UNIVERSITAT POLITÈCNICA DE CATALUNYA

MASTER IN COMPUTATION MECHANICS AND NUMERICAL METHODS IN ENGINEERING

COMPUTATIONAL STRUCTURAL MECHANICS AND DYNAMICS

Assignment 2

by

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1- Introduction

The goal of the first part of the assignment (sections 2.1 and 2.2) is to identify symmetry and antisymmetry lines of 2D structures under loads. Following, the structures are reduced according to the type of symmetry and the suitable boundary conditions are applied in order to keep the overall behavior of the structure. On the second part (section 2.4), the definitions of verification and validation were stated. For last part of the assignment (section 2.5), a problem is solved considering the variational approach. A discussion about each part of the assignment was also considered.

2 – Assignment 2

2.1 – Assignment 2.1 – Part 1

The symmetry and antisymmetry lines of the considered structures [1] are presented below :

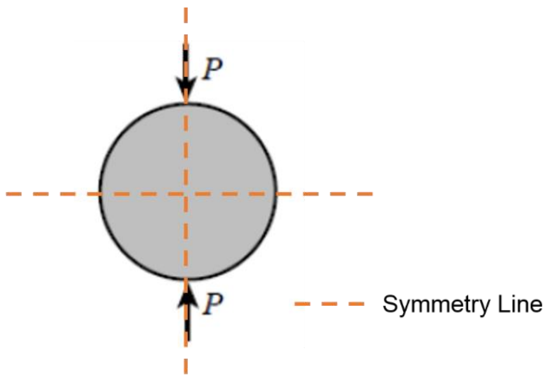

Figure 1. Symmetry Lines for circular disk under two diametrically opposite forces.

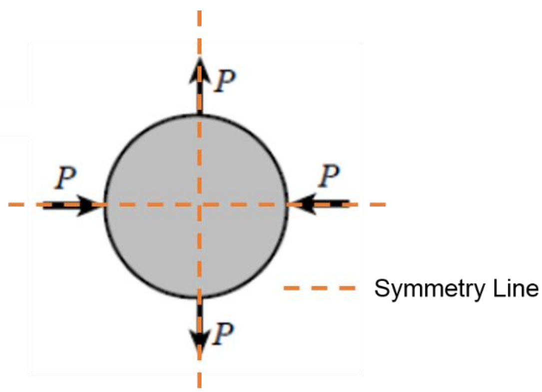

Figure 2. Symmetry Lines for circular disk under two diametrically opposite force pairs.

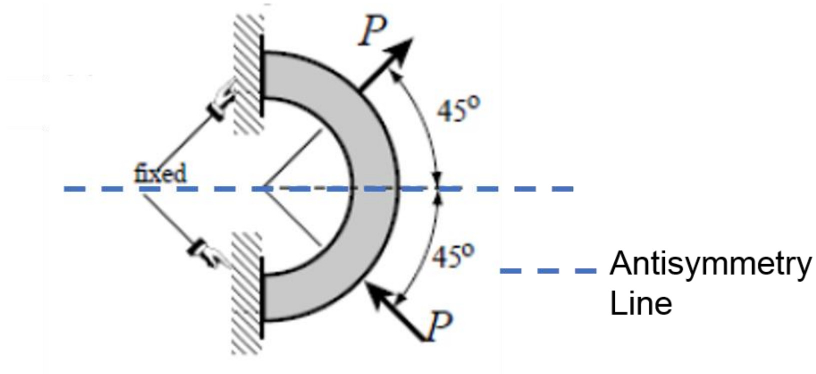

Figure 3. Antisymmetry Line for a clamped semiannulus under force pair.

