INTERNATIONAL CENTRE FOR
NUMERICAL METHODS IN ENGINEERING

UNIVERSITAT POLITÈCNICA DE CATALUNYA

MASTER OF SCIENCE IN COMPUTATIONAL MECHANICS

Computational Structural Mechanics and Dynamics

Assignment 2

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ASSIGNMENT 2.1

On “FEM Modelling”:

1.- Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in the figure. They are:

a) *A circular disk under two diametrically opposite point forces (the famous “Brazilian test” for concrete)*

b) *The same disk under two diametrically opposite force pairs*

c) *A clamped semiannulus under a force pair oriented as shown*

d) *A stretched rectangular plate with a central circular hole.*

e) *and f) are half-planes under concentrated loads.*

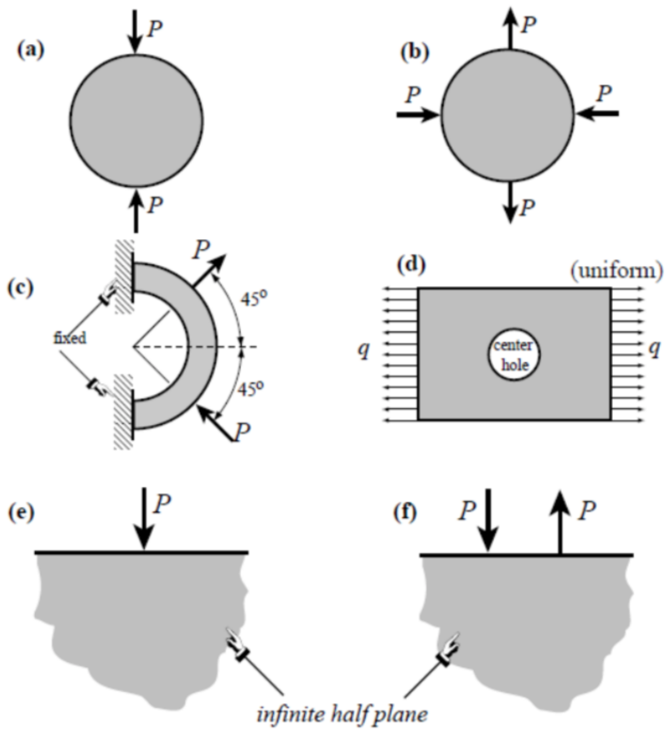

Figure 0.1: Problems for assignment 2.1

Taking into account symmetry and antisymmetry conditions over a problem, it can be reduced the time of calculation in a matricial formulation implemented in a computer as is the FEM. The symmetry condition (in 2D) can be recognized as a mirror of 180º about the symmetry line considering geometry, forces and boundary conditions. In order to reduce the model, some new BCs may be included to correctly imposed how the body will behave after the deformation process and to be consistent with the physics of the problem. In other hand, the antisymmetry condition it is characterized by the "mirror force" in negative direction, so the model reduction and the imposed BCs can be more difficult to reach. The symmetry and antisymmetry lines are depicted next, which can be used to correctly reduce the problems.

