Computational Structural Mechanics & Dynamics

As 2 - FEM modelling & Variational formulation

Prog. in Computational mechanics

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Assignment 2.1 - FEM modelling

① Symmetry / Anti-symmetry lines :

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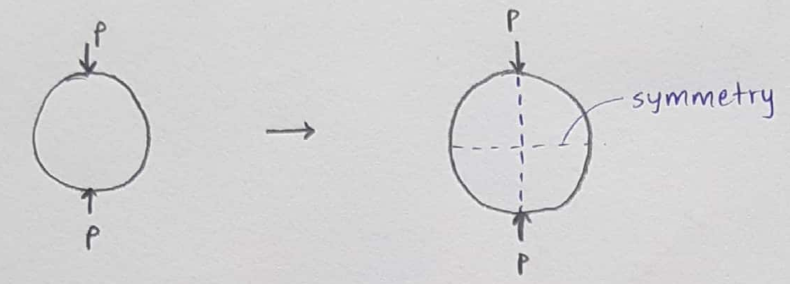

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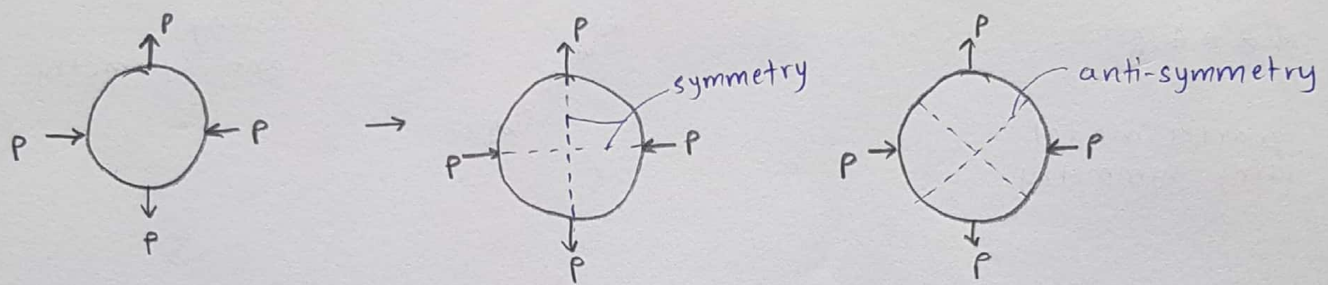

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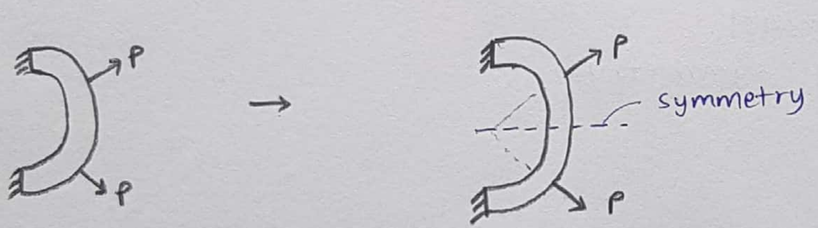

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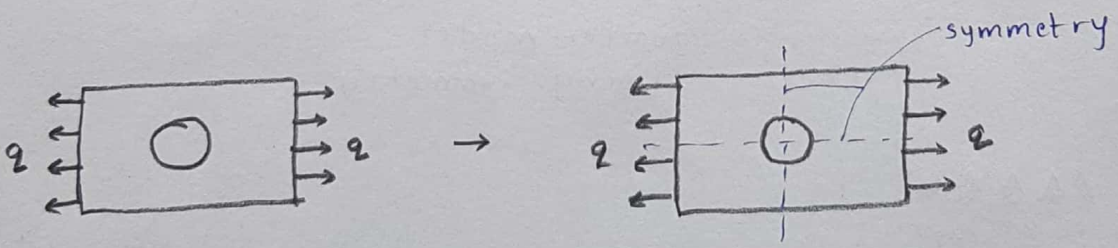

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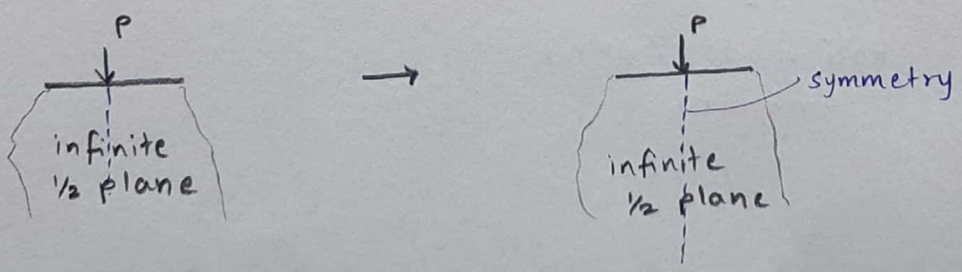

