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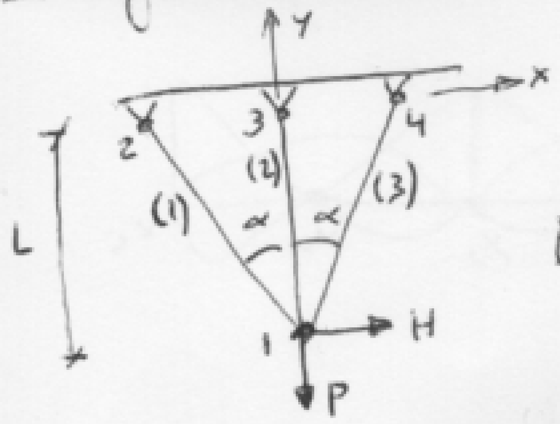
Assignment 1.1



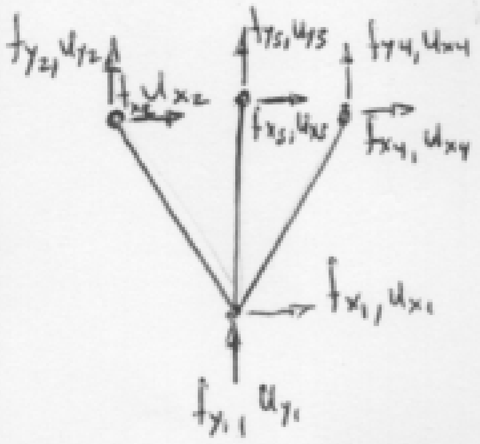
Variables
L, α, E, A, P, H

8 dof

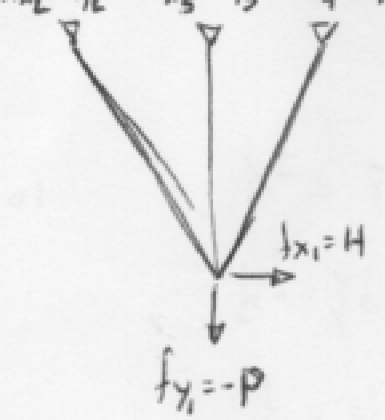
$E^{(1)} = E^{(2)} = E^{(3)} = E \qquad A^{(1)} = A^{(2)} = A^{(3)} = A$

$L^{(1)} = L^{(3)} = L/\cos\alpha = L/c$

$L^{(2)} = L$



$u_{x_2} = u_{y_2} = u_{x_3} = u_{y_3} = u_{x_4} = u_{y_4} = 0$



Element Stiffness equation in local coordinates

Element (1)

$$\begin{bmatrix} \bar{f}_{x_1}^{(1)} \\ \bar{f}_{y_1}^{(1)} \\ \bar{f}_{x_2}^{(1)} \\ \bar{f}_{y_2}^{(1)} \end{bmatrix} = \frac{EAc}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{u}_{x_1}^{(1)} \\ \bar{u}_{y_1}^{(1)} \\ \bar{u}_{x_2}^{(1)} \\ \bar{u}_{y_2}^{(1)} \end{bmatrix} \qquad T^{(1)} = \begin{bmatrix} -s & c & 0 & 0 \\ -c & -s & 0 & 0 \\ 0 & 0 & -s & c \\ 0 & 0 & c & -s \end{bmatrix}$$

Element (2)

$$\begin{bmatrix} \bar{f}_{x_1}^{(2)} \\ \bar{f}_{y_1}^{(2)} \\ \bar{f}_{x_3}^{(2)} \\ \bar{f}_{y_3}^{(2)} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{u}_{x_1}^{(2)} \\ \bar{u}_{y_1}^{(2)} \\ \bar{u}_{x_3}^{(2)} \\ \bar{u}_{y_3}^{(2)} \end{bmatrix} \qquad T^{(2)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Element (3)

$$\begin{bmatrix} \bar{f}_{x_1}^{(3)} \\ \bar{f}_{y_1}^{(3)} \\ \bar{f}_{x_4}^{(3)} \\ \bar{f}_{y_4}^{(3)} \end{bmatrix} = \frac{EAc}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{u}_{x_1}^{(3)} \\ \bar{u}_{y_1}^{(3)} \\ \bar{u}_{x_4}^{(3)} \\ \bar{u}_{y_4}^{(3)} \end{bmatrix} \qquad T^{(3)} = \begin{bmatrix} s & c & 0 & 0 \\ -c & s & 0 & 0 \\ 0 & 0 & s & c \\ 0 & 0 & -c & s \end{bmatrix}$$