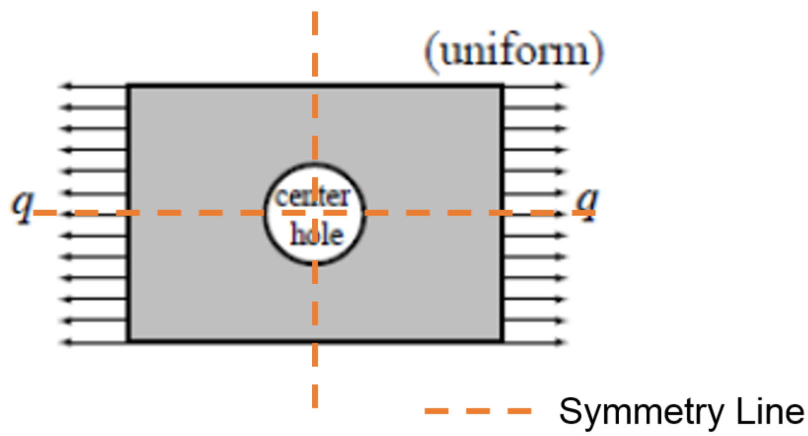

Figure 4. Symmetry Line for stretched plate with central circular hole.

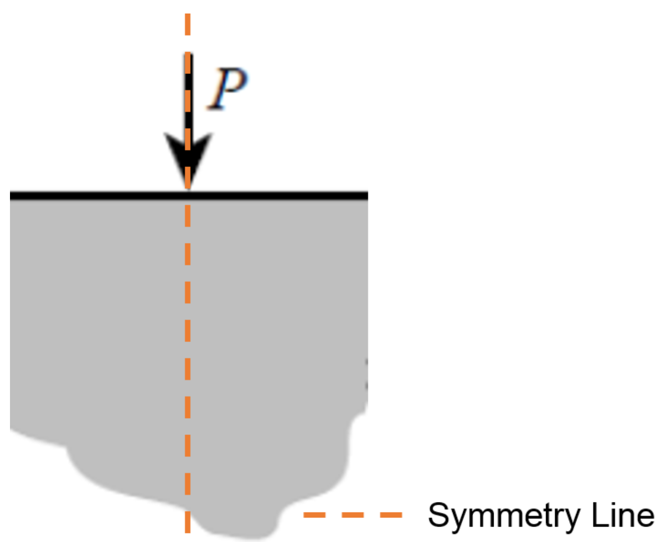

Figure 5. Symmetry line for half plane under concentrated load.

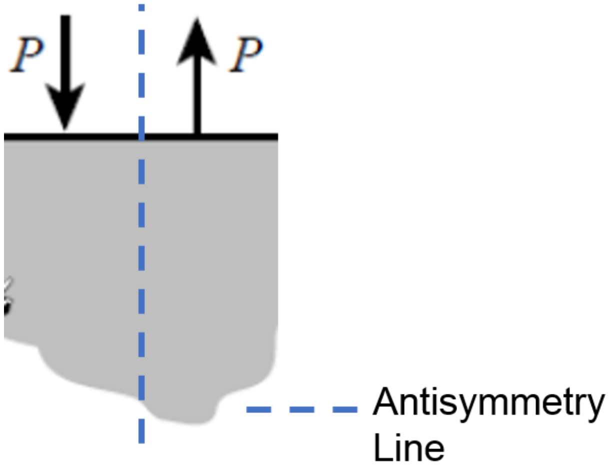

Figure 6. Antisymmetry line for half plane under concentrated loads.

2.2 – Assignment 2.1 – Part 2

Reducing the load cases presented in Figures 1-6 according to the symmetry and antisymetry lines, discretizing coarsely the reduced structures and applying suitable bounday conditions, the load cases become :

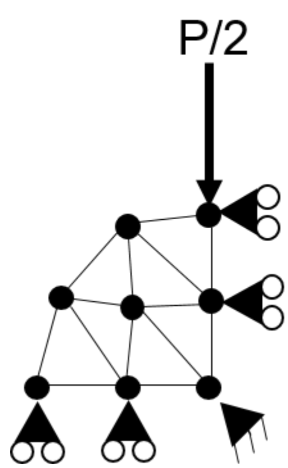

Figure 7. Reduced load case for circular disk under two diametrically opposite forces (symmetry case).

