CSMD Assignment - 01

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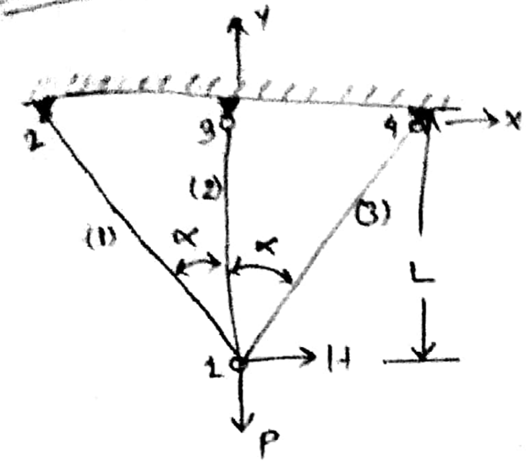
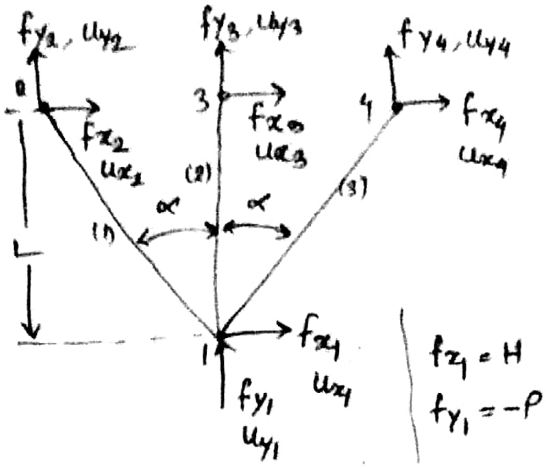
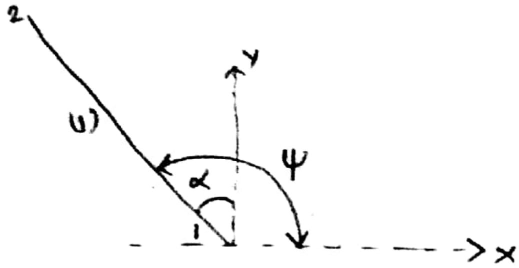
Bar structure





For element 1



Angle with horizontal axis,

$\psi = \pi/2 + \alpha$, $s = \sin\alpha$, $c = \cos\alpha$

Now, general elemental stiffness matrix is written as follows -

$$K^e = \frac{E^e A^e}{L^e}\begin{bmatrix} c'^2 & s'c' & -c'^2 & -s'c' \\ s'c' & s'^2 & -s'c' & -s'^2 \\ -c'^2 & -s'c' & c'^2 & s'c' \\ -s'c' & -s'^2 & s'c' & s'^2 \end{bmatrix} \quad \text{——} \quad (1)$$

where, $c' = \cos\psi$, $s' = \sin\psi$

$L^{(1)} = L/\cos\alpha = L/c$

$\therefore$ For element 1,

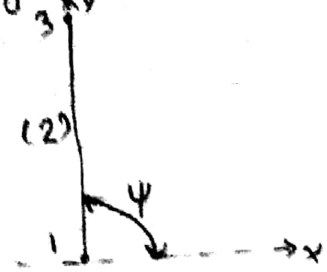
$$K^1 = \frac{EA}{L}\begin{bmatrix} s^2c & -sc^2 & -s^2c & sc^2 \\ -sc^2 & c^3 & sc^2 & -c^3 \\ -s^2c & sc^2 & s^2c & -sc^2 \\ sc^2 & -c^3 & -sc^2 & c^3 \end{bmatrix}$$

As, $s = \sin\alpha$, $c = \cos\alpha$

$\therefore$ $s' = \sin\psi = \sin(\pi/2 + \alpha)$
$= \cos\alpha = c$

$c' = \cos\psi = \cos(\pi/2 + \alpha)$
$= -\sin\alpha = -s$

Similarly, for element 2, $L^{(2)} = L$



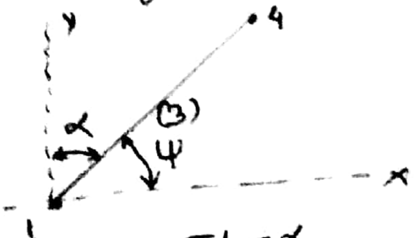
$\psi = \pi/2$

$\therefore c' = \cos \pi/2 = 0$ & $s' = \sin \pi/2 = 1$.