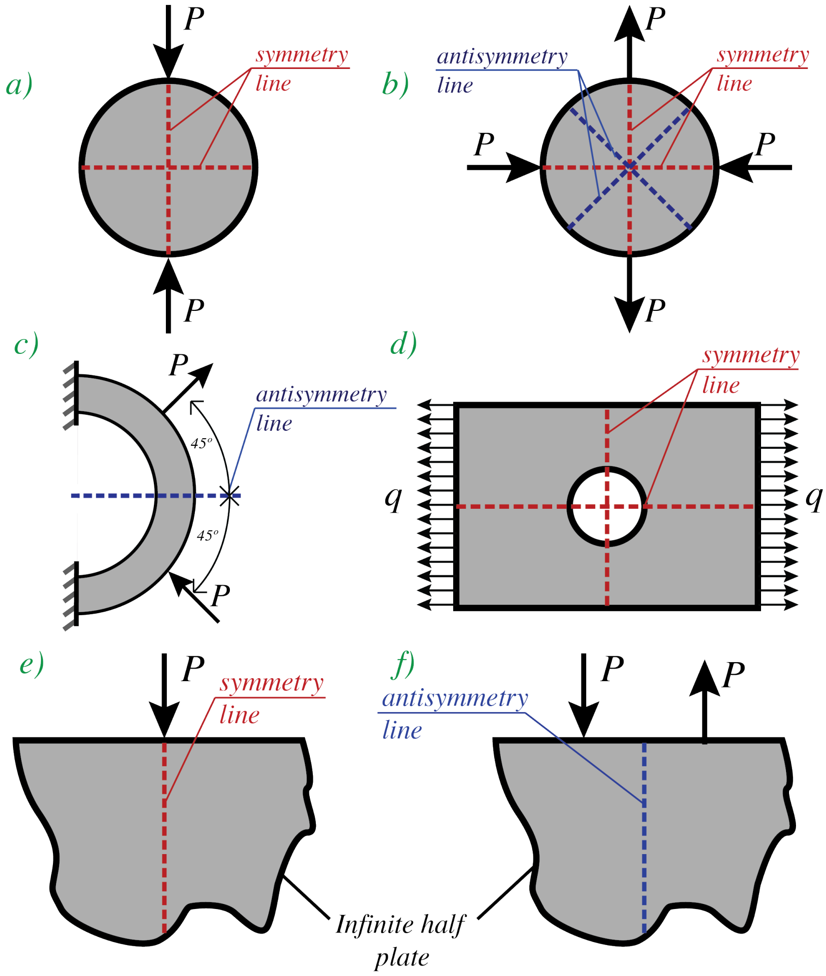

Figure 0.2: Solutions for assignment 2.1.1 - Symmetry / Antisymmetry lines.

2.- Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.

In order to imposed boundary conditions, a coarse discretization is proposed as a conceptual example. Taking into account the symmetry and antisymmetry lines from the previous exercise, the problems are reduced to a quarter or a half of the entire body, also in the corresponding FEM nodes it is located a type of support, and finally, the forces that are in between a symmetry line are considered as a half of the magnitude.

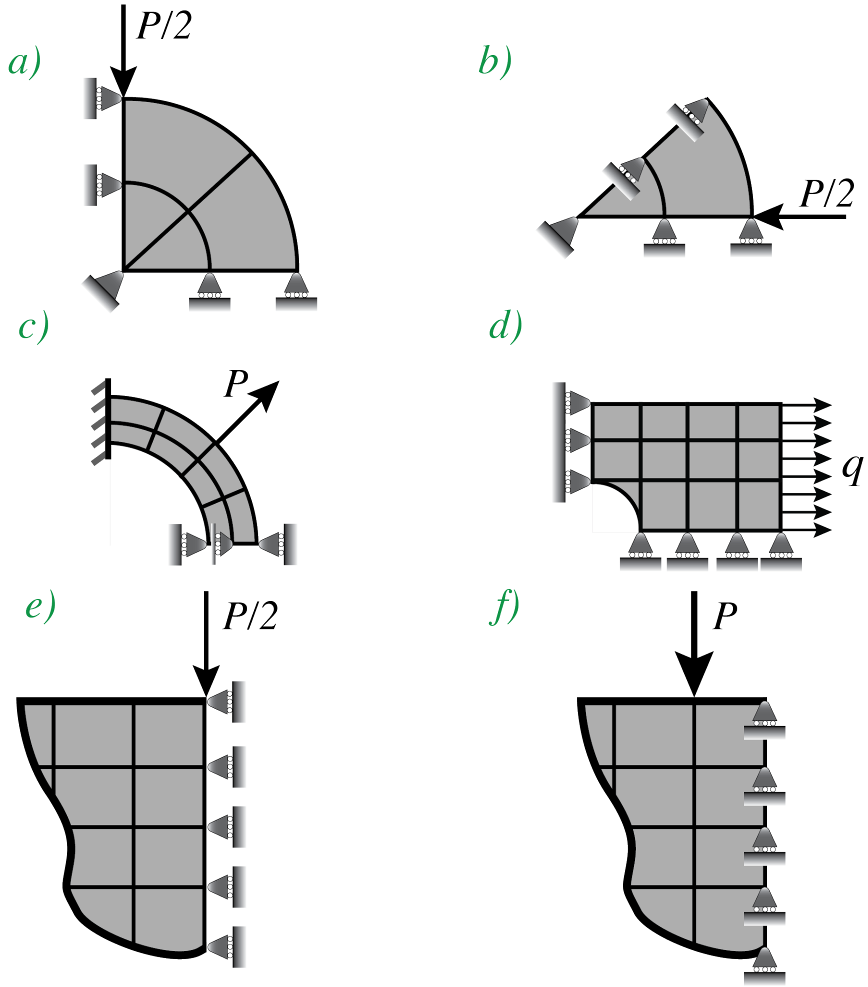

Figure 0.3: Solutions for assignment 2.1.2 - Boundary conditions in a coarse mesh.

