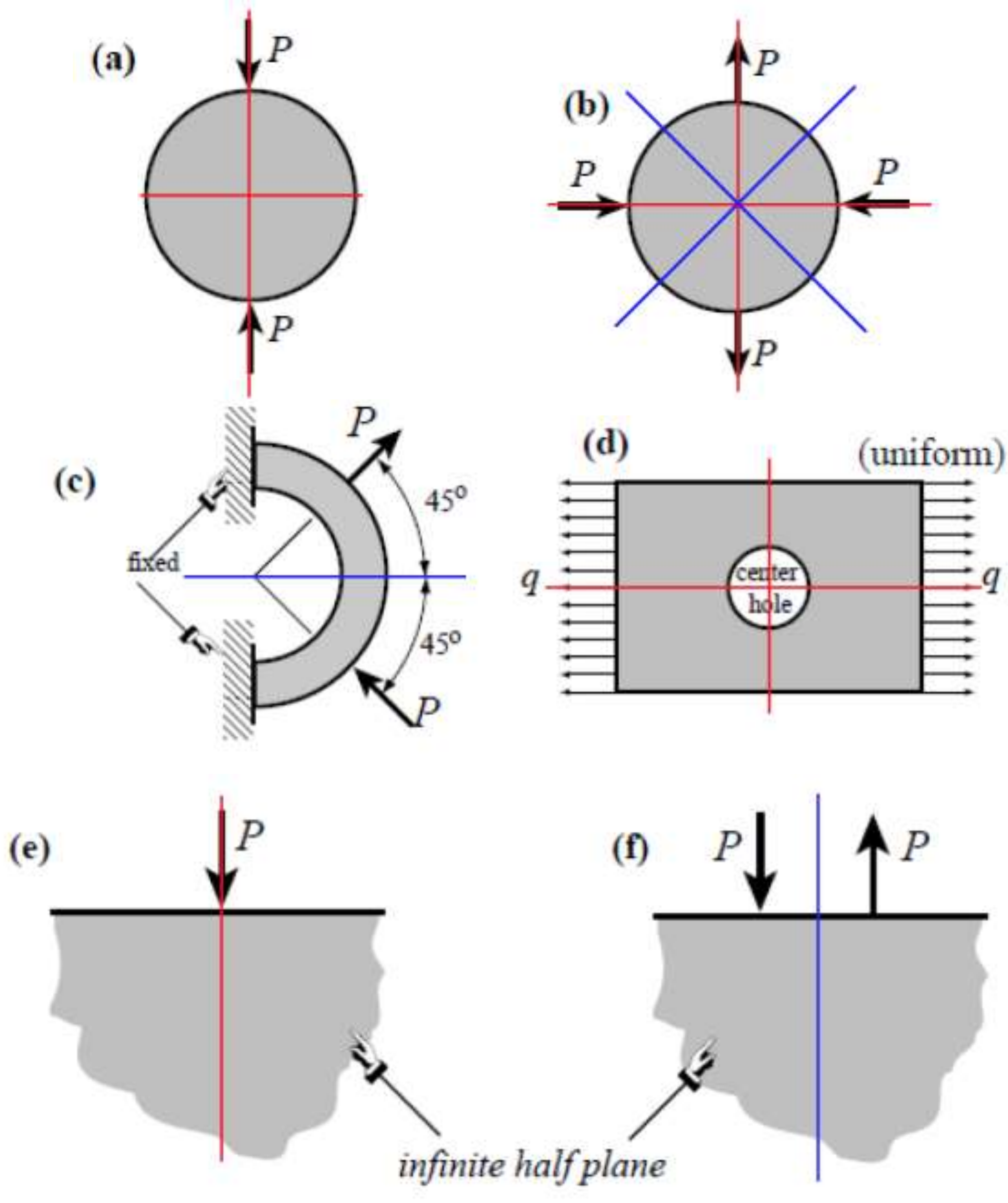
COMPUTATIONAL SUCTURAL MECHANICS AND DYNAMICS
Master of Science in Computational Mechanics/Numerical Methods
Spring Semester 2019

