Universitat Polytechnica de Catalunya
MSc Computational Mechanics
Spring 2018

Computational Structural Mechanics and Dynamics

Assignment 2

Due 19/02/2018

Alexander Keiser

1 Assignment 2.1.1

For the first part of this assignment, we will identify the symmetry (SL) and antisymmetry (ASL) lines in the two-dimensional problems illustrated in the figures below.

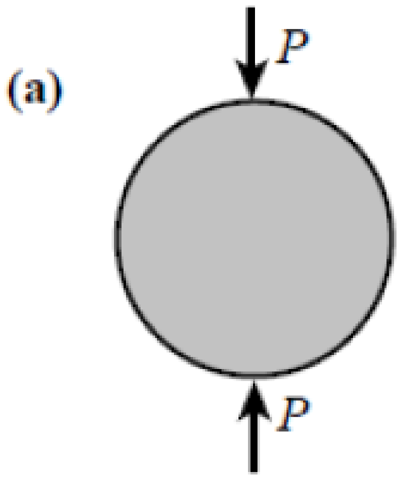


Figure 1a: Brazilian Test

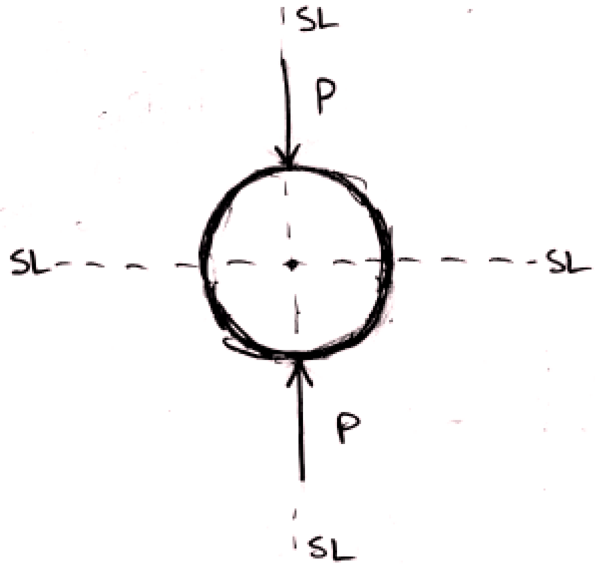


Figure 1b: Brazilian Test w/ SL

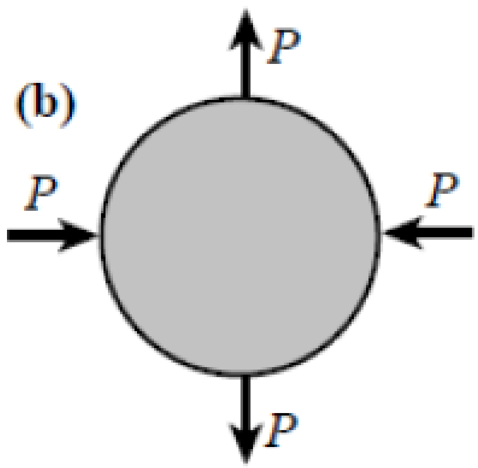


Figure 2a: Disk with opposite force pairs

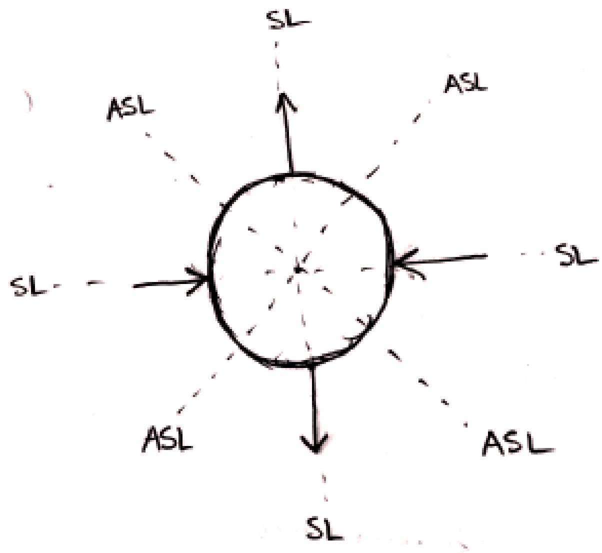


Figure 2b: Disk with opposite force pairs w/ SL and ASL