Assignment 2.2

On “FEM Modelling”:

1.-The plate structure shown in the figure is loaded and deforms in the plane of the paper. The applied load at D and the supports at I and N extend over a fairly narrow area. List what you think are the likely “trouble spots” that would require a locally finer finite element mesh to capture high stress gradients. Identify those spots by its letter and a reason.

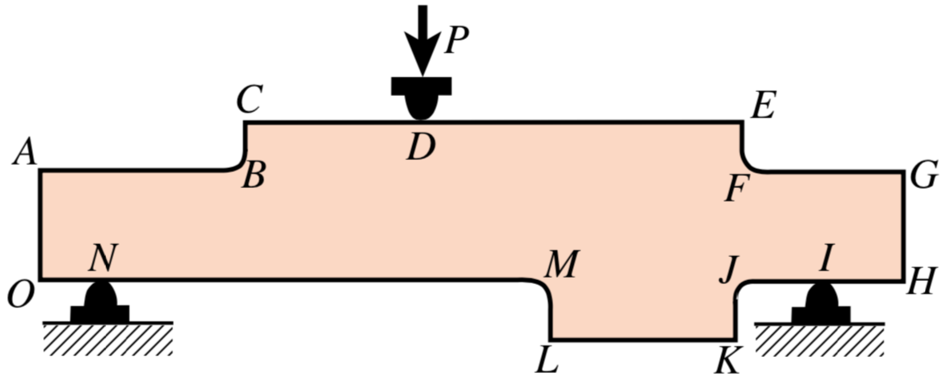

Figure 0.4: Problem for assignment 2.2 - Inplane bent plate.

Listing the “trouble spots” that require a finer mesh, in which the first three places occupy the most relevant regions to consider because of the stress gradients of the BCs applied, and finally with some spots that if are part of the analysis could be important to mesh because present a particularity due entrance of a curve edge of the geometry and the positions in which are located.

1. **Node N** - Support.
2. **Node I** - Support.
3. **Node D** - Applied load.
4. **Node M** - Curved edge and located at the middle of the bar.
5. **Node J** - Curved edge and located near a support over a narrow area.
6. **Node B** - Curved edge.
7. **Node F** - Curved edge.

ASSIGNMENT 2.3

On "Variational Formulation":
1.- A tapered bar element of length l and areas A_i and A_j with A interpolated as:

$$A = A_i(1-\xi) + A_j\xi \tag{0.1}$$

and constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node i. Taking axis x along the rotating bar with origin at node i, the centrifugal axial force is $q(x) = \rho A\omega^2 x$ along the length in which x is the longitudinal coordinate $x = x^e$. Find the consistent node forces as functions of ρ, A_i, A_j, ω and l, and specialize the result to the prismatic bar $A = A_i = A_j$.
Solution:

Consider the *consistent node force vector* which comes from the element contribution to the external work potential W:

$$W^e = \int_{x_1}^{x_2} qu\,dx = \int_0^1 q\mathbf{N}^T\mathbf{u}^e l\,d\xi = (\mathbf{u}^e)^T \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l\,d\xi = (\mathbf{u}^e)^T\mathbf{f}^e \tag{0.2}$$

in which $\xi = (x - x_1)/l$. Consequently,

$$\mathbf{f}^e = \int_{x_1}^{x_2} q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} dx = \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l\,d\xi \tag{0.3}$$

Substituting the value of q and A into the force node vector:

$$\begin{aligned}
\mathbf{f}^e &= \int_0^1 \rho(\xi l)\omega^2[A_i(1-\xi) + A_j\xi] \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l\,d\xi \\
&= \rho\omega^2 l^2 \int_0^1 \begin{bmatrix} A_i\xi(1-\xi)^2 + A_j\xi^2(1-\xi) \\ A_i\xi^2(1-\xi) + A_j\xi^3 \end{bmatrix} d\xi \\
&= \rho\omega^2 l^2 \begin{bmatrix} \frac{1}{12}A_i + \frac{1}{12}A_j \\ \frac{1}{12}A_i + \frac{1}{4}A_j \end{bmatrix}
\end{aligned}$$

Now, in order to consider to a prismatic bar in which the areas are the same in the entire extension $A = A_i = A_j$, the problem reduces in adding the area terms:

$$\mathbf{f}^e = \rho\omega^2 l^2 \begin{bmatrix} \frac{1}{6}A \\ \frac{1}{3}A \end{bmatrix}$$