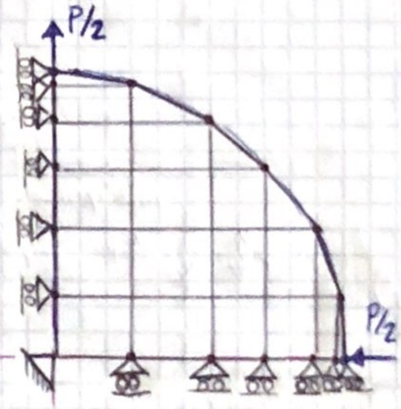


b)



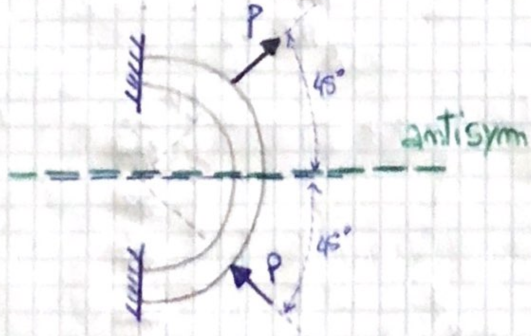
It's possible to cut the structure to one quarter :

or

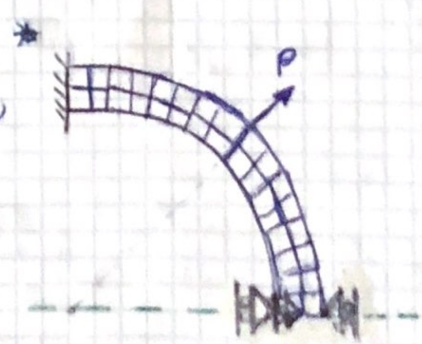


Here there are two antisymmetric lines too.
So I could model it as divided in 8 parts and focus only on one of them. F.E.:

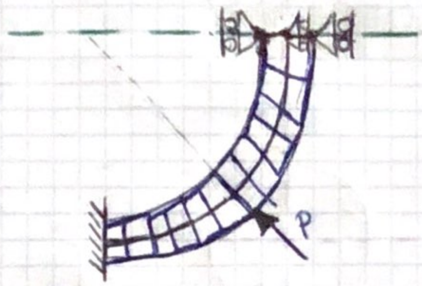
c)



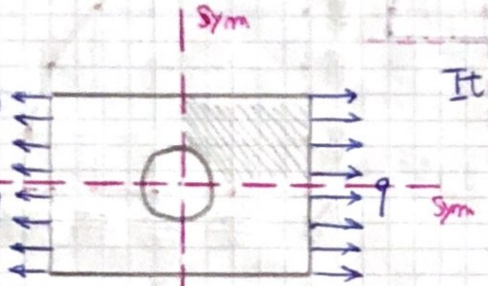
It's possible to cut the structure to one half :



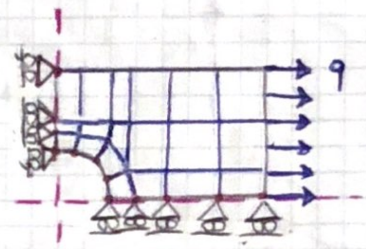
or: **



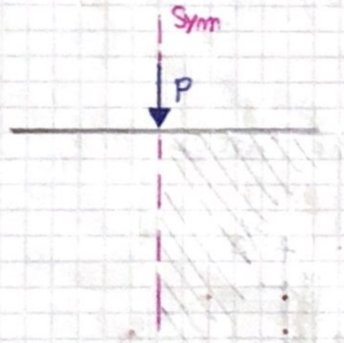
d)



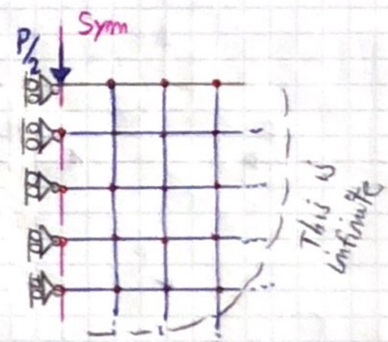
It can be divided into a quarters:



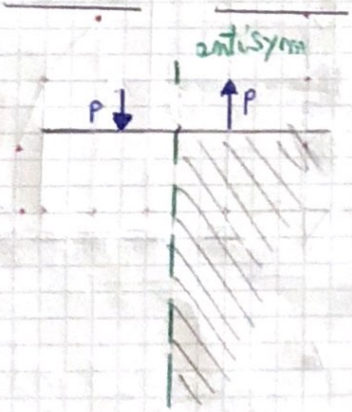
e)



It can be divided into two halfs:

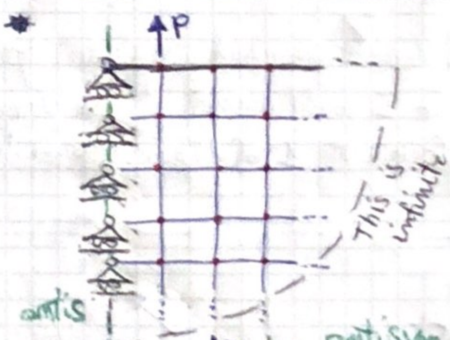


f)

