Universitat Politècnica de Catalunya

Computational Structural Mechanics and Dynamics

Master's Degree in Numerical Methods in Engineering

Assignment on FEM Modelling

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1 Assignment 2.1

When symmetry applies, it means that the structure is identical on either side of the dividing line. Along the line, boundary conditions must be applied so that the out-of-plane displacement is 0. However, when there is anti-symmetry, the loading of the model is oppositely balanced on either side of the dividing line, so that the two in-plane displacements are zero. Figures 1,2,3,4,5 and 6 show the different lines of the proposed figures. As an extension of the proposed problem, note that the figures have been properly drawn with a suitable latex package.

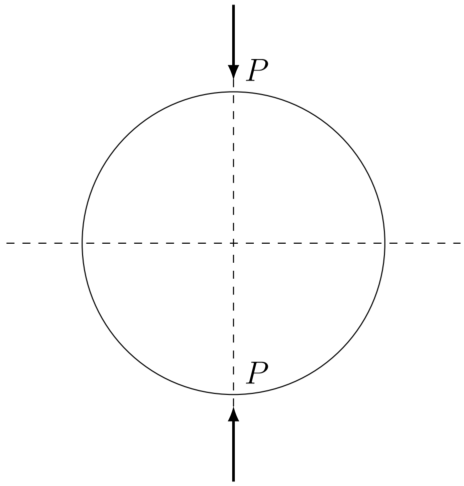

Figure 1: Two lines of symmetry.

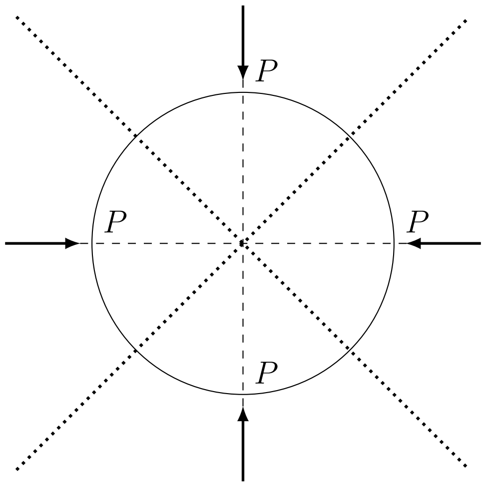

Figure 2: Two lines of symmetry and two of anti-symmetry.

Now the meshed for each figures are drawn. As mentioned before, the boundary conditions have to be applied to those parts of the structure that are divided by lines of symmetry or anti-symmetry. All structures can be divided if there is a symmetry line and further divided if there is an anti-symmetry line. Note that in Figures 7, 8, 9 and 10 the meshed have been

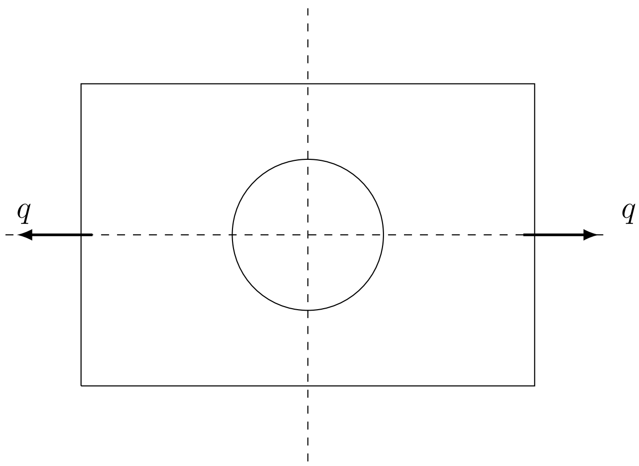

Figure 3: Two lines of symmetry.

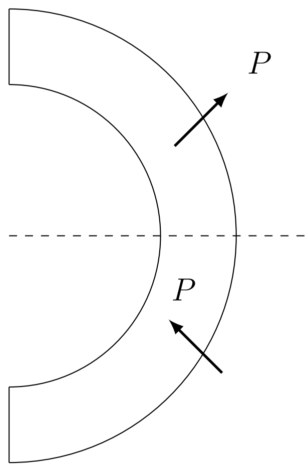

Figure 4: One lie of anti-symmetry.

drawn with a proper software and are depicted straightforward. To indicate the displacement BCs, squares or circles have been used surrounding the nodes. The square means that the node has both displacements prescribed.

In Fig. 7, as there are two symmetry lines, only a quarter of the structure can be analyzed. For the Brazilian case, only an eight-part of the circle is needed, and the mesh is equal to that in Fig. 7 but now the angle is not 90ž but 45ž, and the nodes located at the line of 45ž do not have zero displacement perpendicular to the lines of symmetry but zero displacement on the direction of the line of anti-symmetry.