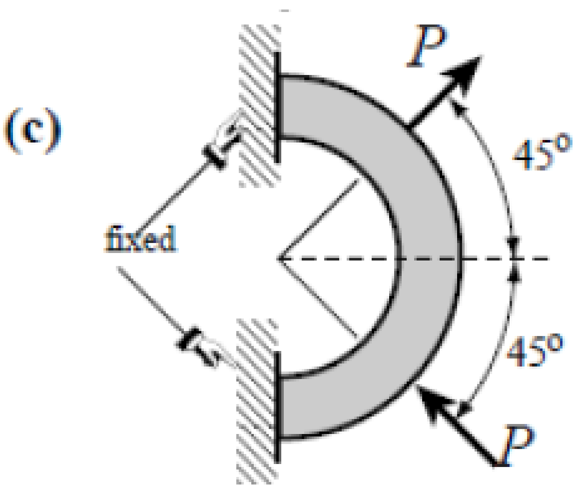


Figure 3a: Clamped Semiannulus

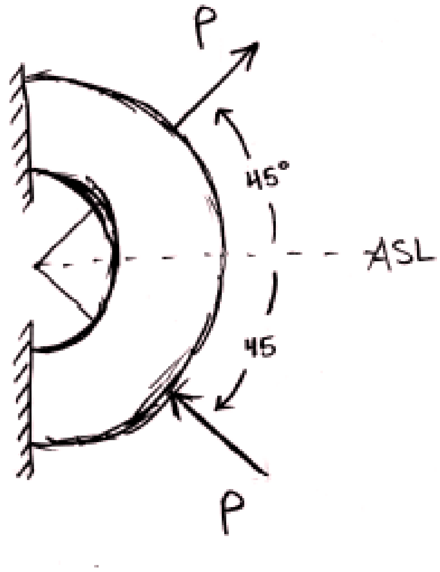


Figure 3b: Clamped Semiannulus w/ ASL

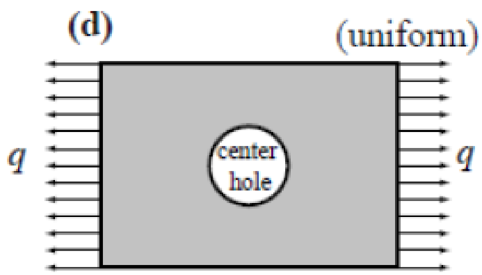


Figure 4a: Rectangular Plate with Central Hole

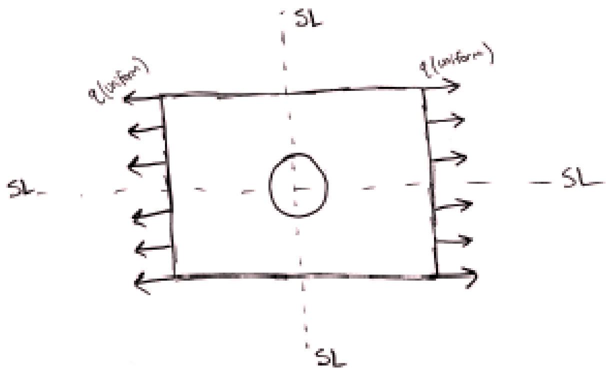


Figure 4b: Rectangular Plate with Central Hole w/ SL

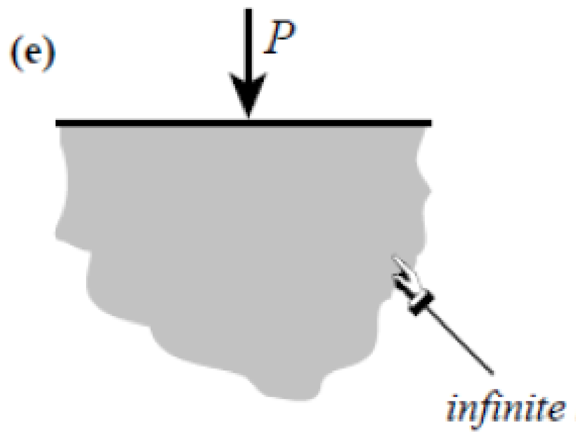


Figure 5a: Half Plane with Load

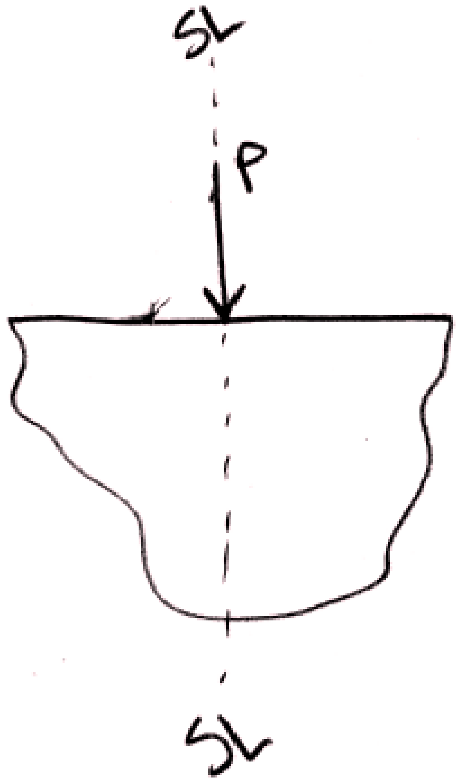


Figure 5b: Half Plane with Load w/ SL

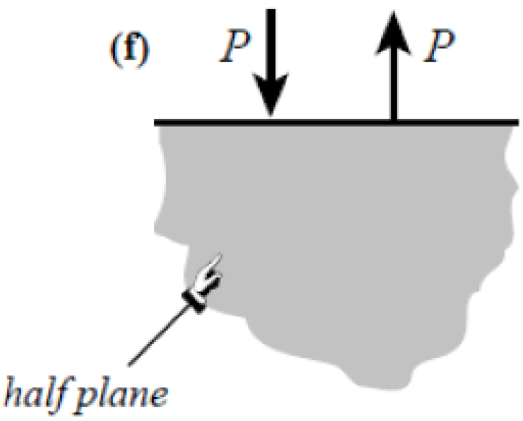


Figure 6a: Half Plane with Loads

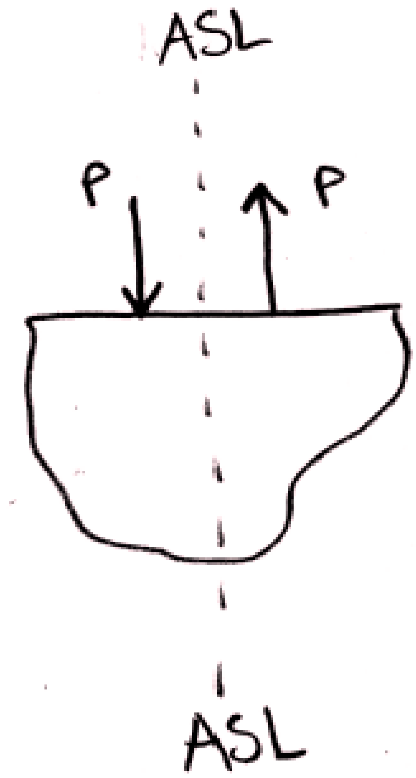


Figure 6b: Half Plane with Loads w/ ASL

2 Assignment 2.1.2

Now that we have identified the symmetry and anti-symmetry lines, we will now say whether it is possible to cut these structures into halves or quarters, lay down a mesh, and explain the boundary conditions associated with each.

The first problem is a circular disk under two diametrically opposite point forces. This problem has symmetry lines in the vertical and horizontal directions. It is possible to cut this into fourths and solve. The appropriate boundary conditions are as follows. We will fix the orgin in both x and y directions. We will allow only x-direction movement along the horizontal symmetry line using rollers. We will allow only y-direction movement along the vertical symmetry line using rollers. This can be seen below in Figure 7b.

