Computational Structural Mechanics and Dynamics

Assignment 2

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MS in Computational Mechanics

Assignment 2.1

a) *Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in the figure.*
b) *Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.*

In two-dimensional it is defined:

- Symmetry line, the one which a 180º rotation of the body about it reproduces exactly the original problem.
- Antisymmetry line, the one which a 180º rotation of the body about itself reproduces exactly the original problem except the loads, that are reversed.

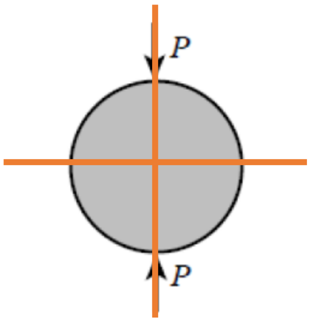
Using this criterium the symmetry and antisymmetry lines can be identify in the following cases.
Once we have defined the symmetry/antisymmetry lines, it will be possible to cut the structure in one half or one quarter depending on the stablished lines, defining for each case the appropriate boundary conditions and loads.

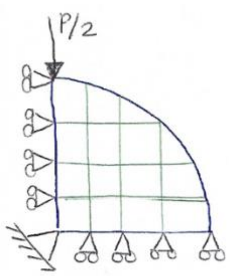
a)

Lines
In case a, it can be defined two symmetry lines. The first one matches with the diameter that is perpendicular to the forces direction and the second one matches the loads direction.
For this case it is possible to work just with a quarter of the initial structure. Talking about the BC, as it is symmetric:

- No displacement in the point corresponding with the center of the circle.
- Nodes in the symmetry lines, will have displacement in the corresponding lines direction.

Finally, as the loads direction matches with one of the symmetry lines, it will be P/2.





b)

Lines
Case b has symmetry and antisymmetry lines. The symmetry lines (the orange ones) are those corresponded with the diameters that have the two forces directions.

On the other side, antisymmetry lines (the green ones) can be defined as those corresponded with the circle diameters at 45º grades from the forces directions.

Symmetry case

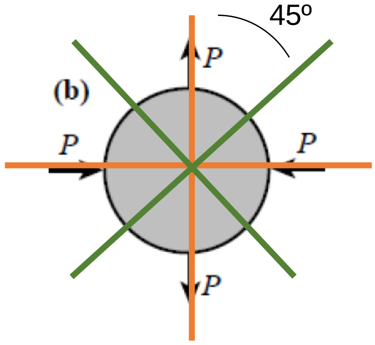
We apply jus the same condition that in the previous case, taking into account that now there are loads in both symmetry axis.

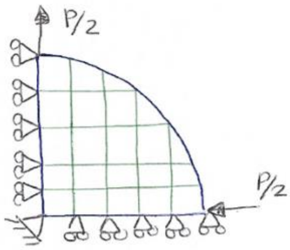
Antisymmetry case

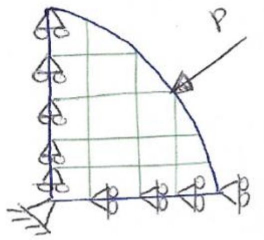
For the antisymmetric case, we use just a quarter of the circle too.

About the BC:

- The node corresponding with the center of the cylinder has no displacement.
- Nodes on the antisymmetry lines have no displacement in the lines directions.







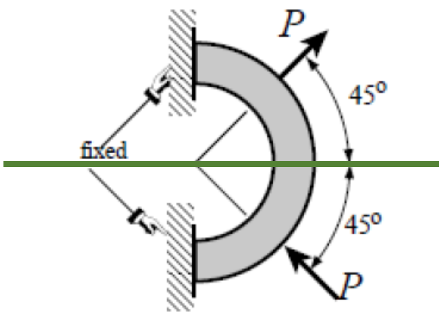
c)

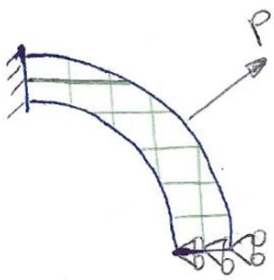
Lines

For case c, one antisymmetry line can be defined. It will be a horizontal line dividing the structure into two identical parts.

In this case it is going to be use a half of the structure.

About the BC, as in the previous case, we impose no displacement in the direction of the line for the nodes contained in the antisymmetry line.



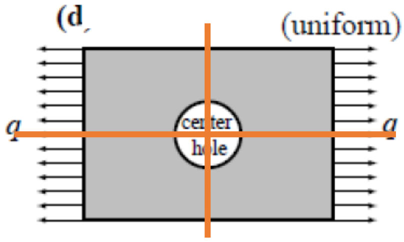


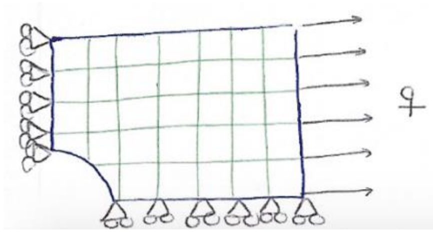
d)

Lines

At case d, two symmetry lines can be defined. One of them paralleled to y-direction passing through the hole center. The other one, paralleled to x-direction, passing through the hole center too.

For the calculation, due to the 2 symmetry lines just a quarter of the structure is going to be used. About the BC, those nodes in the symmetry lines, will have displacement in the corresponding lines direction. Finally, as the load is uniform, it doesn't suffer any change.

