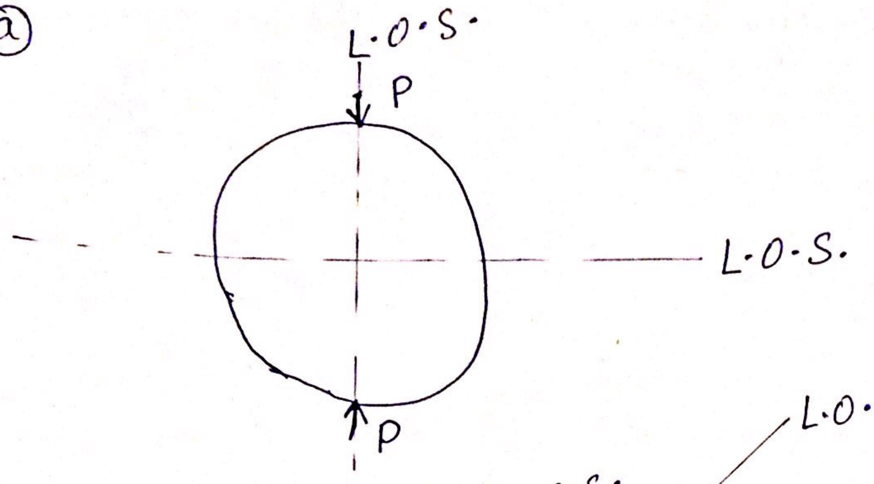
PRADEEP KUMAR BAL

①

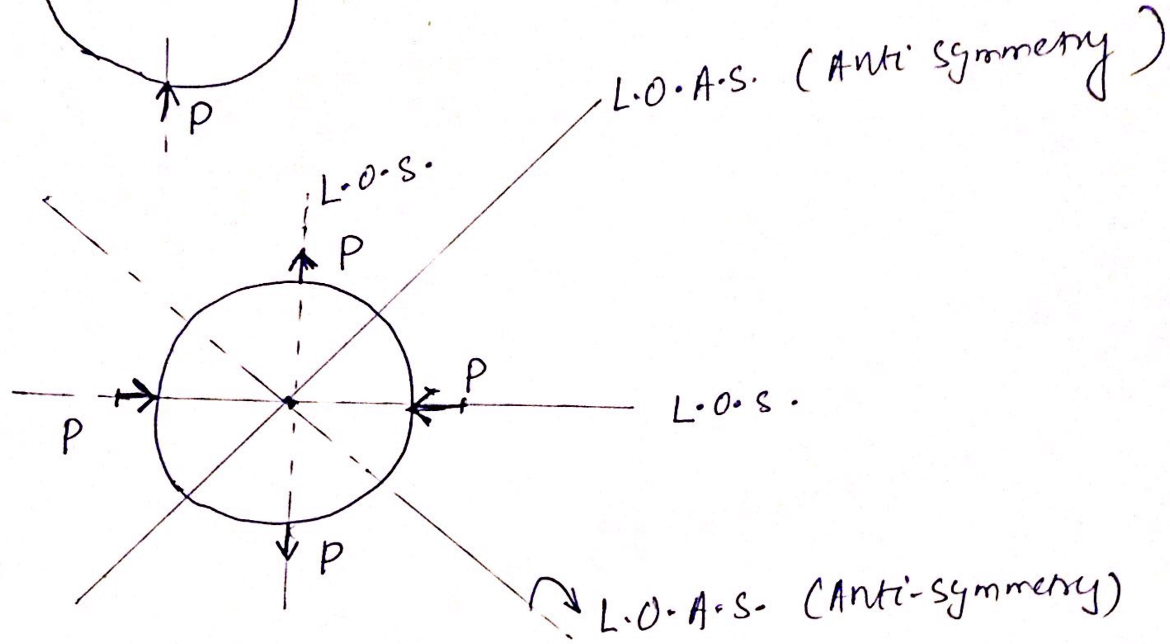
L.O.S. → Line of symmetry

L.O.AS. → Line of Antisymmetry

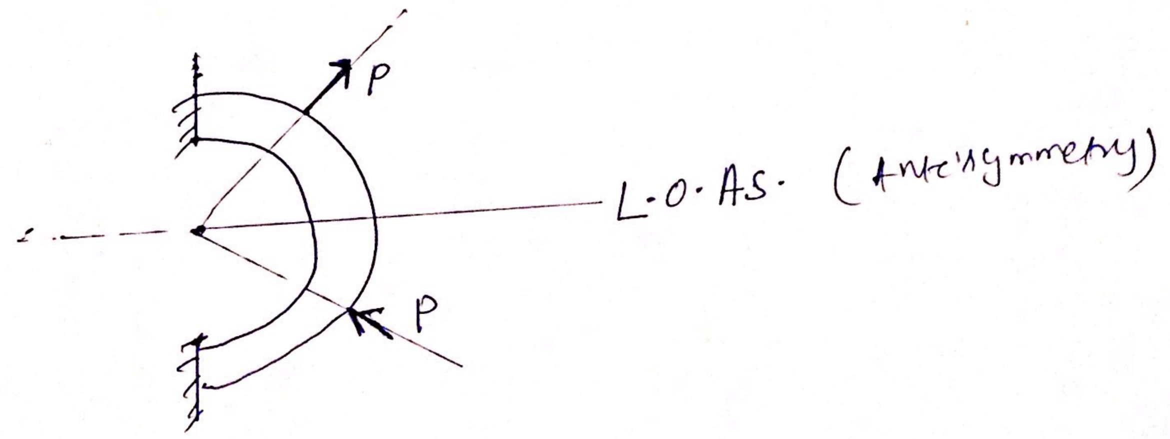
