MASTER OF SCIENCE IN COMPUTATIONAL MECHANICS

COMPUTATIONAL STRUCTURAL MECHANICS AND DYNAMICS

Assignment 2: FEM Modelling and Variational Formulation

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1   Assignment 2.1 - FEM Modelling: Introduction

1. Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in the figure. They are:

(a) a circular disk under two diametrically opposite point forces (the famous "Brazilian test" for concrete)

(b) the same disk under two diametrically opposite force pairs (c) a clamped semi-annulus under a force pair oriented as shown

(d) a stretched rectangular plate with a central circular hole

(e) and (f) are half-planes under concentrated loads.

2. Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.
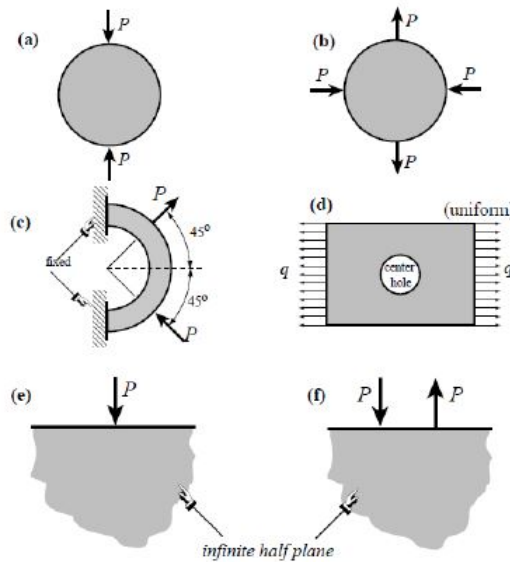

Figure 2.1.- Problems for assignment 2.1

Figure 1.1 – Problems for assignment 2.1

Figure (1.2) depicts the symmetry and antisymmetry lines for the problems of Figure (1.1).

Furthermore, taking into the account the properties of symmetry and antisymmetry in structural analysis, a solution of the problems can be obtained by computing the needed values in a reduced subdomain. Figure (1.3) shows the reduced domains and mesh obtained after considering symmetry and antisymmetry with their corresponding boundary conditions to be applied on each one of their boundaries.
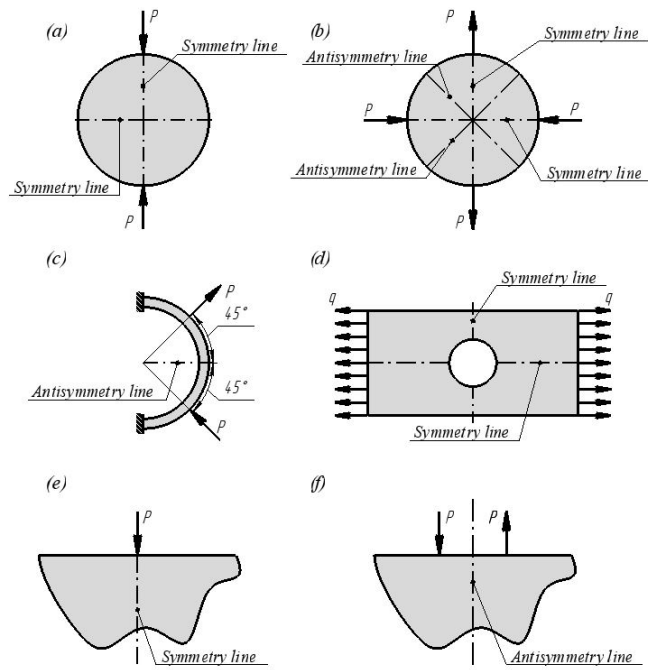
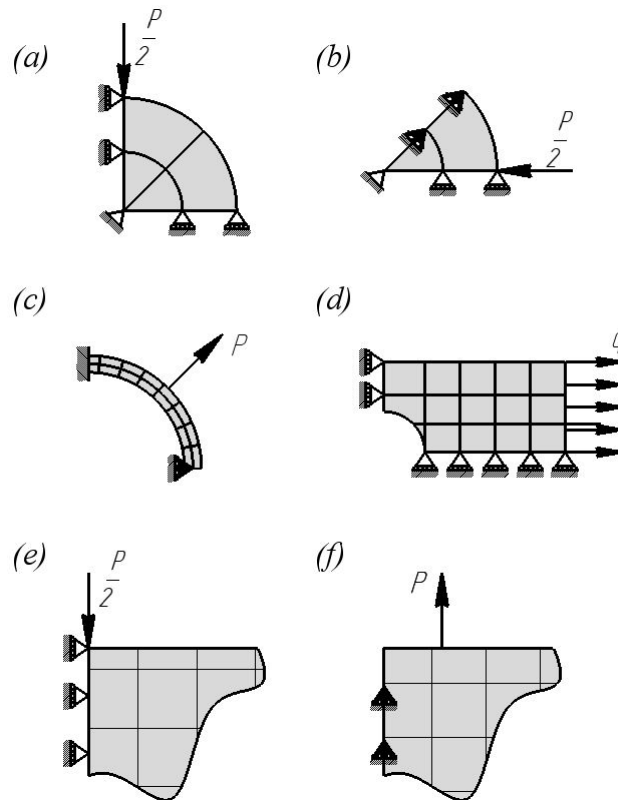Figure 1.2 – Symmetry and antisymmetry identification for problem 2.1