Rafel Perelló i Ribas

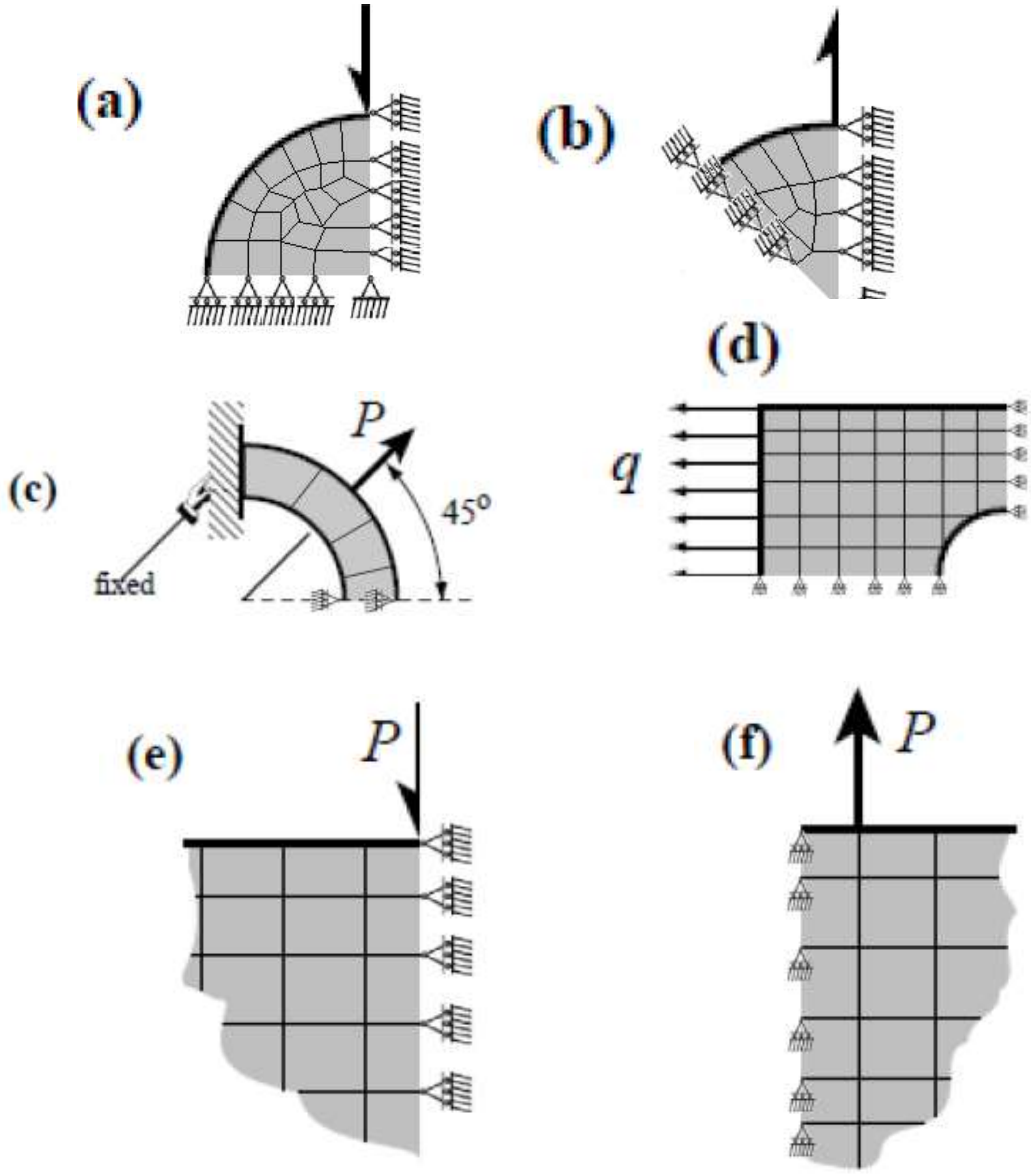
Assignment 2: FEM Modelling: Introduction

1. **Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in the figure. They are:**

Symmetry lines are marked with red lines while antisymmetry lines are marked in blue.