ⓐ



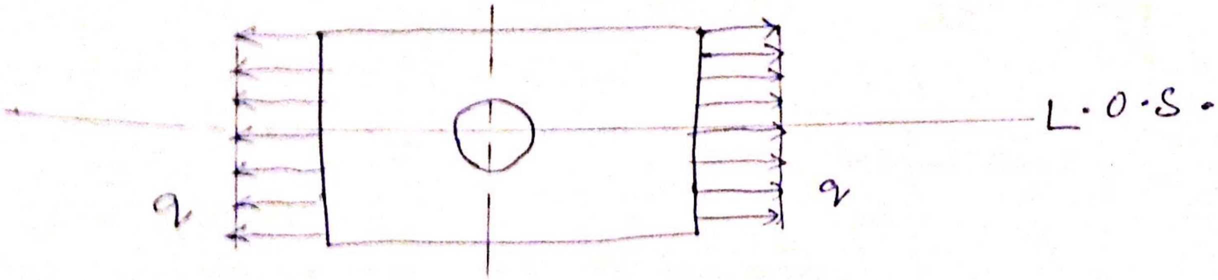
ⓑ



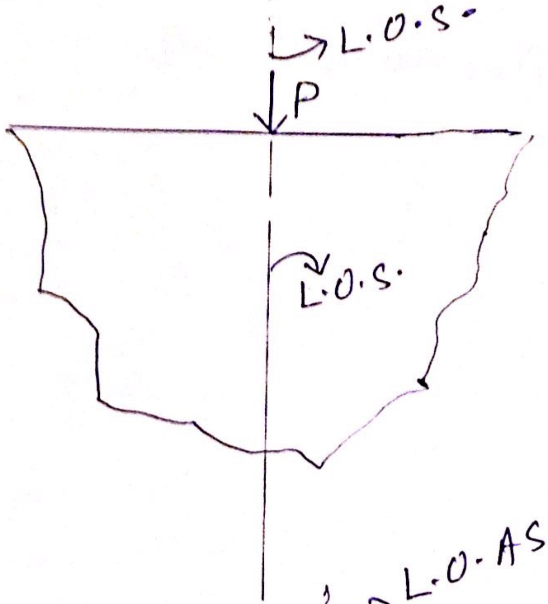
ⓒ



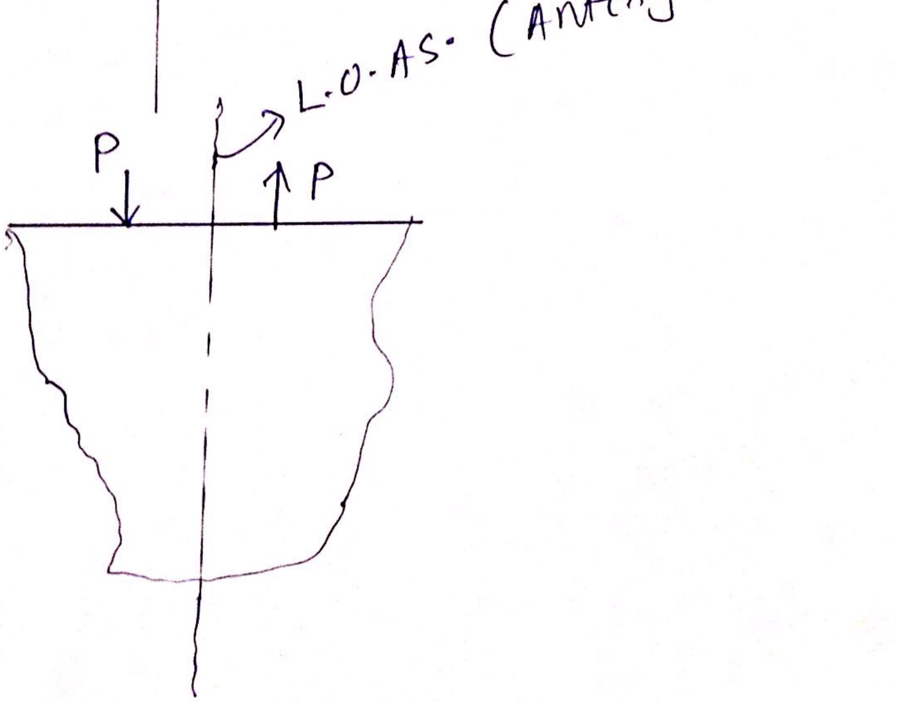
(a)



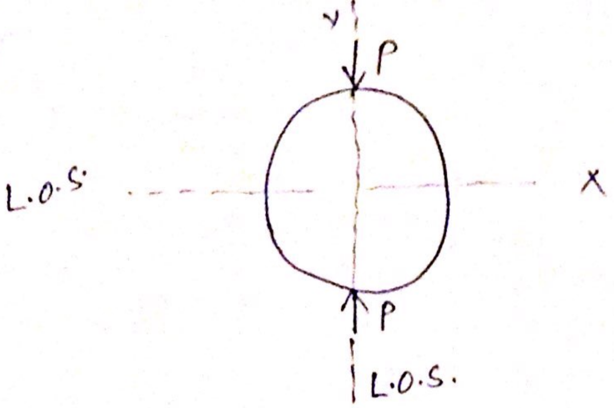
(c)