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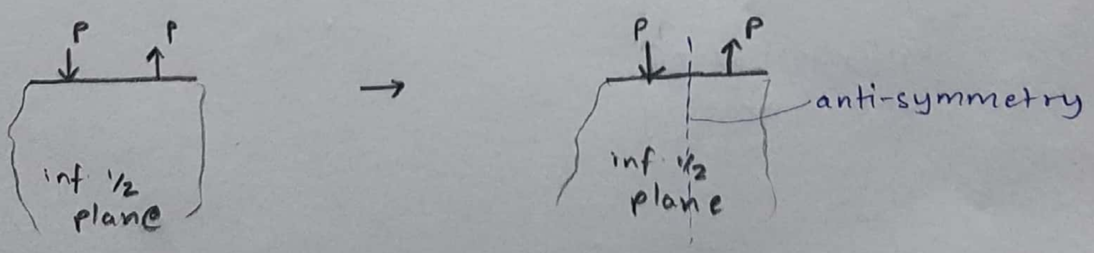

② Coarse FE mesh & using symmetry -

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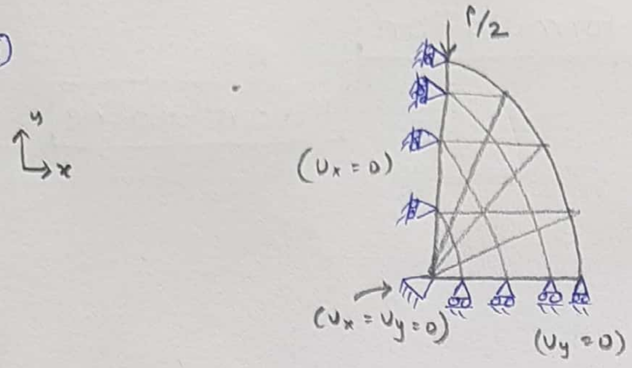

quarter model using symmetry.

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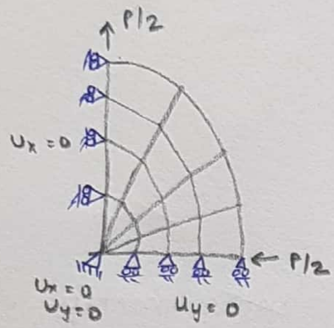
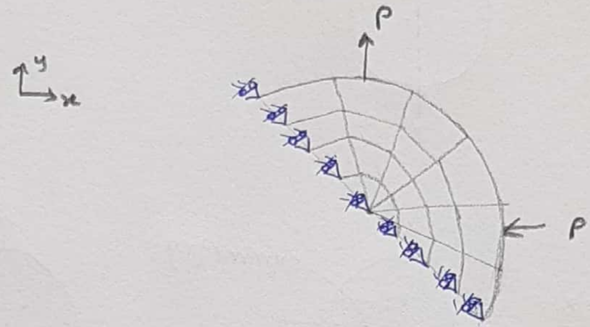


quarter model using symmetry



half - model using anti - symmetry

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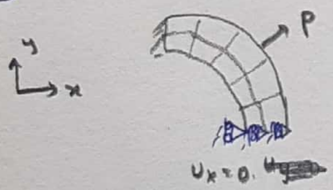

half model using symmetry

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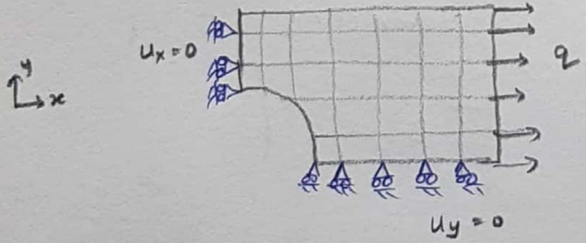

quarter model using symmetry

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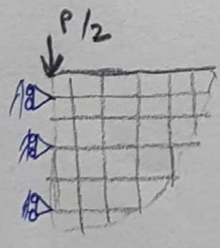

half model using symmetry

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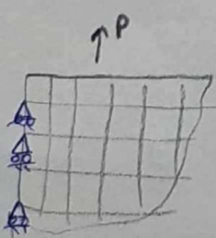

half model using anti-symmetry

Assignment 2.2

Verification:

Verification could be seen as the process which determines whether a computational model accurately represents the underlying mathematical model & its sol. In simple words, verification means constantly checking the errors between numerical sol. & analytical sol. The outputs of the developed codes are verified with the analytical results. Hence, it is totally a domain of mathematics.