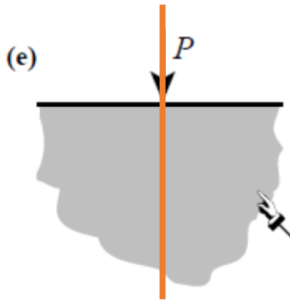


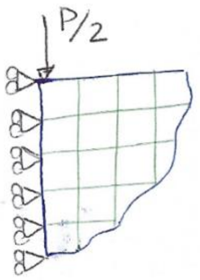


e)

Lines. For case d the only line that could be defined is a vertical symmetry line which matches with the load direction.

In this case, a half of the structure is going to be used. About the BC, we impose in those nodes at the symmetry line no displacement in x direction.
As the load matches with the symmetry line, P/2 is going to be used in the calculation.



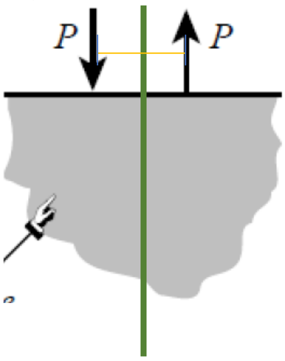


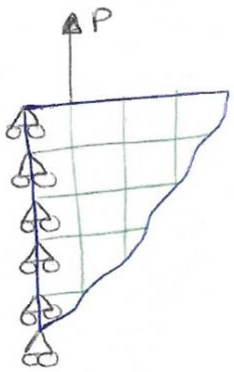
f)

Lines

In this last case, an antisymmetry line can be defined. It would be a vertical line fixed at the middle x point between both loads.

As it is just one antisymmetry line, a half of the structure is going to be used. Applying the antisymmetric BC, those nose at the antisymmetry line will not have y-displacement.





Assignment 2.2

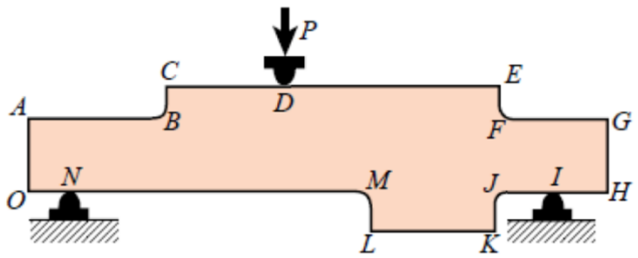
The plate structure shown in the figure is loaded and deforms in the plane of the paper. The applied load at D and the supports at I and N extend over a fairly narrow area. List what you think are the likely "trouble spots" that would require a locally finer finite element mesh to capture high stress gradients. Identify those spots by its letter and a reason.

A fine discretization is used in these zones a high gradient of stresses and/or strains is expected. In this example, these regions will be:

D → The concentrated load P is applied in point D, so that a refined mesh is used to check the behaviour in this zone.

N and I → At these point are placed fixed supports, so that concentrated reactions will appear. The mesh is refined in order to have better knowledge about the behaviour in the area.

B, F, M and J → These points are known as entrant corners, regions in which the principal stresses bunch up. So that, it is important to use a finer mesh.

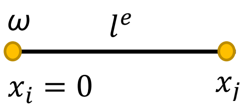


Assignment 2.3

A tapered bar element of length l and areas Ai and Aj with A interpolated as

$$A = A_i(1 - \xi) + A_j\xi$$

and constant density ρ rotates on a plane at uniform angular velocity ω(rad/sec) about node i. Taking axis x along the rotating bar with origin at node i , the centrifugal axial force is q(x)=ρAω²x along the length in which x is the longitudinal coordinate $x = x^e$. Find the consistent node forces as functions of ρ, Ai, Aj, ω and l, and specialize the result to the prismatic bar A = Ai = Aj.



The expression of the element nodal force is taking as

$$f^e = \int_{x_i^e}^{x_j^e} N^T q dx$$

Looking to the area A expression, it is decided to work with the natural coordinates ξ. According to that, a change of variables is done

$$\xi = \frac{x - x_i}{l^e} = \frac{x}{l^e}$$

$$d\xi = \frac{dx}{l^e}$$

Now, the previous expression can be written and solved as function of the different parameters as

$$f^e = \int_0^1 \rho\omega^2\big(A_i(1-\xi) + A_j\xi\big)\xi l^{e\,2}\begin{bmatrix}1-\xi\\ \xi\end{bmatrix} d\xi =$$

$$= \rho\omega^2 l^{e\,2}\int_0^1 \begin{bmatrix}\big(A_i\xi - A_i\xi^2 + A_j\xi^2\big)(1-\xi)\\ \big(A_i\xi - A_i\xi^2 + A_j\xi^2\big)\xi\end{bmatrix} d\xi =$$

$$= \rho\omega^2 l^{e\,2}\begin{bmatrix}\left(A_i\left(\frac{\xi^2}{2} - \frac{\xi^3}{3} - \frac{\xi^3}{3} + \frac{\xi^4}{4}\right) + A_j\left(\frac{\xi^3}{3} - \frac{\xi^4}{4}\right)\right)\\ A_i\left(\frac{\xi^3}{3} - \frac{\xi^4}{4}\right) + A_j\left(\frac{\xi^4}{4}\right)\end{bmatrix}_0^1 = \rho\omega^2 l^{e\,2}\begin{bmatrix}A_i\frac{1}{12} + A_j\frac{1}{12}\\ A_i\frac{1}{12} + A_j\frac{1}{4}\end{bmatrix}$$

Finally, if the solution is generalised for the prismatic bar in which $A = A_i = A_j$, sustiuying in the previous equation the following nodal force is obtained

$$f^e = \rho\omega^2 l^{e\,2}\begin{bmatrix}A\frac{1}{6}\\ A\frac{1}{3}\end{bmatrix}$$