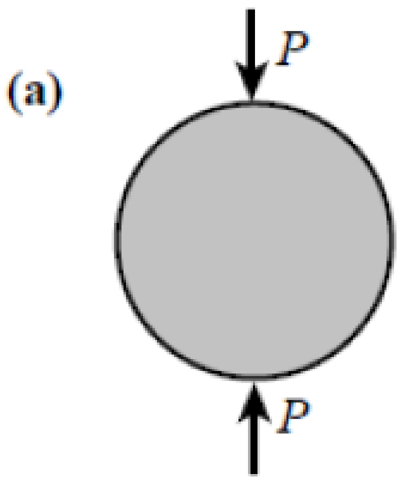


Figure 7a: Brazilian Test

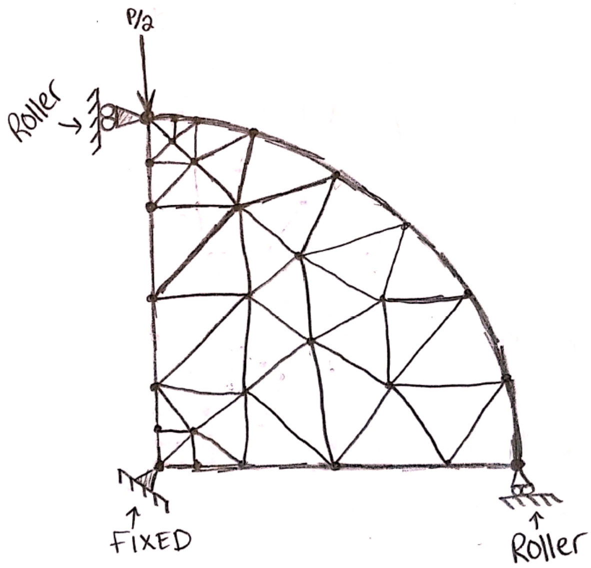


Figure 7b: Brazilian Test w/ Mesh and BCs

The second problem is the same disk under two diametrically opposite force pairs. This problem has symmetry lines in the vertical and horizontal directions, as well as anti-symmetry lines on the diagonals. It is possible to cut this into fourths and solve. The appropriate boundary conditions are as follows and is the same as in the previous example. We will fix the orgin in both x and y directions. We will allow only x-direction movement along the horizontal symmetry line using rollers. We will allow only y-direction movement along the vertical symmetry line using rollers. This can be seen below in Figure 8b.

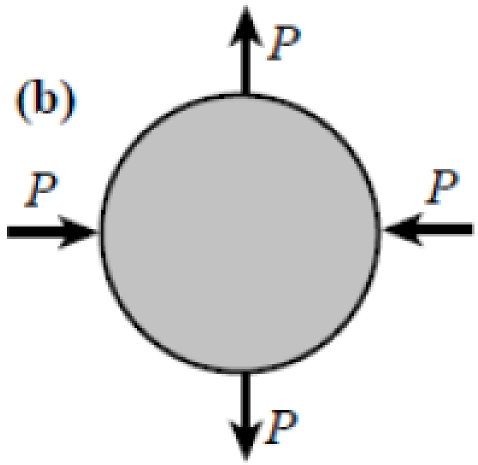


Figure 8a: Disk with opposite force pairs

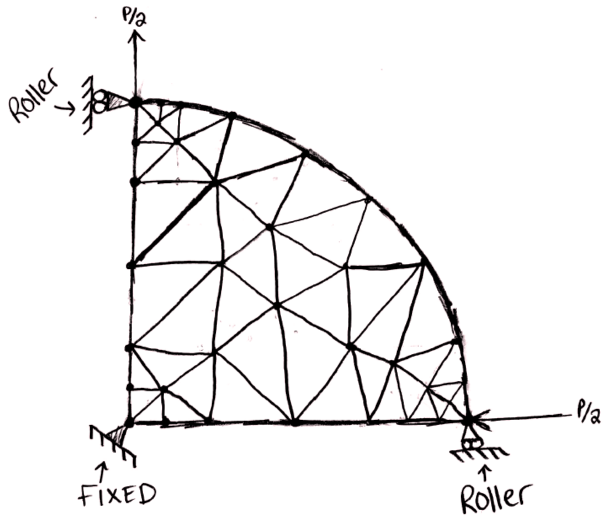


Figure 8b: Disk with opposite force pairs w/ Mesh and BCs

The third problem is a clamped semiannulus under a force pair. This problem has an anti-symmetry line in only the horizontal direction. It is possible to cut this into halves and solve. The appropriate boundary conditions are as follows. We will fix the wall touching part of the semiannulus in both x and y directions. We will allow only y-direction movement along the horizontal anti-symmetry line using rollers. This can be seen below in Figure 9b.