Validation:

Validation is the process of determining how accurate the model represents the real world from the perspective of the intended uses of the model. In simple words, validation is associated with comparison of the computational results of the model with the experimental results achieved. It resembles physical modelling of the problem. This process also addresses the uncertainties that arise from both simulation results & experimental data.

Assignment 2.3 - Variational formulation.

$A = A_i(1-\xi) + A_j\xi$ $\qquad q(x) = \rho A \omega^2 x$

Special case $\longrightarrow$ $A = A_i = A_j$.

Solution :

The consistent node forces are given by -

$$f = \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l \, d\xi$$

Here, $\xi = \dfrac{x - x_1}{l}$

$\therefore \; x = \xi l + x_1$

But as only 1 element is considered, $\therefore \; x_1 = 0$.

$\therefore \; x = \xi l$.

$$\therefore f = \int_0^1 \rho A \omega^2 \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} (\xi l) \, l \, d\xi$$

$$= \rho\omega^2 l^2 \int_0^1 A \begin{bmatrix} \xi(1-\xi) \\ \xi^2 \end{bmatrix} d\xi$$

$$= \rho\omega^2 l^2 \int_0^1 A_i(1-\xi) + A_j \cdot \xi \begin{bmatrix} \xi(1-\xi) \\ \xi^2 \end{bmatrix} d\xi$$

$$= \rho\omega^2 l^2 \int_0^1 \begin{bmatrix} A_i \cdot \xi(1-\xi)^2 + A_j \cdot \xi^2(1-\xi) \\ A_i \cdot \xi^2(1-\xi) + A_j \cdot \xi^3 \end{bmatrix} d\xi$$

$$= \rho\omega^2 l^2 \int_0^1 \begin{bmatrix} A_i \cdot (\xi - 2\xi^2 + \xi^3) + A_j \cdot (\xi^2 - \xi^3) \\ A_i \cdot (\xi^2 - \xi^3) + A_j \cdot \xi^3 \end{bmatrix} d\xi$$

$$= \rho\omega^2 l^2 \begin{bmatrix} A_i \cdot \left(\frac{\xi^2}{2} - \frac{2\xi^3}{3} + \frac{\xi^4}{4}\right) + A_j \cdot \left(\frac{\xi^3}{3} - \frac{\xi^4}{4}\right) \\ A_i \cdot \left(\frac{\xi^3}{3} - \frac{\xi^4}{4}\right) + A_j \cdot \frac{\xi^4}{4} \end{bmatrix}_0^1$$

$$= \rho\omega^2 l^2 \begin{bmatrix} \frac{A_i}{12} + \frac{A_j}{12} \\ \frac{A_i}{12} + \frac{A_j}{4} \end{bmatrix}$$

Implementing $A = A_i = A_j$.

$$\therefore f = \rho\omega^2 l^2 \begin{bmatrix} \frac{A}{6} \\ \frac{A}{3} \end{bmatrix} \longrightarrow \boxed{f = \frac{\rho\omega^2 A l^2}{6} \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$$

Discussions :

Assignment 2.1 -

If a geometry is cut symmetrically, the forces remain in the same direction when the half geometry is rotated by 180°. while, if a geometry is cut anti-symmetrically, the forces are in opposite direction when the half geometry is rotated by 180°.

In symmetry conditions, the displacements at axis of symmetry of the geometry are restricted in the direction perpendicular to the cutting plane & the loads at nodes become half of the original value.

Where as, in anti-symmetry conditions, the displacements at the axis of the geometry are restricted in the direction of cutting plane.

Assignment 2.2 -

Verification & validation process is important because it ensures the correctness of the model by allowing a comparison of physical observations (experimentation) with the selected computational models to achieve minimal error & converged results. If the computational results & experimental results go hand-in-hand, then nothing can be more fruitful for a certain model to be implied in the real world.

~~The &~~ Verification could be identified as 'prediction' while validation can be seen as 'confirmation'.