In Fig. 8, there are imposed fixed displacements, denoted with a square, and the structure is also cut into a half due to the line of anti-symmetry. On that line, the displacements are prescribed.

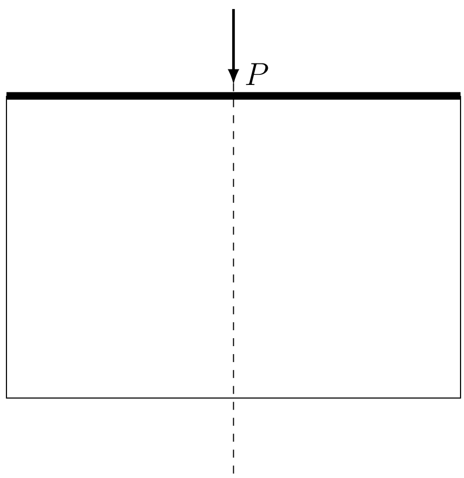

Figure 5: One line of symmetry.

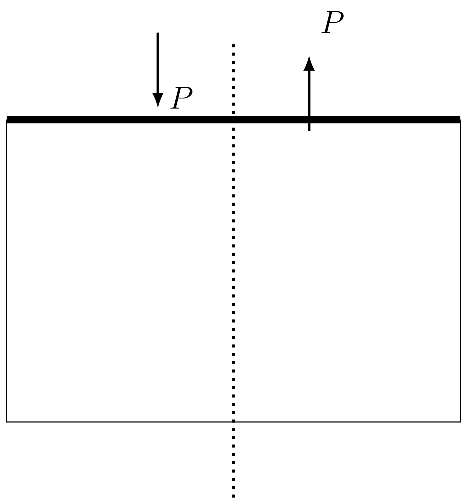

Figure 6: One line of anti-symmetry.

In Fig. 9, the structure is cut into a quarter due to the two lines of symmetry.

Eventually, for the infinite half-planes (Fig. 10, the situation is the same. When the symmetry applies, the structure is cut into a half, as well as in the anti-symmetry case. However, the conditions are diverse.

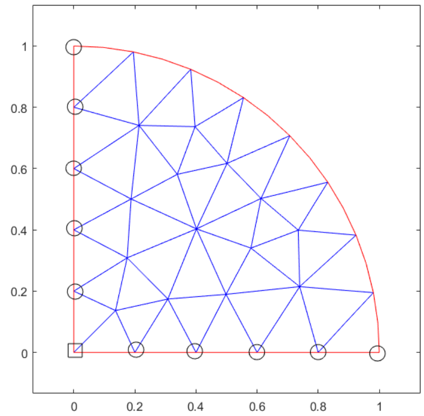

Figure 7: Here the circles mean that the nodes associated have zero displacement perpendicular to the lines of symmetry.

2 Assignment 2.2

The finite element developers have to face models of a real problem, and the geometry of this model is divided into different sub-domains of a certain accuracy. In like manner, there will be two sources of errors. One will account for the error produced by the structural model and its ability to capture the behavior of the structure, and the other will account for the quality of the mesh. Therefore, in order to quantify those errors, validation and verification are essential steps when evaluating the quality of a finite mesh discretization.

- On one hand, validation explores how similar is a structural and computational model with the physical behavior observed in reality through experimental tests made on a laboratory (for instance). Therefore, the numerical results of the simulation are compared against experimental data.
- On the other hand, verification is focused on reducing the possible sources of numerical errors (such the discretization or numerical errors) through the use of estimation techniques. This allows to check on the accuracy of the resolution, and weather it requires of a finer mesh or of higher order elements. It has no relation with the real nature of the problem.

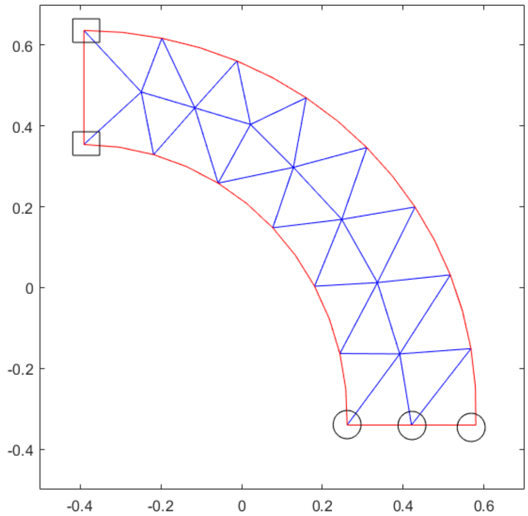

Figure 8: Here the circles mean that the nodes associated have zero displacement on the direction of the line of anti-symmetry.