Globalized Element Stiffness equations

$K^e = (T^e)^T \bar{K}^e T^e$

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} cs^2 & -c^2s & -cs^2 & c^2s \\ -c^2s & c^3 & c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -c^2s \\ c^2s & -c^3 & -c^2s & c^3 \end{bmatrix} \begin{bmatrix} u_{x_1}^{(1)} \\ u_{y_1}^{(1)} \\ u_{x_2}^{(1)} \\ u_{y_2}^{(1)} \end{bmatrix}$$

$$\begin{bmatrix} f_{x_1}^{(2)} \\ f_{y_1}^{(2)} \\ f_{x_3}^{(2)} \\ f_{y_3}^{(2)} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{x_1}^{(2)} \\ u_{y_1}^{(2)} \\ u_{x_3}^{(2)} \\ u_{y_3}^{(2)} \end{bmatrix}$$

$$\begin{bmatrix} f_{x_1}^{(3)} \\ f_{y_1}^{(3)} \\ f_{x_4}^{(3)} \\ f_{y_4}^{(3)} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} cs^2 & c^2s & -cs^2 & -c^2s \\ c^2s & c^3 & -c^2s & -c^3 \\ -cs & -c^2s & s^2 & c^2s \\ -c^2s & -c^3 & c^2s & c^3 \end{bmatrix}$$

Master Stiffness Equation

$f = f^{(1)} + f^{(2)} + f^{(3)} = (K^{(1)} + K^{(2)} + K^{(3)})\, u = Ku$

$$\begin{bmatrix} f_{x_1} = H \\ f_{y_1} = -P \\ f_{x_2} \\ f_{y_2} \\ f_{x_3} \\ f_{y_3} \\ f_{x_4} \\ f_{y_4} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ 0 & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ c^2s & -c^3 & -c^2s & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -cs^2 & -c^2s & 0 & 0 & 0 & 0 & cs^2 & c^2s \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix}$$

verified ✓

The 5th row and column contain only zeros because the elements are just subjected to axial load. The element (2) is vertical so the tension is vertical and the reaction $f_{x_3} = 0$.

2.- Boundary Conditions

$$U_{x_2}=U_{y_2}=U_{x_3}=U_{y_3}=U_{x_4}=U_{y_4}=0 \qquad f_{x_1}=H \quad f_{y_1}=-P$$

$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^2 \end{bmatrix}\begin{bmatrix} U_{x_1} \\ U_{y_1} \end{bmatrix}=\begin{bmatrix} H \\ -P \end{bmatrix}$$

$$U_{x_1}=\frac{HL}{2EAcs^2}$$

$$U_{y_1}=\frac{-PL}{EA(1+2c^3)}$$

3.- when $\alpha \to 0 \begin{cases} c=1 \\ s=0 \end{cases}$

$$U_{x_1}=\frac{HL}{2EA(0)}=\infty$$

$$U_{y_1}=\frac{-PL}{3EA}$$

when $\alpha \to \pi/2 \begin{cases} c=0 \\ s=1 \end{cases}$

$$U_{x_1}=\frac{HL}{2EA(0)}=\infty$$

$$U_{y_1}=\frac{-PL}{EA}$$

If $H \neq 0$ and $\alpha \to 0$ the nodes 2, 3 and 4 coincide and there is not restriction to rotate at the nodes so the bars will tend to rotate freely. (the structure becomes unstable)

4.- Element (1)

$$\begin{bmatrix} \bar{U}_{x_1}^{(1)} \\ \bar{U}_{y_1}^{(1)} \\ \bar{U}_{x_2}^{(1)} \\ \bar{U}_{y_2} \end{bmatrix}=\begin{bmatrix} -s & c & & \\ -c & -s & & \\ & & -s & c \\ & & c & -s \end{bmatrix}\begin{bmatrix} \frac{HL}{2EAcs^2} \\ -\frac{PL}{EA(1+2c^3)} \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} -\frac{HL}{2EAcs}-\frac{PLc}{EA(1+2c^3)} \\ -\frac{HL}{2EAs^2}+\frac{PLs}{EA(1+2c^3)} \\ 0 \\ 0 \end{bmatrix}$$

$$d^{(1)}=\bar{U}_{x_2}^{(1)}-\bar{U}_{x_1}^{(1)}$$

$$d^{(1)}=\frac{HL}{2EAcs}+\frac{PLc}{EA(1+2c^3)}$$

$$F^{(1)}=\frac{EA}{L/c}\left(\frac{HL}{2EAcs}+\frac{PLc}{EA(1+2c^3)}\right)$$

$$\boxed{F^{(1)}=\frac{H}{2s}+\frac{Pc^2}{1+2c^3}}$$

Element (2)

$$\begin{bmatrix} \bar{U}_{x_1}^{(2)} \\ \bar{U}_{y_1}^{(2)} \\ \bar{U}_{x_3}^{(2)} \\ \bar{U}_{y_3}^{(2)} \end{bmatrix}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} \frac{HL}{2EAcs^2} \\ -\frac{PL}{EA(1+2c^3)} \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} -\frac{PL}{EA(1+2c^3)} \\ -\frac{HL}{\;} \\ 0 \\ 0 \end{bmatrix}$$

$$d^{(2)}=\frac{PL}{EA(1+2c^3)}$$

$$F^{(2)}=\frac{EA}{L}\left(\frac{PL}{EA(1+2c^3)}\right)$$

$$\boxed{F^{(2)}=\frac{P}{1+2c^3}}$$

Element (3)