(b)



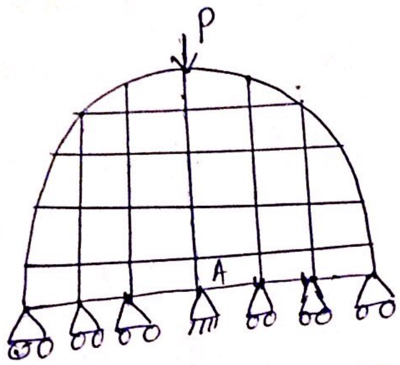
2 (a)



It is possible to cut the complete structure to both one half and one quarter.

One Half :-

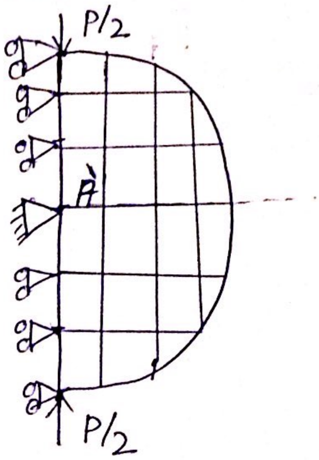
(a) It can be cut along x axis



→ No displacement in the y direction for any point on the x Axis.

→ To preclude horizontal rigid body motion node 'A' is constrained in the x, y directions

(b) It can be cut along y axis



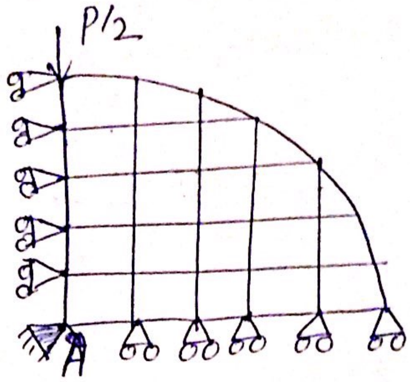
→ No displacement in the x direction for any point on the y axis

→ To preclude vertical rigid body motion node A' is constrained in the x, y directions.

('A' is constrained both in x, y directions)

2.(a)

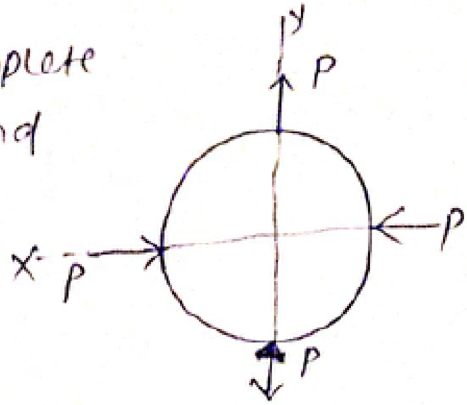
One Quarter :-



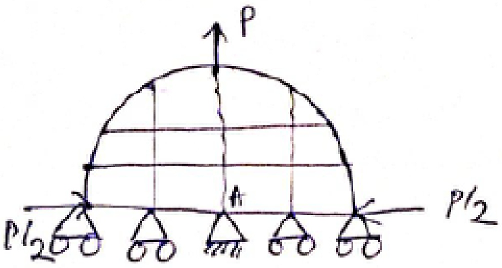
As the structure is doubly symmetric in both geometry and loading, it is evident that no y displacements are possible for points on the x axis and no x displacements are possible for any points on the y axis.

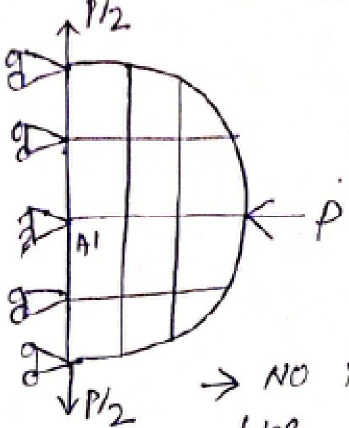
(b)

It is possible to cut the complete structure to both one half and one quarter.



One Half





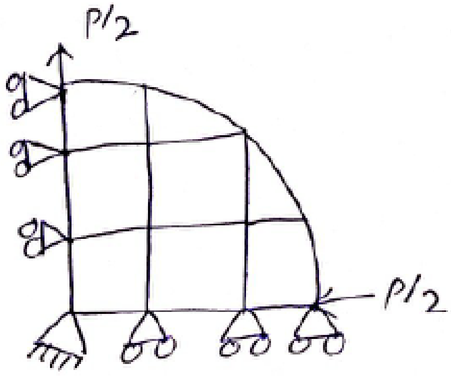
→ No y displacements of the points on the x-axis.

→ To preclude Horizontal rigid body motion node 'A' is constrained in the x direction.