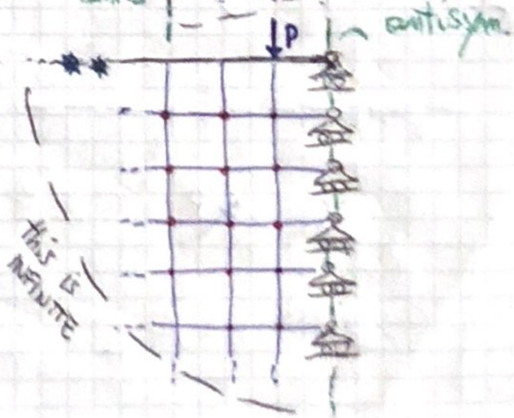


It can be divided into two halfs

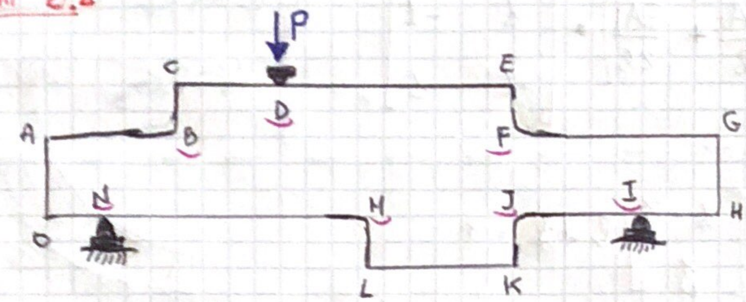
*



or: **



Assignment 2.2



B - F - J - M] → all are entrant corners, regions where the isostatic stresses gather.

N - I] → vicinity of sharp (pointy) contact areas.

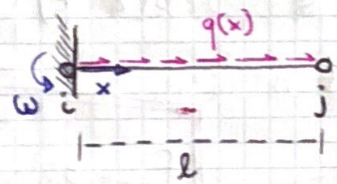
D] → vicinity of concentrated load.

Assignment 2.3

for simplicity

$$A = A(\xi) = A_i(1-\xi) + A_j\,\xi \quad , \quad \xi = \frac{x - x_i}{\ell} \; (x_i \neq 0)$$

$$q(x) = \rho A\,\omega^2 x$$



I write down the shape forms : $N_1^e = 1 - \frac{x}{\ell}$ $\quad N_2^e = \frac{x}{\ell}$ $\quad \underline{N}^e = [N_1^e \;\; N_2^e]$

So that $\underline{u}(x) = \underline{N}^e\,\underline{u}^e = \left(1 - \frac{x}{\ell}\right) u_i^e + \frac{x}{\ell} u_j^e$

once derivated, $\underline{\varepsilon}(x) = \frac{d\underline{u}}{dx} = -\frac{u_i^e}{\ell} + \frac{u_j^e}{\ell} = \underline{B}\,\underline{u}^e$, $\underline{B} = \left[\frac{dN_1^e}{dx} \;\; \frac{dN_2^e}{dx}\right] = \frac{1}{\ell}[-1 \;\; 1]$

1) the bar stiffness matrix.

$$\underline{K}^e = \int_0^\ell EA\,\underline{B}^T\underline{B}\,dx = \frac{E}{\ell^2}\int_0^\ell \left[A_i\left(1 - \frac{x}{\ell}\right) + A_j\frac{x}{\ell}\right] \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} dx$$

$$= \frac{E}{\ell^2}\left[A_i \cdot \ell - A_i\frac{\ell^2}{2\ell} + A_j\frac{\ell^2}{2\ell}\right] \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} dx =$$

EXTRA WORK

$$\underline{K}^e = \left(\frac{EA_i}{2l} + \frac{EA_j}{2l}\right) \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

EXTRA WORK

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2) $$\underline{f}_{ext} = \int_0^1 q(\xi) \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l \, d\xi = *$$

$(x_i = \emptyset)$

$$q(x) = \rho A \omega^2 x \quad \text{if } \xi = \frac{x - x_i}{l} \quad q(\xi) = \rho l A(\xi) \omega^2 \xi$$

$$= \rho l \omega^2 \cdot [A_i (1-\xi) + A_j \xi] \xi$$

$$* = \int_0^1 \rho l^2 \omega^2 \begin{bmatrix} A_i(\xi - 2\xi^2 + \xi^3) + A_j(\xi^2 - \xi^3) \\ A_i(\xi^2 - \xi^3) + A_j(\xi^3) \end{bmatrix}$$

$$= \rho l^2 \omega^2 \begin{bmatrix} A_i\left[\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right] + A_j\left[\frac{1}{3} - \frac{1}{4}\right] \\ A_i\left[\frac{1}{3} - \frac{1}{4}\right] + A_j \frac{1}{4} \end{bmatrix}$$

$$\boxed{\underline{f}_{ext} = \rho l^2 \omega^2 \begin{bmatrix} \frac{A_i}{12} + \frac{A_j}{12} \\ \frac{A_i}{12} + \frac{A_j}{4} \end{bmatrix}}$$

if $A = A_i = A_j$

$$\boxed{\underline{f}_{ext} = \rho l^2 \omega^2 A \begin{bmatrix} 1/6 \\ 1/3 \end{bmatrix}}$$

[answer to question 2.3]

So the problem is:

$$\underline{K}^e \underline{u}^e = \underline{f}^e$$

$$\left(\frac{EA_i}{2l} + \frac{EA_j}{2l}\right) \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_i^e \\ u_j^e \end{Bmatrix} = \rho l^2 \omega^2 \begin{bmatrix} \frac{A_i}{12} + \frac{A_j}{12} \\ \frac{A_i}{12} + \frac{A_j}{4} \end{bmatrix}$$

I consider the BC, so $\hat{K}^e u_j^e = \hat{f}^e$

$$\left(\frac{EA_i}{2l} + \frac{EA_j}{2l}\right) u_j^e = \rho l^2 \omega^2 \left[\frac{A_i}{12} + \frac{A_j}{4}\right]$$

and I solve the equation

EXTRA WORK