$$\begin{bmatrix} \bar{U}_{x_1}^{(3)} \\ \bar{U}_{y_1}^{(3)} \\ U_{x_4}^{(3)} \\ U_{y_4}^{(3)} \end{bmatrix}=\begin{bmatrix} s & c & 0 & 0 \\ -c & s & 0 & 0 \\ 0 & 0 & s & c \\ 0 & 0 & -c & s \end{bmatrix}\begin{bmatrix} \frac{HL}{2EAcs^2} \\ -\frac{PL}{EA(1+2c^3)} \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} \frac{HL}{2EAcs}-\frac{PLc}{EA(1+2c^3)} \\ -\frac{HL}{2EAs^2}-\frac{PLs}{EA(1+2c^3)} \\ 0 \\ 0 \end{bmatrix}$$

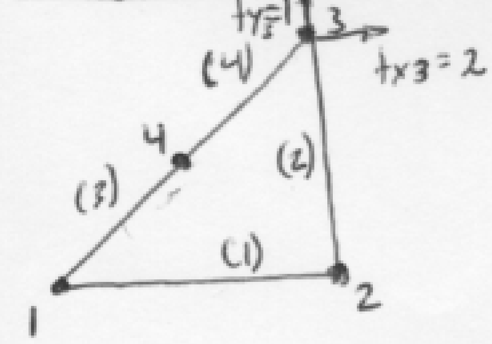
$$d^{(3)}=-\frac{HL}{2EAcs}+\frac{PLc}{EA(1+2c^3)}$$

$$F^{(3)}=\frac{EA}{L/c}\left(-\frac{HL}{2EAcs}+\frac{PLc}{EA(1+2c^3)}\right)$$

$$\boxed{F^{(3)}=-\frac{H}{2s}+\frac{Pc^2}{1+2c^3}}$$

If $H \neq 0$ $\alpha \to 0$ $\begin{cases} F^{(1)}=\frac{H}{2(0)}+\frac{P}{3} & F^{(1)} \to \infty \\ F^{(3)}=-\frac{H}{2(0)}+\frac{P}{3} & F^{(3)} \to \infty \end{cases}$ $\Rightarrow$ The structure becomes unstabe

Assignment 1.2



$L^{(3)} = L^{(4)} = 5\sqrt{2}$

$E^{(3)}A^{(3)} = E^{(4)}A^{(4)} = 200\sqrt{2}$

$L^{(1)} = 10$

$E^{(1)}A^{(1)} = 100$

$L^{(2)} = 10$

$E^{(2)}A^{(2)} = 50$

$$K^e = \frac{E^eA^e}{L^e}\begin{pmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{pmatrix}$$

Globalized Element Stiffness equations

Element (1)

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \end{bmatrix} = 10\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} u_{x_1}^{(1)} \\ u_{y_1}^{(1)} \\ u_{x_2}^{(1)} \\ u_{y_2}^{(1)} \end{bmatrix}$$

Element (2)

$$\begin{bmatrix} f_{y_2}^{(2)} \\ f_{y_2}^{(2)} \\ f_{x_3}^{(2)} \\ f_{y_3}^{(2)} \end{bmatrix} = 5\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} u_{x_2}^{(2)} \\ u_{y_2}^{(2)} \\ u_{x_3}^{(2)} \\ u_{y_3}^{(2)} \end{bmatrix}$$

Note that equations for (1) and (2) don't change

Elemen (3)

$$\begin{bmatrix} f_{x_1}^{(3)} \\ f_{y_1}^{(3)} \\ f_{x_4}^{(3)} \\ f_{y_4}^{(3)} \end{bmatrix} = 40\begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix}\begin{bmatrix} u_{x_1}^{(3)} \\ u_{y_1}^{(3)} \\ u_{x_4}^{(3)} \\ u_{y_4}^{(3)} \end{bmatrix}$$

Element (4)

$$\begin{bmatrix} f_{x_3}^{(4)} \\ f_{y_3}^{(4)} \\ f_{x_4}^{(4)} \\ f_{y_4}^{(4)} \end{bmatrix} = 40\begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix}\begin{bmatrix} u_{x_3}^{(4)} \\ u_{y_3}^{(4)} \\ u_{x_4}^{(4)} \\ u_{y_4}^{(4)} \end{bmatrix}$$

Master Stiffness Equation

$$\begin{bmatrix} f_{x_1} \\ f_{y_1} \\ f_{x_2} \\ f_{y_2} \\ f_{x_3} \\ f_{y_3} \\ f_{x_4} \\ f_{y_4} \end{bmatrix} = \begin{bmatrix} 30 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & -5 & 20 & 25 & -20 & -20 \\ -20 & -20 & 0 & 0 & -20 & -20 & 20 & 20 \\ -20 & -20 & 0 & 0 & -20 & -20 & 20 & 20 \end{bmatrix}\begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix}$$

$\det(K) = 0 \Rightarrow K$ is singular

K is singular because the truss becomes unstable if it is subdivided as the statement proposes