→ No x displacement of the points on the y axis

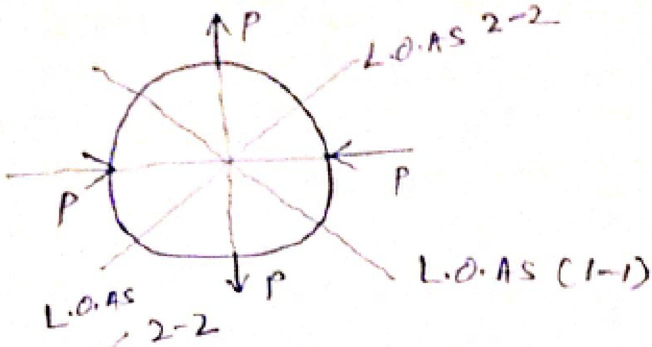
→ To preclude vertical rigid body motion node A' is constrained in the y direction.

One Fourth



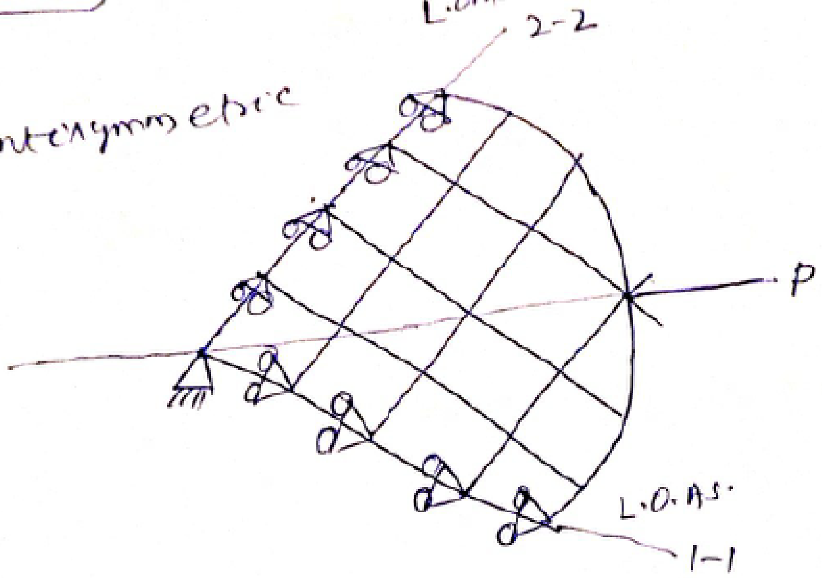
→ As the structure is doubly symmetric in both geometry and loading, It is evident that no y displacements are possible for points on the x axis and No x displacements are possible for any points on the y axis.

It also can be cut to both one half and one quarter along the line of Anti-symmetries.

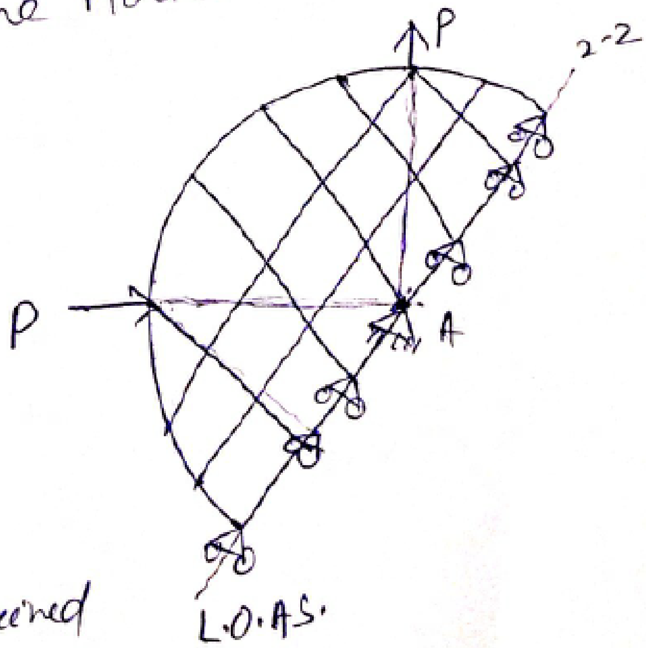


one quarter

1-1 are antisymmetric
2-2 Lines



The displacement of the nodes which are on 1-1 are zero along 1-1; the displacement of the nodes which are on 2-2 are zero along 2-2.



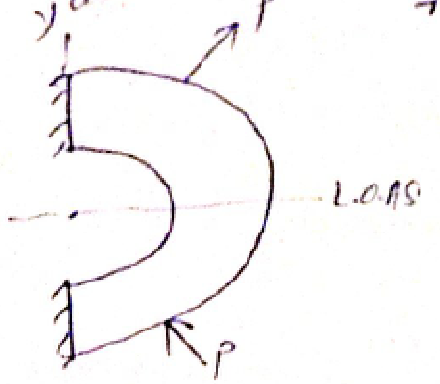
one half

The displacement of nodes on 2-2 are zero along 2-2.

