Computational Structural Mechanics and Dynamics, Assignment 2

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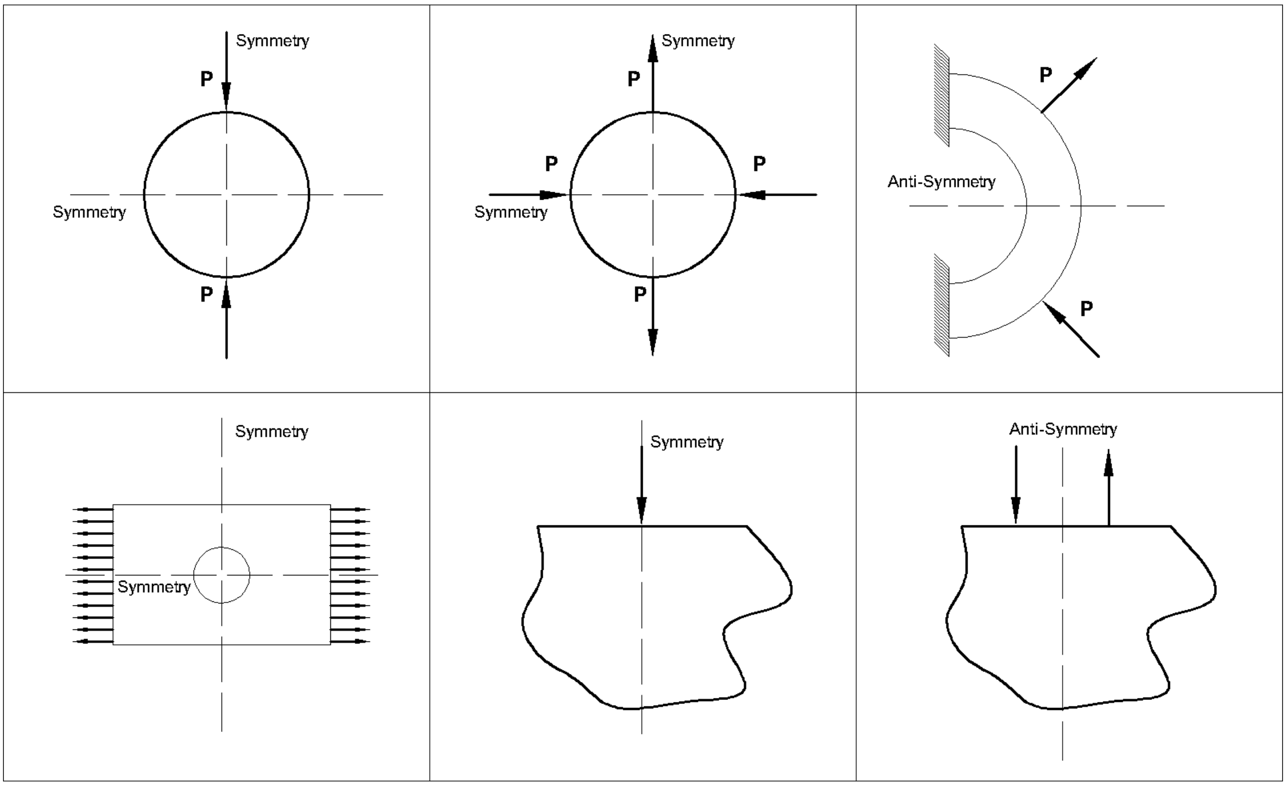
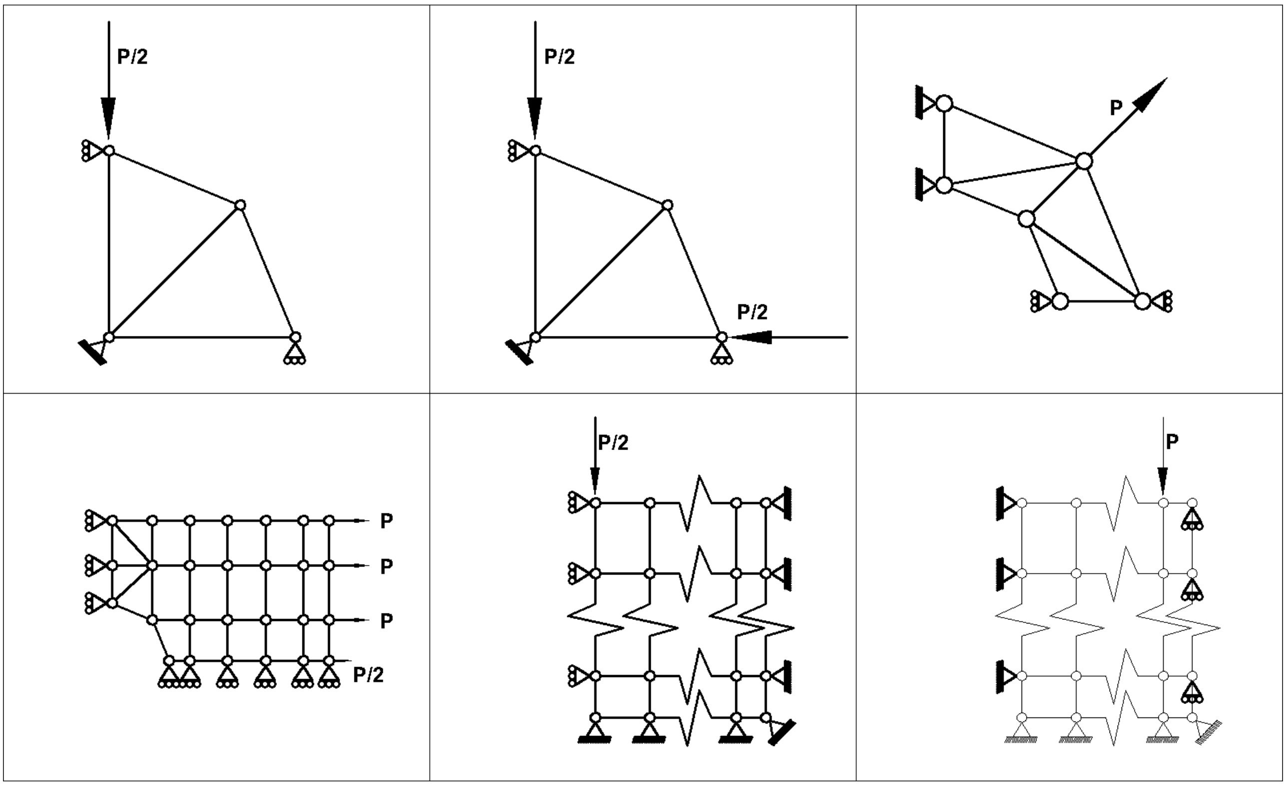
Assignment 2.1