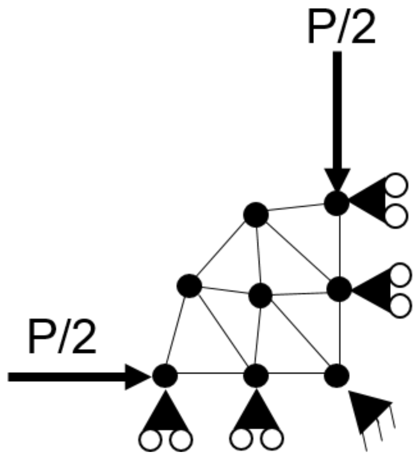

Figure 8. Reduced load case for circular disk under two diametrically opposite force pairs (symmetry case).

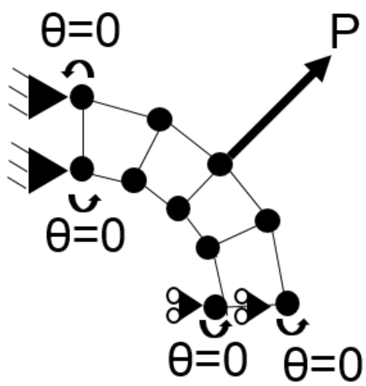

Figure 9. Reduced load case for a clamped semiannulus under force pair (antisymmetry case).

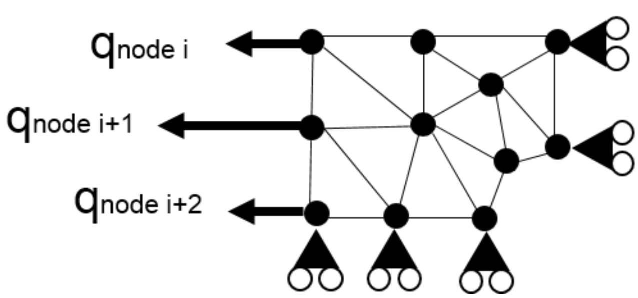

Figure 10. Reduced load case for stretched plate with central circular hole (symmetry case).

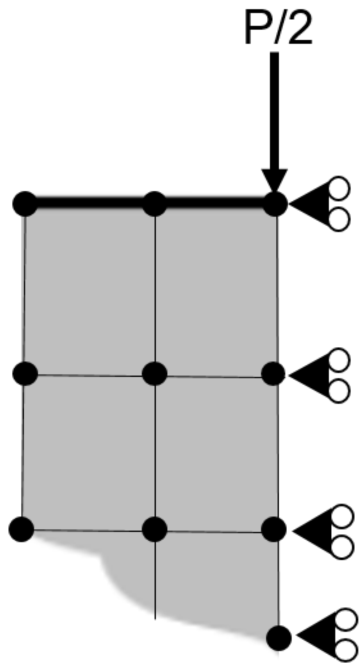

Figure 11. Reduced load case for half plane under concentrated load (symmetry case).

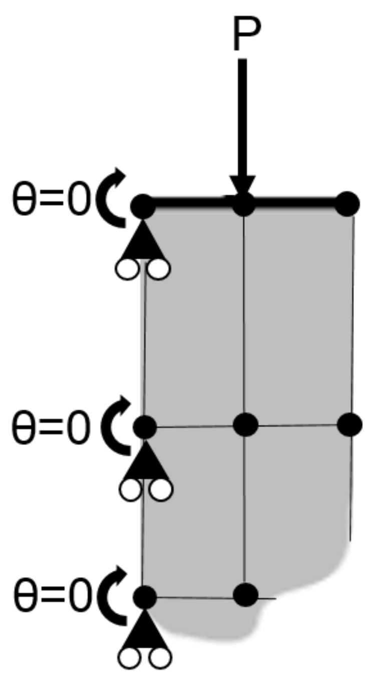

Figure 12. Reduced load case for half plane under concentrated loads (antisymmetry case).