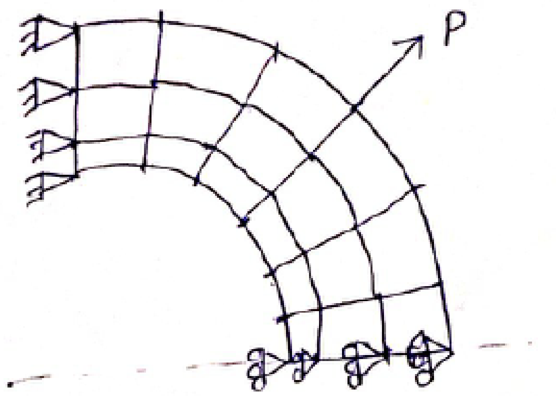
The node A is constrained in both 1-1 and 2-2 to prevent rigid body motion,

2. ©

It is possible to cut the structure to one half along the line of antisymmetry. (x axis)



One half

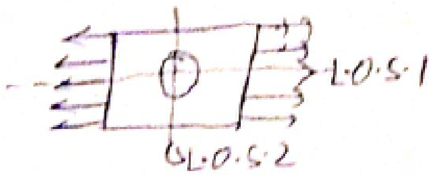


→ No displacements in both the x and y axis for the points on the y axis. (fixed position)

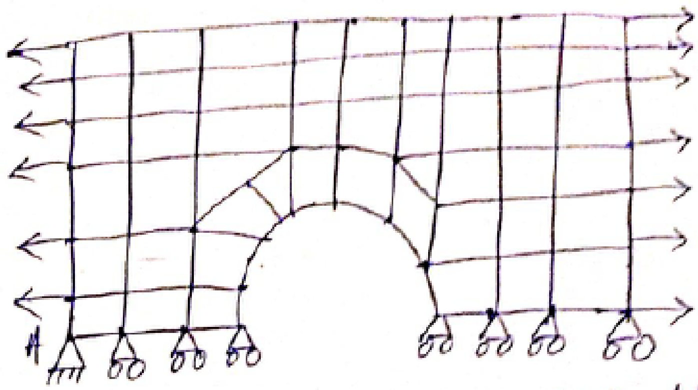
→ No x displacements ~~for~~ the points on the line of Antisymmetry (x axis) of the structure.

It is possible to cut the complete structure to both one half and one quarter.

ONE HALF :-

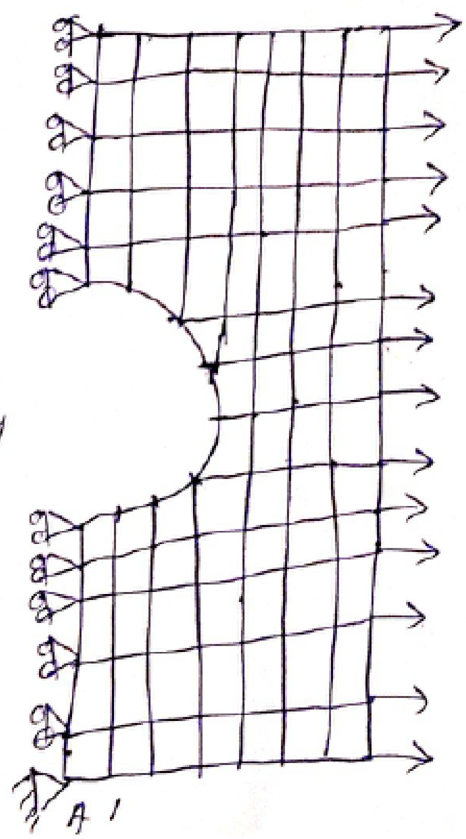


Along L.O.S:-1



The y displacements of the nodes of the structure on the x axis are zero. To prevent rigid body motion in the x direction, both the x, y displacements of the node A are constrained.

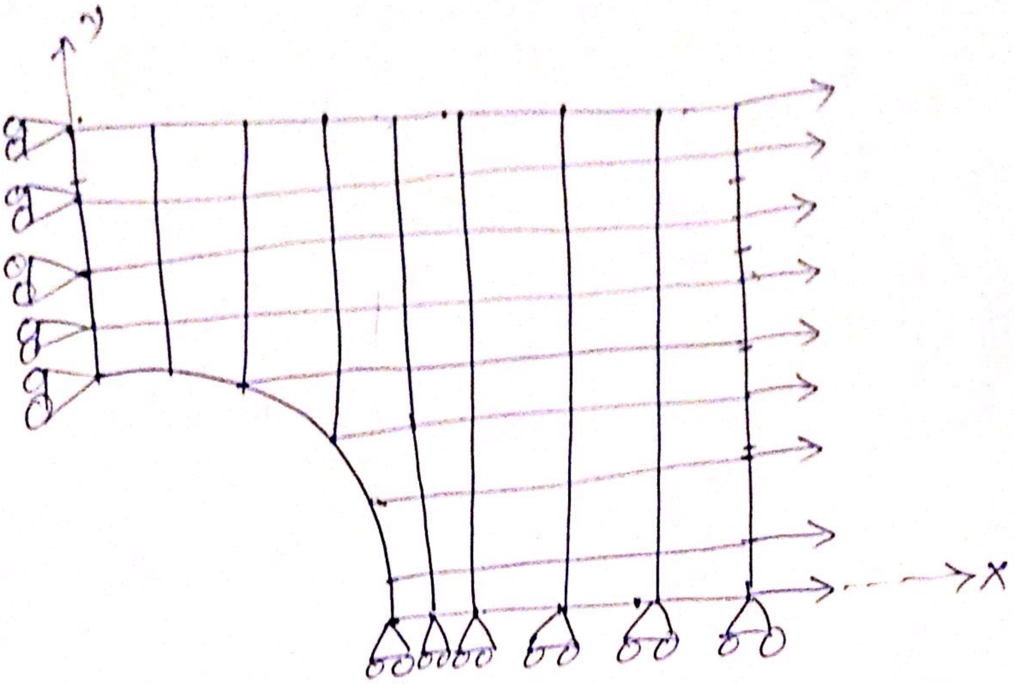
Along L.O.S:-2



→ The x displacements of the nodes of the structure on the y axis are zero.

