Master on Numerical Methods in Engineering

Computational Structural Mechanics and Dynamics

Assignment 10

Solid and structural dynamics

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On “Solid and Structural Dynamics”:

1. In the dynamic system of slide 6, let $r(t)$ be a constant force F. What is the effect of F on the time-dependent displacement $u(t)$ and the natural frequency of vibration of the system?

2. A weight whose mass is m is placed at the middle of a uniform axial bar of length L that is clamped at both ends. The mass of the bar may be neglected. Estimate the natural frequency of vibration in terms of m, L, E and A. Suggestion: First determine the effective k.

3. Use the expression on slide 18 to derive the mass matrix of slide 17.

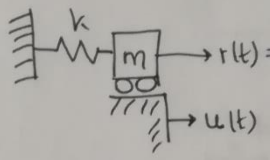
4. Obtain also the mass matrix of a two-node, linear displacement element with a variable cross-sectional area that varies from A_1 to A_2.

5. A uniform two-node bar element is allowed to move in a 3D space. The nodes have only translational d.o.f. What is the diagonal mass matrix of the element?

Assignment 10: SOLID AND STRUCTURAL DYNAMICS

MOC

Section 10.1:



Newtons second law (f=ma) reads:

$$m\ddot{u} + ku = F$$

Displacement differential equation when $r = F = 0$ leads to:

$m \cdot \ddot{u} + ku = 0$ It is the case of free undamped vibration.

On the other hand, if $F \neq 0$: Particular case for forced undamped vibrations:

$$ku = F \longrightarrow u_p(t) = F/k$$

Total force balance equation is equal to:

$$u(t) = u(t)_{(F=0)} + u(t)_{(F\neq 0)}$$

$$[1] \quad u(t) = [A\sin(\omega t) + B\cos(\omega t)] + F/k \quad ; \quad \omega = \sqrt{k/m}$$

Giving initial condition values as:

$$\begin{cases} t = 0 \\ u(0) = 0 \end{cases}$$

$$0 = 0 + B \cdot 1 + F/k \longrightarrow \begin{cases} B = -F/k \\ A = 0. \end{cases}$$

$$u(t) = F/k + \frac{F}{k}\cos(\omega t)$$

$$\boxed{u(t) = [1 + \cos(\omega t)] \cdot \frac{F}{k}}$$ TIME DEPENDENT DISPLACEMENT.

(1)

Section 10.2.

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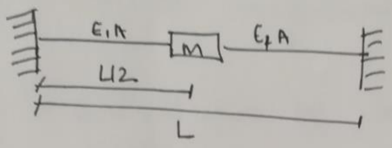
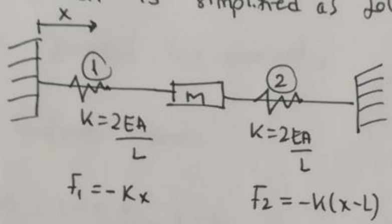
The problem description is as follows:



Which is simplified as follows:



From Newton's 2nd Law: $\sum f = ma$:

$$m\ddot{x} = f_1 + f_2 = -K(2x - L)$$

For this problem, equilibrium position is expected in the middle of the bar: where $f_1 + f_2 = 0$.

$$f_1 + f_2 = 0 \rightarrow -K(2x - L) = 0 \rightarrow x_{equilibrium} = L/2$$

If $u = x - x_{eq} \rightarrow u = x - L/2$

Then:

$$m\ddot{u} = -2Ku - \cancel{2K\left(\tfrac{L}{2}\right)} + \cancel{KL}$$

$$m\ddot{u} = -2Ku$$

$$\omega = \sqrt{\frac{2K}{m}}$$

$$\omega = \sqrt{\frac{2 \cdot \frac{2EA}{L}}{m}} \longrightarrow \boxed{\omega = \sqrt{\frac{4EA}{Lm}}}$$ NATURAL FREQUENCY OF VIBRATION.

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Section 10.3 :

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Expression in slide 20: $m = \int N^T N \rho \, dV$.

Expression to use in slide 19: $m = \begin{bmatrix} \frac{\rho A L}{3} & \frac{\rho A L}{6} \\ \frac{\rho A L}{6} & \frac{\rho A L}{3} \end{bmatrix}$.

Case of a 2-noded bar element:

The shape functions are:

$$N_1 = 1 - \frac{x - x_1}{l} = \frac{1-\xi}{2}$$
$$N_2 = \frac{x - x_1}{l} = \frac{(1+\xi)}{2}$$

; ξ is the isoparametric natural coordinate $(-1, 1)$

$$m = \rho A \int_0^l N^T N \, dx = \frac{1}{4} \rho l A \int_{-1}^{1} \begin{bmatrix} 1-\xi \\ 1+\xi \end{bmatrix} [1-\xi] [1+\xi] \, d\xi =$$

$$= \frac{1}{6} \rho l A \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\boxed{m = \frac{\rho A L}{3} \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix}}$$

Section 10.4 :

To solve this case, Direct Mass Lumping procedure can be used.
It will lead to:

$$\boxed{m = \frac{1}{2} \rho L \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}}$$

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Section (10.5):

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Uniform 2-noded bar element
3D space movement.
Only translational dof.

What is the diagonal mass matrix of the element?

$$m_L = \frac{1}{2}\rho A L \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

MATRIX CONSTRUCTED
BY DIAGONALIZED LUMPING.

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