2.3 – Discussion on Symmetry and Antisymmetry

Identifying lines of symmetry and antisymmetry has great advantages for solving structural problems. If the problem presents such lines, it can be reduced and solved with much more efficiency. The computational cost is reduced due to the reduction of number of nodes. Therefore, it allows a deeper study of mesh convergence on the reduced structure regarding which type of elements could provide a more reliable solution and how dense the mesh should be to achive such solution. Nevertheless, making use of symmetry and antisymmetry lines require attention when it comes to applying suitable boundary conditions to the reduced structure. The reduced structure must present the same behavior as if the whole structure was to be solved and applying the right boundary conditions guarantees that.

2.4 – Assignment 2.2

Validation – the validation of the model refers to how close the model represents the real problem. In that sense, validation is related to solving the right mathematical equations considering the right boundary conditions regarding the physics of the problem [2]. The model should present a similar behavior of real physical problem to be considered as validated.

Verification – the verification of the model is related to how the chosen mathematical equations to represent the real physical problem are being solved. In that sense, verification inquires if the chosen model is providing the best or most reliable solution it could provide. If the Finite Element Method is considered to solve the mathematical equations, verifying how the type of element and its size (mesh convergence) influence the final solution could be considered as a verification process of the model [2]. Once the best or most reliable solution of the model has been determined, it can compared to the real physical problem for its validation.

2.5 – Discussion on Verification and Validation

The verification and validation procedures are of great importance for modeling real physical problems. They guarantee that the solution to an idealized problem represents the real physical problem with certain fidelity. When the verification and validation conditions are met for a certain computational solution, the computational model becomes reliable and it can be expanded/extrapolated to other conditions.

2.6 – Assignment 2.3

To find the consistent node forces as required in assignment 2 .3 [1], we apply the following equation [3]:

$$\boldsymbol{f}_{ext} = \int_0^1 q \begin{bmatrix} 1 - \zeta \\ \zeta \end{bmatrix} l d\zeta \tag{1}$$

Where l is the length of the bar, q is the distributed load and ζ is the local coordinate defined as :

$$\zeta = \frac{x - x_i}{l}$$

It is worth mentioning that the component f_1 of vector $\mathbf{f}_{ext}$ refers to the consistent force at node i and the component f_2 refers to the consisten force at node j.

Considering Equation (1) and the data provided in assignment 2.3 [1], the consistent node forces are :

$$f_i = \frac{1}{12} \rho \omega^2 l^2 (A_i + A_j)$$

$$f_j = \frac{1}{4} \rho \omega^2 l^2 \left(\frac{A_i}{3} + A_j\right)$$

Specializing the result for a prismatic bar ($A = A_i = A_j$), the components of the consistent node force vector $\mathbf{f}_{ext}$ are :

$$f_i = \frac{1}{6} \rho \omega^2 l^2 A$$

$$f_j = \frac{1}{3} \rho \omega^2 l^2 A$$

2.7 – Discussion on the Variational Approach

The Variational Aprroach offers a different perspective on how to obtain the master stiffness equations. Instead of applying direct equilibium between the domain and the boundary conditions (Direct Stiffness Method), it calculates the Total Potencional Energy (TPE) functional of the structure, defined as the difference between the internal energy and the external work acting on the structure [3]. With the TPE functional, the Minimum Potential Energy Principle is applied (minimization of TPE functional) to obtain expressions for the elemental stiffness matrix and the element consistent force vector. The resulting element stiffness matrix and consistent force vector are the same as the ones obtained by applying the Direct Stiffness Method [3]. Nevertheless, the Variational Approach, or Energy Approach, can be considered as more efficient when the structure is more complicated, such as 2D structures in 2D/3D spaces. Its weak form provides the elemental stiffness equation in a more general and straight-forward manner in comparison with the disconnection and localization steps found in the Direct Stiffness Method.

3 - References

[1] – Assignment-2, Computational Structural Mechanics, Master of Science in Computational Mechanics, 2020.

[2] – Presentation ‘Introduction to V&V’, Finite Elements, Master of Science in Computational Mechanics, 2019.

[3] – Presentation ‘Variational Fomulation of Bar Element’, Computational Structural Mechanics and Dynamics, Master of Science in Computational Mechanics, 2020.