→ To preclude rigid body motion in the y direction, both the x and y displacements of the node A' are constrained.

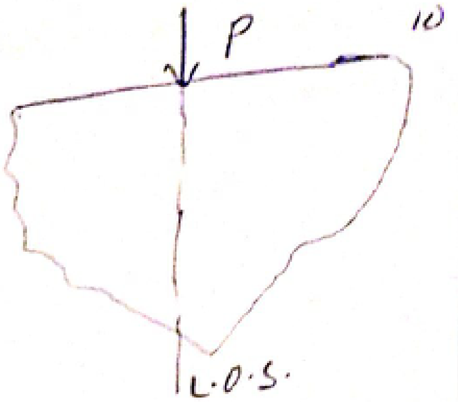
one quarter



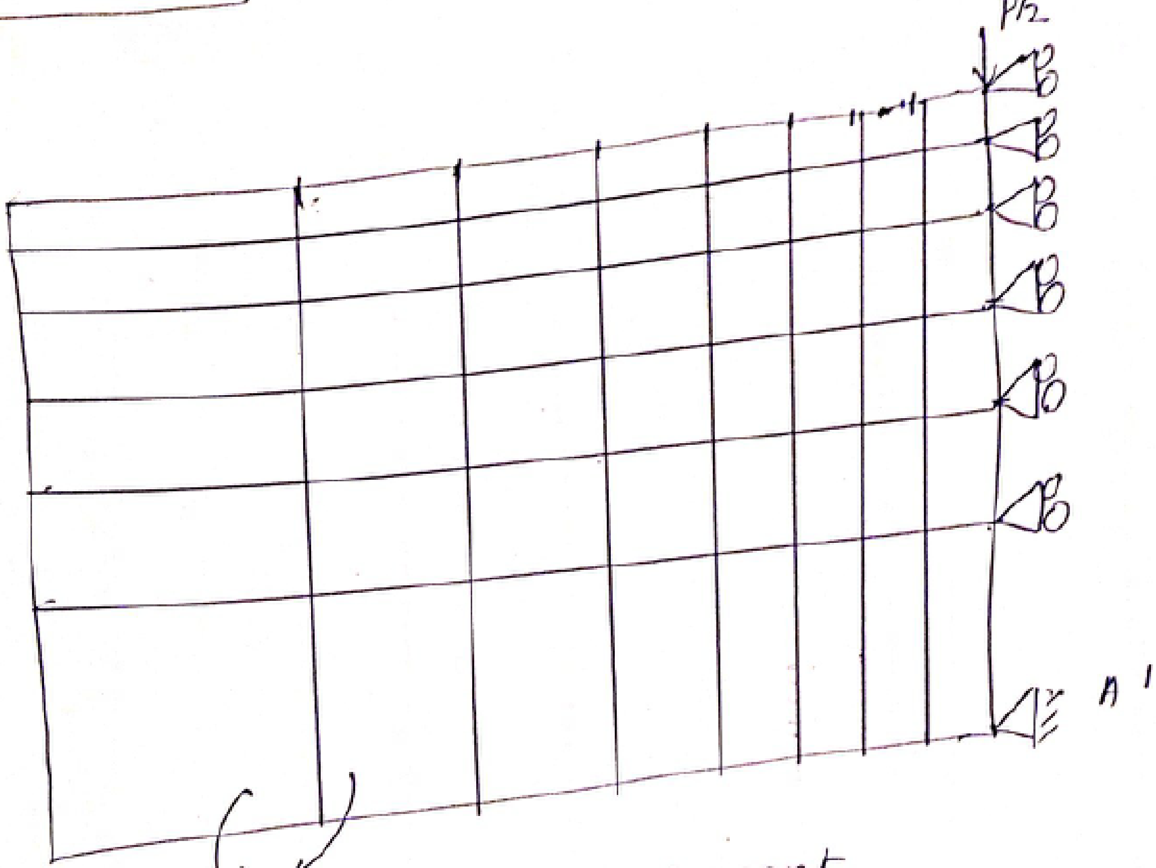
The vertical displacements of the nodes of the structure on the x axis are zero.

The horizontal displacements of the nodes of the structure on the y axis are zero.

2.(e) It is possible to cut the complete structure to one half along the line of symmetry.



ONE HALF



Infinite Boundary Element

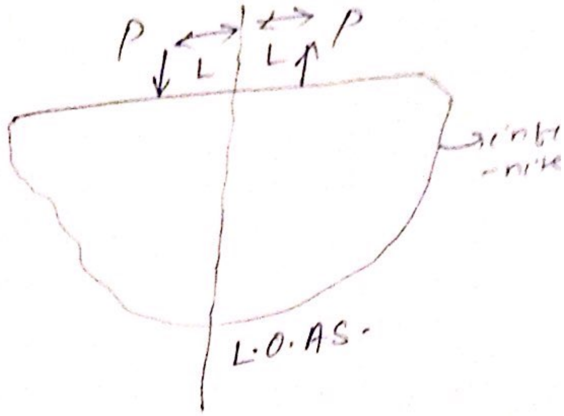
X displacements of the Nodes on the line of symmetry are zero.

(The nodes represent infinite Element will also have very small X, Y displacements).