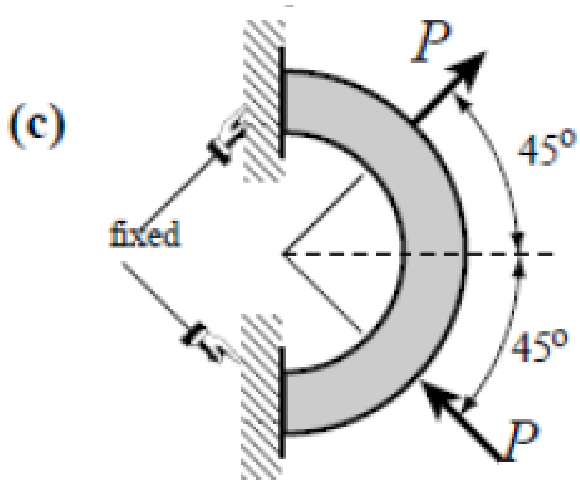


Figure 9a: Clamped Semiannulus

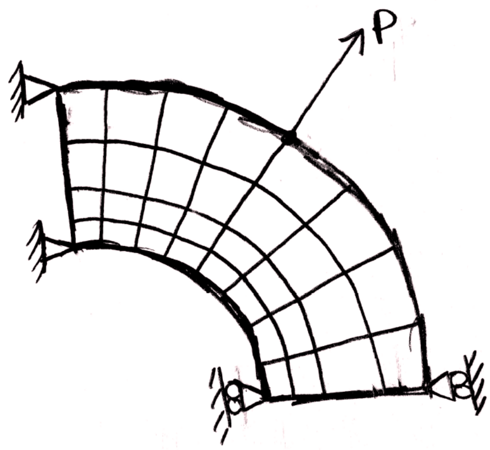
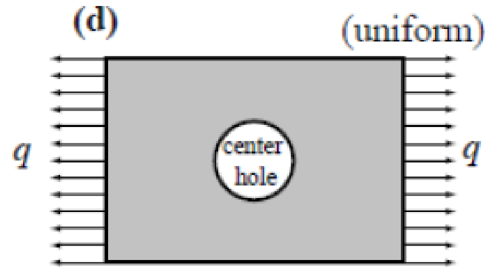


Figure 9b: Clamped Semiannulus w/ Mesh and BCs

The fourth problem is a stretched rectangular plate with a central circular hole. This problem has symmetry lines in the vertical and horizontal directions. It is possible to cut this into fourths and solve. The appropriate boundary conditions are as follows. We will allow only y-direction movement along the vertical symmetry line using rollers. We will also only allow x-direction movement along the horizontal symmetry line using rollers. This can be seen below in Figure 10b.



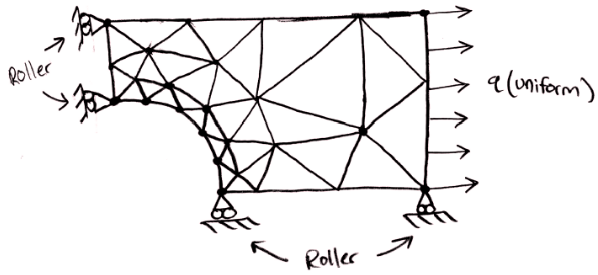
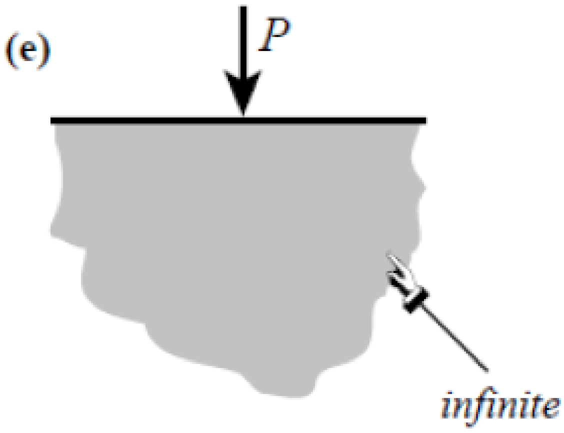