On "FEM Modelling Introduction":

Consider the symmetry and anti symmetry lines in the two dimensional problems illustrated in the figures:



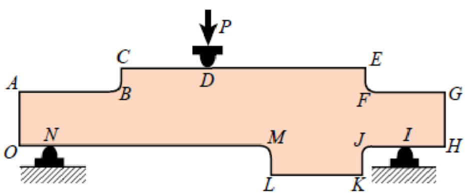


Notice that for the case of the infinite half-planes, the discretization is done is such a way that the supports are sufficiently far away to let deformations occur without mayor restrictions. This is enough if the problem to deal with is static. For the case of dynamic problems, one also needs to consider non-reflecting boundaries.

Assignment 2.2

On "FEM Modelling Introduction":

The plate structure shown in the figure is loaded and deforms in the plane of the paper. The applied load at D and the supports at I and N extend over a fairly narrow area. List what you think are the likely "trouble spots" that would require a locally finer finite element mesh to capture high stress gradients. Identify those spots by its letter and a reason.



Points where refinement is needed are the following:

- B, F, J, M . They are entrant corners
- D, N, I. They are zones of concentrated load.

Assignment 2.3
On "FEM Modelling Introduction":
A tapered bar element of length l and areas A_i and A_j with A interpolated as

$$A = A_i(1-\xi) + A_j(\xi)$$

and constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node i. Taking axis x along the rotating bar with origin at node i , the centrifugal axial force is $q(x) = \rho A\omega^2 x$ along the length in which x is the longitudinal coordinate $x = x^e$.

Find the consistent node forces as functions of ρ, A_i, A_j, ω and l, and specialize the result to the prismatic bar $A = A_i = A_j$.

Finding the Jacobian:

$$\xi = \frac{x - x_1}{l} \qquad J = \frac{dx}{d\xi} = l$$

Expressing the centrifugal axial force in terms of the non-dimensional variable:

$$q(x) = \rho A\omega^2 x \quad \rightarrow \quad q(\xi) = \rho[A_i(1-\xi) + A_j(\xi)]\omega^2 \xi l$$

Using the expression of the nodal force

$$\boldsymbol{f}^e = \int_0^1 \rho[A_i(1-\xi) + A_j(\xi)]\omega^2 \xi l \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} d\xi$$

$$\rho l\omega^2 \begin{bmatrix} \int_0^1 A_i(\xi-\xi^2)(1-\xi)d\xi + \int_0^1 A_j\xi^2(1-\xi)d\xi \\ \int_0^1 A_i(\xi-\xi^2)(\xi)d\xi + \int_0^1 A_j\xi^3 d\xi \end{bmatrix}$$

$$\boldsymbol{f}^e \rho l\omega^2 \begin{bmatrix} \frac{A_i+A_j}{12} \\ \frac{A_i}{12} + \frac{A_j}{4} \end{bmatrix}$$

For the case $A = A_i = A_j$:

$$\boldsymbol{f}^e = A\rho l\omega^2 \begin{bmatrix} \frac{1}{6} \\ \frac{1}{3} \end{bmatrix}$$