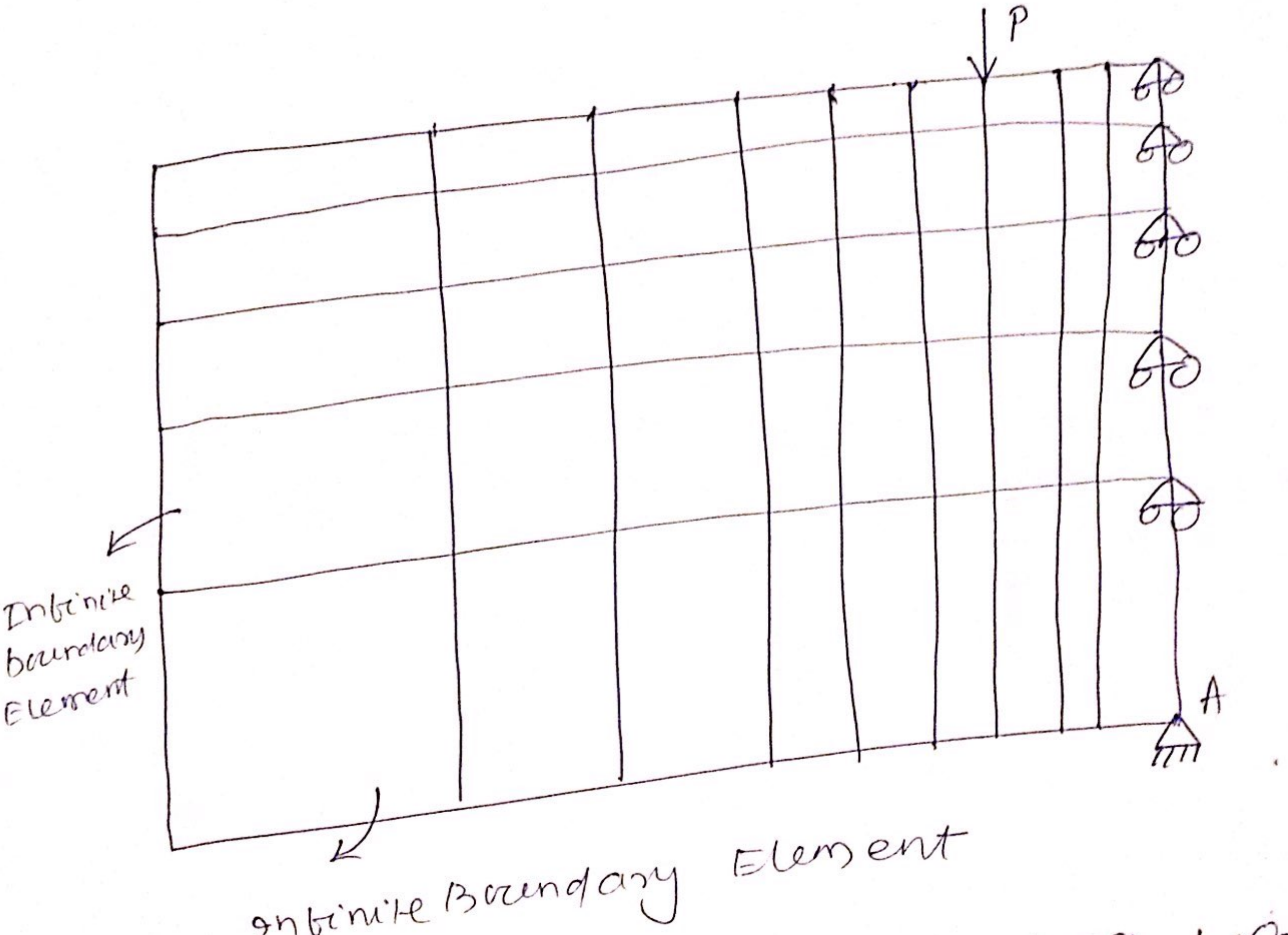
A' is constrained in both X, Y directions to prevent the rigid body motion.

Scanned by CamScanner

2.(f) It is possible to cut the complete "structure to one half along the line of antisymmetry.



One Half

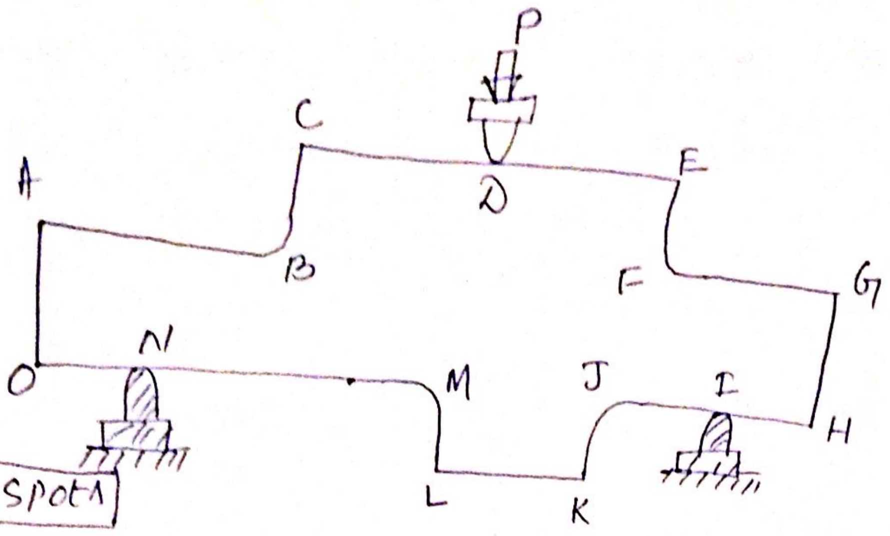


y displacements of the nodes along L.O.AS. are zero.