Figure 10a: Rectangular Plate with Central Hole

Figure 10b: Rectangular Plate with Central Hole w/ Mesh and BCs

The fifth problem is a half-plane under concentrated load. This problem has a symmetry line in the vertical direction. It is possible to cut this into halves and solve. The appropriate boundary conditions are as follows. We will allow only y-direction movement along the vertical symmetry line using rollers. We will also fix a node far from the load in both directions to anchor the problem. This can be seen below in Figure 11b.



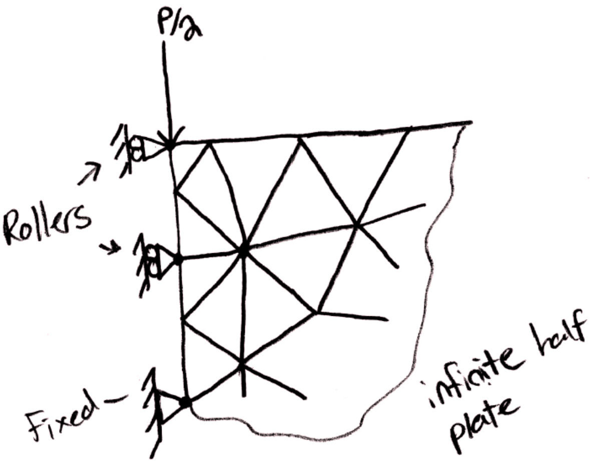


Figure 11a: Half Plane with Load

Figure 11b: Half Plane with Load w/ Mesh and BCs

The sixth and final problem is a half-plane under concentrated loads. This problem has an anti-symmetry line in the vertical direction. It is possible to cut this into halves and solve. The appropriate boundary conditions are as follows. We will allow only x-direction movement along the vertical anti-symmetry line using rollers. We will also fix a node far from the to anchor the problem. This can be seen below in Figure 12b.

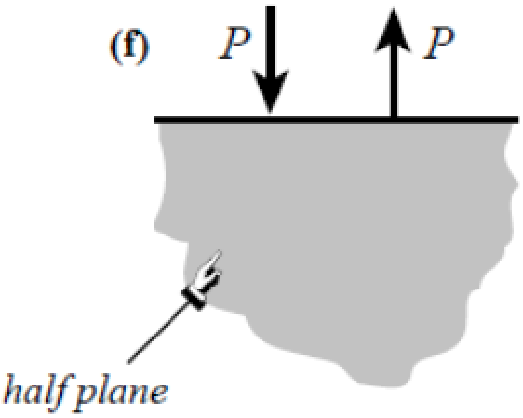


Figure 12a: Half Plane with Loads

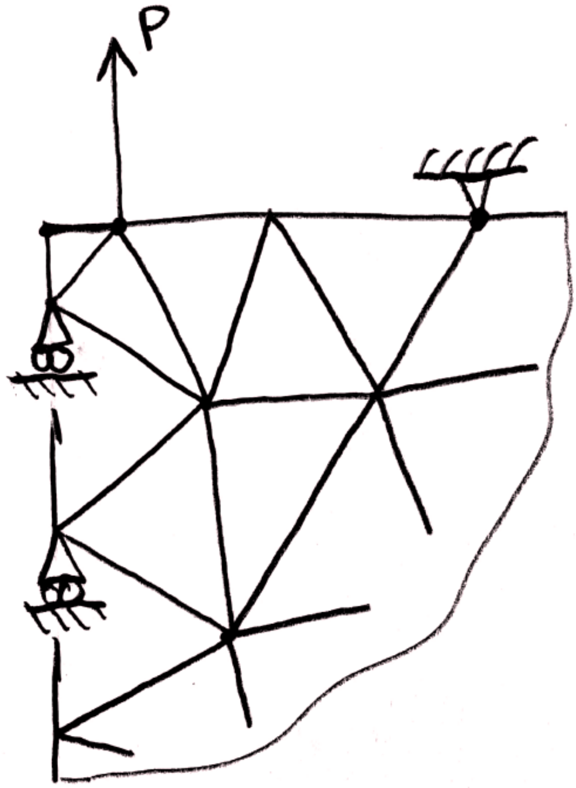


Figure 12b: Half Plane with Loads w/ Mesh and BCs

3 Assignment 2.2

In this assignment, we will identify points on the following figure that will have higher stress concentrations and require a finer local mesh.