2. **Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.**

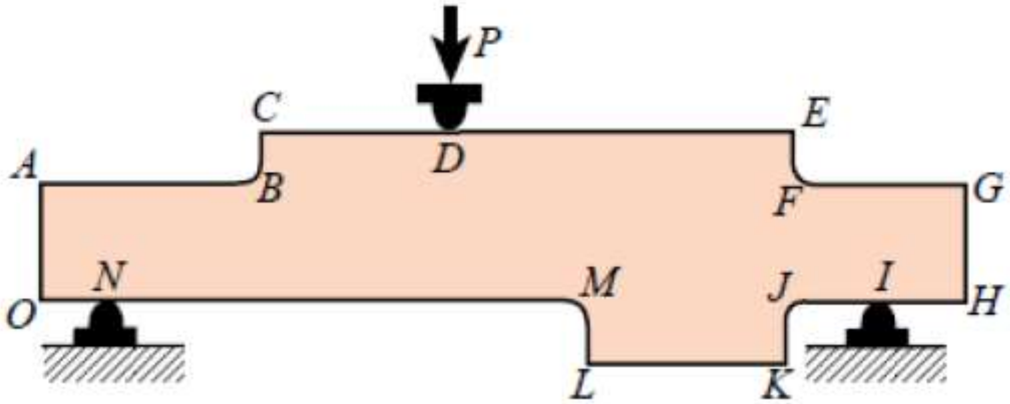


Here, in the a), b) and e) cases, the force to apply is P/2.

3. The plate structure shown in the figure is loaded and deforms in the plane of the paper. The applied load at *D* and the supports *I* and *N* extend over a fairly narrow area. List what you think are the likely "trouble spots" that would require a locally finer element mesh to capture high stress gradients. Identify those spots by its letter and a reason.

The stress concentration may be due to two causes:

- **Point loads or loads applied on a narrow area:** This includes the points *D*, *N* and *I.*
- **Non-convex domains:** For that reason, the mesh should be refined around *B, F, M* and *J.*



4. A tapered bar element of length *l* and areas A_i and A_j with *A* interpolated as:

$$A = A_i(1-\xi) + A_j\xi$$

And constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node *i*. Taking axis x along the rotating bar with origin at node *I*, the centrifugal axial force is $q(x) = \rho A \omega^2 x$ along the length in which x is the longitudinal coordinate $x = x^e$.
Find the consistent node forces as functions of ρ, A_i, A_j, ω and *l*, and specialize the result to the prismatic bar $A = A_i = A_j$.

First of all, the ξ coordinate is expressed in terms of the element coordinate *x*:

$$\xi = \frac{x}{l}$$

Now, the unknown variables are expressed in function of ξ:

$$\boldsymbol{N} = \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix}$$

The force vector is calculated as follows:

$$\boldsymbol{f} = \int_0^1 \boldsymbol{N} \cdot q \cdot J^{-1} d\xi = \int_0^1 \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} \rho\big(A_i(1-\xi) + A_j\xi\big)\omega^2 \xi \cdot l \cdot l \, d\xi$$

$$= \rho\omega^2 l^2 \int_0^1 A_i \begin{bmatrix} (1-\xi)^2\xi \\ \xi^2 - \xi^3 \end{bmatrix} + A_j \begin{bmatrix} \xi^2 - \xi^3 \\ \xi^3 \end{bmatrix} d\xi$$

$$= \rho\omega^2 l^2 \left(A_i \left(\begin{bmatrix} 1/12 \\ 1/12 \end{bmatrix} \right) + A_j \left(\begin{bmatrix} 1/12 \\ 1/4 \end{bmatrix} \right) \right)$$

For the case of the prismatic bar:

$$\boldsymbol{f} = \rho\omega^2 l^2 A \left(\begin{bmatrix} 1/6 \\ 1/3 \end{bmatrix} \right)$$