Also, the nodes which are at infinite will have very small x, y displacements.

The constraint at point A prevent rigid body motion.

2.2



D :- vicinity of point Load

B, M, J, F :- Entrant corners or sharply curved Edges.

N, I :- vicinity of concentrated reactions

$\boxed{2\text{-}3}$

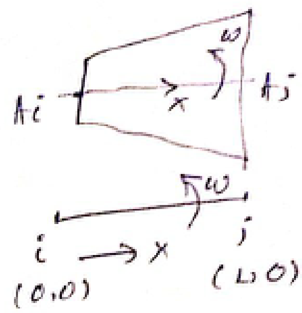
The length of the given tapered bar $= L$

Area is interpolated as $A = A_i(1-\xi) + A_j\xi$

where $\xi = \frac{x-x_1}{L} = \frac{x-0}{L} = \frac{x}{L}$

$\Rightarrow x = \xi L$

$\Rightarrow dx = L\,d\xi$

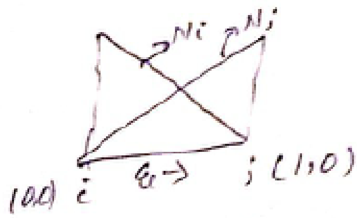


The consistent node force vector $\underline{f}^e$ comes from the element contribution to the external work potential W:

$$W^e = \int_{x_1}^{x_2} q u\,dx = \int_0^1 q N^T \underline{u}^e L\,d\xi = (u^e)^T \int_0^1 q \begin{bmatrix}1-\xi\\ \xi\end{bmatrix} L\,d\xi \overset{\text{def}}{=} (u^e)^T f^e$$

Since, $\underline{u}^e$ is arbitrary

$$\underline{f}^e = \int_0^1 q\begin{bmatrix}1-\xi\\ \xi\end{bmatrix} L\,d\xi$$



$N_i = 1-\xi$

$N_j = \xi$

$q(x) = \rho A\omega^2 x$

$q(\xi) = \rho A\omega^2 \xi L$

$$\underline{f}^{(e)} = \int_0^1 \rho A\omega^2 L^2 \begin{bmatrix}\xi-\xi^2\\ \xi^2\end{bmatrix} d\xi$$

$$= \rho\omega^2L^2 \int_0^1 [A_i(1-\xi)+A_j\xi]\begin{bmatrix}\xi-\xi^2\\ \xi^2\end{bmatrix}d\xi$$

$$= \rho\omega^2L^2\int_0^1 \begin{bmatrix} A_i(\xi-\xi^2-\xi^2+\xi^3)+A_j[\xi^2-\xi^3] \\ A_i(\xi^2-\xi^3)+A_j\xi^3\end{bmatrix} d\xi$$

(14/2)

$$= \rho \omega^2 L^2 \int_0^1 \begin{bmatrix} (A_i(\xi_e - 2\xi_i^2 + \xi_i^3) + A_j(\xi_i^2 - \xi_i^3)) \\ (A_i(\xi_i^2 - \xi_i^3) + A_j \xi_i^3) \end{bmatrix} d\xi_e$$

$$= \rho \omega^2 L^2 \begin{bmatrix} \left[A_i\left(\frac{\xi_i^2}{2} - \frac{2}{3}\xi_i^3 + \frac{\xi_i^4}{4}\right) + A_j\left(\frac{\xi_i^3}{3} - \frac{\xi_i^4}{4}\right)\right]_0^1 \\ \left[A_i\left(\frac{\xi_i^3}{3} - \frac{\xi_i^4}{4}\right) + A_j \frac{\xi_i^4}{4}\right]_0^1 \end{bmatrix}$$

$$= \rho \omega^2 L^2 \begin{bmatrix} A_i\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right) + A_j\left(\frac{1}{3} - \frac{1}{4}\right) \\ A_i\left(\frac{1}{3} - \frac{1}{4}\right) + A_j \frac{1}{4} \end{bmatrix}$$

$$\Rightarrow \underline{f}^{(e)} = \rho \omega^2 L^2 \begin{bmatrix} \frac{A_i}{12} + \frac{A_j}{12} \\ \frac{A_i}{12} + \frac{A_j}{4} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} f_i^e \\ f_j^e \end{bmatrix} = \rho \omega^2 L^2 \begin{bmatrix} \frac{A_i + A_j}{12} \\ \frac{A_i + 3A_j}{12} \end{bmatrix}$$ (Ans)

For prismatic bar $(A_i = A_j)$; $A_i = A_j = A$ (say)

$$\begin{bmatrix} f_i^e \\ f_j^e \end{bmatrix} = \rho \omega^2 L^2 \begin{bmatrix} \frac{A}{6} \\ \frac{A}{3} \end{bmatrix} = \begin{bmatrix} \frac{\rho \omega^2 L^2 A}{6} \\ \frac{\rho \omega^2 L^2 A}{3} \end{bmatrix}$$

(Ans)