The first places to get a finer mesh will be points B,F,J,M because they are classified as entrant corners where isostatics bunch up.

The final places to receive a finer mesh are D,N,I because they are concentrated areas of stress.

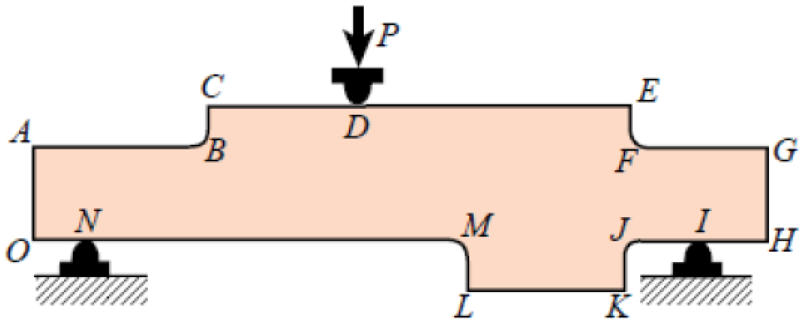


Figure 2.2.- Inplane bent plate

4 Assignment 2.3

The problem statement for this assignment is as follows.

On "Variational Formulation":

1. A tapered bar element of length l and areas A_i and A_j with A interpolated as

$$A = A_i(1-\xi) + A_j\xi$$

and constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node i. Taking axis x along the rotating bar with origin at node i, the centrifugal axial force is $q(x)=\rho A\omega^2 x$ along the length in which x is the longitudinal coordinate $x=x^e$.

Find the consistent node forces as functions of ρ, A_i, A_j, ω and l, and specialize the result to the prismatic bar $A = A_i = A_j$.

To solve this problem we will first solve for the force at each node of the element, we will then add these together to get the consistant node forces. After this is complete, we will set A=Ai=Aj to obtain the specialized result. This process can be seen on the following page.

$$f^e = f^1 + f^2 \qquad A = A_i(1-\xi) + A_j\xi$$

$$f^e = \int_0^l \rho\omega^2 A x \begin{pmatrix} 1-\xi \\ \xi \end{pmatrix} dx \qquad \xi = \frac{x}{l} \quad d\xi = \frac{dx}{l}$$

For f^1 replacing x & A gives us

$$f^1 = \int_0^1 \rho\omega^2 l^2 \xi\left((1-\xi)(A_i(1-\xi) + A_j\xi\right) d\xi$$

$$f^1 = \rho\omega^2 l^2 \left[\tfrac{1}{2}A_i - \tfrac{1}{3}A_i + \tfrac{1}{3}A_j - \tfrac{1}{3}A_i + \tfrac{1}{4}A_i - \tfrac{1}{4}A_j\right]$$

simplifying gives us

$$f^1 = \frac{\rho\omega^2 l^2}{12}\left(A_i + A_j\right)$$

Now for solving f^2 replacing x & A gives us

$$f^2 = \int_0^1 \rho\omega^2 l^2 \left((\xi^2)(A_i(1-\xi) + A_j\xi)\right) d\xi$$

$$f^2 = \rho\omega^2 l^2 \left[\tfrac{1}{3}A_i - \tfrac{1}{4}A_i + \tfrac{1}{4}A_j\right] = \frac{\rho\omega^2 l^2}{12}\left[A_i + 3A_j\right]$$

now remembering that $f^e = f^1 + f^2$ gives us

$$\boxed{f^e = f^1 + f^2 = \frac{\rho\omega^2 l^2}{12}\begin{bmatrix} A_i + A_j \\ A_i + 3A_j \end{bmatrix}}$$

and setting $A = A_i = A_j$ gives us

$$\boxed{f^e = \frac{\rho\omega^2 l^2 A}{6}\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$$