Figure 1.3 – Reduced domains and corresponding BC's for problem 2.1

2 Assignment 2.2 - FEM Modelling: Introduction

1. **The plate structure shown in the figure is loaded and deforms in the plane of the paper. The applied load at D and the supports at I and N extend over a fairly narrow area. List what you think are the likely "trouble spots" that would require a locally finer finite element mesh to capture high stress gradients. Identify those spots by its letter and a reason.**
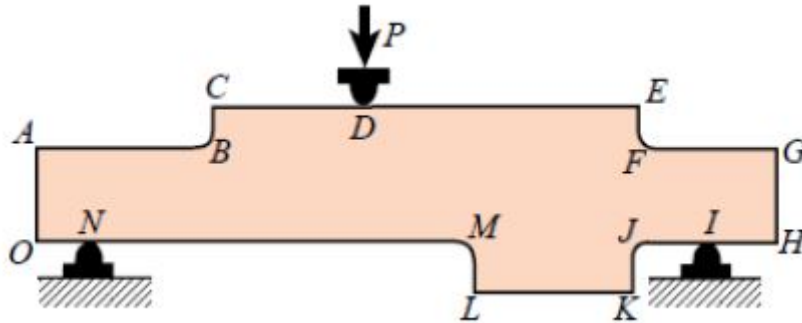

Figure 2.1 – In-plane bent plate

Critical areas are "trouble spots" where high or low gradient of strains or stress are expected. These points are usually present in the vicinity of geometrical zones with sharp corners or intersection of meshes. Additionally, discontinuities can appear at nodes with applied point loads or at nodes where point supports are located. In the case of the problem depicted in Figure (2.1), the following "trouble spots" can be identified:

1. Point B - Entrant sharply curved edge

2. Point D - Node with applied point load

3. Point F - Entrant sharply curved edge

4. Point I - Node with point support

5. Point J - Entrant sharply curved edge

6. Point M - Entrant sharply curved edge

7. Point N - Node with point support

3   Assignment 2.2 - Variational Formulation

1. **A tapered bar element of length l and areas A_i and A_j with A interpolated as**

$$A = A_i(1 - \xi) + A_j\xi \tag{1}$$

and constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node i. Taking axis x along the rotating bar with origin at node i, the centrifugal axial force is $q(x) = \rho A\omega^2 x$ along the length in which x is the longitudinal coordinate $x = x^e$.

Find the consistent node forces as functions of ρ, A_i, A_j, ω and l, and specialize the result to the prismatic bar $A = A_i = A_j$.

From the variational formulation of a linear element, the consistent node force can be computed as:

$$f^e = \int_0^1 q \begin{bmatrix} 1 - \xi \\ \xi \end{bmatrix} l d\xi$$

where the local natural coordinate ξ can be defined as $\xi = \frac{x - x_1}{l}$. Using this transformation, the centrifugal axial force becomes:

$$q = \rho A\omega^2 x$$
$$= \rho A\omega^2 \xi l$$

Thus,

$$f^e = \int_0^1 q \begin{bmatrix} 1 - \xi \\ \xi \end{bmatrix} l d\xi$$
$$= \int_0^1 \rho(A_i(1 - \xi) + A_j\xi)\omega^2 \xi l \begin{bmatrix} 1 - \xi \\ \xi \end{bmatrix} l d\xi$$
$$= \rho\omega^2 l^2 \int_0^1 (A_i(1 - \xi) + A_j\xi)\xi \begin{bmatrix} 1 - \xi \\ \xi \end{bmatrix} d\xi$$
$$= \rho\omega^2 l^2 \int_0^1 \begin{bmatrix} A_i\xi(1 - \xi)^2 + A_j\xi^2(1 - \xi) \\ A_i\xi^2(1 - \xi) + A_j\xi^3 \end{bmatrix} d\xi$$
$$= \rho\omega^2 l^2 \begin{bmatrix} \frac{1}{12}A_i + \frac{1}{12}A_j \\ \frac{1}{12}A_i + \frac{1}{4}A_j \end{bmatrix}$$

For a prismatic bar. for which $A = A_i = A_j$, the result reduces to:

$$f^e = \rho\omega^2 l^2 \begin{bmatrix} 1/6A \\ 1/3A \end{bmatrix}$$
$$= \frac{1}{6}\rho\omega^2 l^2 A \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$