3 Assignment 2.3

The consistent load vector of an element e acting on a node i can be calculated as

$$f_i^e = \int_0^{l^e} N_i^e q dx \tag{1}$$

Where N is the shape function and q is simply the section at point x multiplied by the body force at that point. As we are given an area which is a function of a dimensionless variable, it is needed to integrate as a function of this variable. In this sense, $N = 1 - \xi$ and $dx = l d\xi$. Then, if integrating

$$f_i^e = \int_0^{l^e} N_i^e q dx = \int_0^1 N_i^e \rho A(\xi) l \xi \omega^2 l d\xi = \rho l^2 \omega^2 \frac{A_i + A_j}{12} \tag{2}$$

As this expression is valid for any node in the element, then particularizing for a prismatic bar it is clear that both nodes will experience the same force, i.e. $f = \rho l^2 \omega^2 \frac{A}{6}$.

4 Assignment 2.3 (extra)

4.1 a)

For a typical element e of length h^e with associated nodes i, j

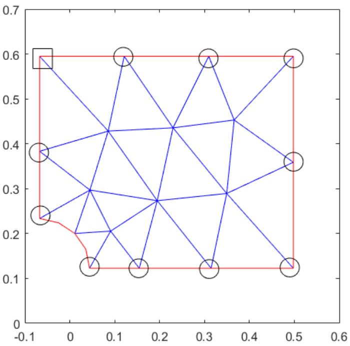

Figure 9: Here the circles mean that the nodes associated have zero displacement perpendicular to the lines of symmetry.

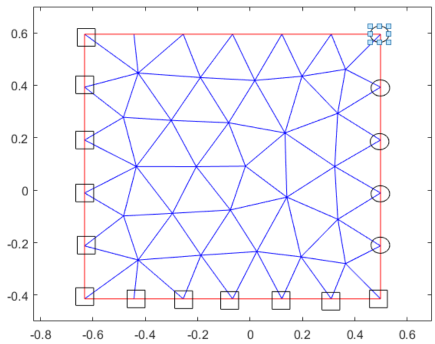

Figure 10: Here the circles mean that the nodes associated have zero displacement perpendicular to the line of symmetry or zero displacement on the line of anti-symmetry, depending on the case.

$$K_{ij}^{e} = \int_{0}^{1} EA \frac{dN_i^e}{dx} \frac{dN_j^e}{dx} h^e d\xi \tag{3}$$

If we assume EA to be constant

$$\mathbf{K} = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{pmatrix} \mathrm{K}_{ii}^{(e)} & \mathrm{K}_{ij}^{(e)} \\ \mathrm{K}_{ji}^{(e)} & \mathrm{K}_{jj}^{(e)} \end{pmatrix} \end{array}$$

Where $K_{ii}^{(e)} = K_{jj}^{(e)} = -K_{ij}^{(e)} = -K_{ji}^{(e)} = EA/h^e$. For a tapered element with cross section area $A = (A_i + A_j)/2$ the matrix is obtained straightforward with the stiffness element matrix.

However, if the cross section is not constant, it is needed to integrate in (4). Clearly $\frac{dN_i^e}{dx} = -1/h^e = -\frac{dN_j^e}{dx}$.

$$K_{ij}^e = -\frac{E}{h^2}\int_0^1 (A_i(1-\xi) + A_j\xi)hd\xi = -\frac{E(A_i + A_j)}{2h} \tag{4}$$

$$\mathbf{K} = \frac{E(A_i + A_j)}{2h} \begin{matrix} & {\scriptstyle 1} & {\scriptstyle 2} \\ {\scriptstyle 1} & 1 & -1 \\ {\scriptstyle 2} & -1 & 1 \end{matrix}$$

4.2 b)

The consistent load vector of an element e acting on a node i can be calculated as

$$f_i^e = \int_0^{h^e} N_i^e q dx = \int_0^1 N_i^e \rho g A(\xi) h d\xi = \int_0^1 (1-\xi)\rho g(A_i(1-\xi) + A_j\xi)hd\xi = \rho g h \frac{2A_i + A_j}{6} \tag{5}$$

If $A_i = A_j$, the force is simply divided equally between the two nodes, i.e. $f_i = f_j = \rho g h A_i/2$. If $A_j = 0$, then $f_i = \rho g h A_i/3$, which means that this nodes receives one third of the total force.

4.3 c)

The consistent load vector of an element e acting on a node i can be calculated as

$$f_i^e = \int_x^{h^e} {}_i N_i^e q dx = \int_0^1 N_i^e Q\delta(x-a)dx = \frac{Q(h-a)H(a)H(h-a)}{h} \tag{6}$$

Where H is the Heaviside function. If $a = h$ then clearly the force is 0 at the node, whereas if $a = h/2$ the force is one half of Q and Q for $a = 0$.