Substituting these values in eq^n (1), we get,

$$K^{(2)} = \frac{EA}{L} \begin{array}{c} \begin{array}{cccc} 1 & 1 & 3 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 1 \\ 3 \\ 3 \end{array}$$

Similarly, for element 3, $L^{(3)} = L/\cos\alpha = L/c$



$\psi = \pi/2 - \alpha$

$\therefore c' = \cos\psi = \cos(\pi/2 - \alpha)$
$= \sin\alpha = s$

$s' = \sin\psi = \sin(\pi/2 - \alpha) = \cos\alpha = c$

$\therefore$ From eq^n (1), we have,

$$K^{(3)} = \frac{EA}{L} \begin{array}{c} \begin{array}{cccc} 1 & 1 & 4 & 4 \end{array} \\ \begin{bmatrix} s^2c & s^2c^2 & -s^2c & -sc^2 \\ sc^2 & c^3 & -sc^2 & -c^3 \\ -s^2c & -sc^2 & s^2c & sc^2 \\ -sc^2 & -c^3 & sc^2 & c^3 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 1 \\ 4 \\ 4 \end{array}$$

Now, Expanding elemental stiffness equation,

$$K = K^{(1)} + K^{(2)} + K^{(3)} \longleftarrow (2)$$

$$K^{(1)} = \frac{EA}{L} \begin{array}{c} \begin{array}{cccccccc} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \end{array} \\ \begin{bmatrix} s^2c & -sc^2 & -s^2c & sc^2 & 0 & 0 & 0 & 0 \\ -sc^2 & c^3 & sc^2 & -c^3 & 0 & 0 & 0 & 0 \\ -s^2c & sc^2 & s^2c & -sc^2 & 0 & 0 & 0 & 0 \\ sc^2 & -c^3 & -sc^2 & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 1 \\ 2 \\ 2 \\ 3 \\ 3 \\ 4 \\ 4 \end{array}$$

$$K^{(2)} = \frac{EA}{2} \begin{array}{c} \begin{array}{cccccccc} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 1 \\ 2 \\ 2 \\ 3 \\ 3 \\ 4 \\ 4 \end{array}$$

$$K^{(3)} = \frac{EA}{L} \begin{array}{c} \begin{array}{cccccccc} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \end{array} \\ \begin{bmatrix} s^2c & sc^2 & 0 & 0 & 0 & 0 & -s^2c & -sc^2 \\ sc^2 & c^3 & 0 & 0 & 0 & 0 & -sc^2 & -c^3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -s^2c & -sc^2 & 0 & 0 & 0 & 0 & s^2c & sc^2 \\ -sc^2 & -c^3 & 0 & 0 & 0 & 0 & sc^2 & c^3 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 1 \\ 2 \\ 2 \\ 3 \\ 3 \\ 4 \\ 4 \end{array}$$

Master stiffness matrix eqn.

$$K u = f$$

$$K = K^{(1)} + K^{(2)} + K^{(3)} , \quad f = f^{(1)} + f^{(2)} + f^{(3)}$$

$$\frac{KA}{L}\begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ 0 & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -cs^2 & 0 & 0 & 0 & 0 \\ c^2s & -c^3 & -c^2s & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -cs^2 & -c^2s & 0 & 0 & 0 & 0 & cs^2 & c^2s \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix} = \begin{bmatrix} H \\ -P \\ f_{x_2} \\ f_{y_2} \\ f_{x_3} \\ f_{y_3} \\ f_{x_4} \\ f_{y_4} \end{bmatrix}$$

1/(a) Here, sum of applied forces on nodes 2, 3 & 4 = 0.

$$\Rightarrow \frac{KA}{L}\begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ & & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & & 1 & 0 & 0 \\ & & & & & & cs^2 & c^2s \\ \text{symmetric} & & & & & & & c^3 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix} = \begin{bmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

It can be inferred from the structure that the horizontal force acting at node '3' will be zero & as a result there will be no horizontal displacement. As there is no horizontal force on f_{x_3}, so the corresponding values of the stiffness matrices in the fifth row has to be zero.

1/1(b)

From the structure, it is clear that

$u_{x_2} = u_{y_2} = u_{x_3} = u_{y_3} = u_{x_4} = u_{y_4} = 0$

∴ We ~~can~~ eliminating the known displacements, we have

$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix}\begin{bmatrix} u_{x_1} \\ u_{y_1} \end{bmatrix} = \begin{bmatrix} H \\ -P \end{bmatrix} \quad \text{—— (3)}$$

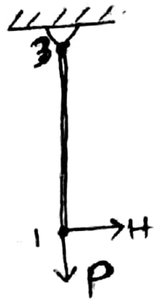
This is the desired 2-equation modified ~~sy~~ stiffness system.

1/1 (c) From eq^n (3),

$\frac{EA}{L}\left(2cs^2 u_{x_1}\right) = H$ &, $\frac{EA}{L}(1+2c^3)u_{y_1} = -P$

$\Rightarrow u_{x_1} = \frac{HL}{EA(2cs^2)}$ $\Rightarrow u_{y_1} = \frac{-PL}{EA(1+2c^3)}$

Now, Limiting cases



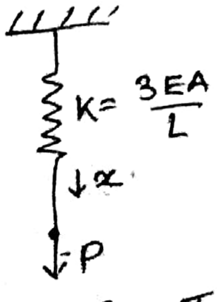
(i) $\alpha = 0$

∴ $c = \cos\alpha = 1$

$s = \sin\alpha = 0$

∴ $u_{x_1} \to \infty$ & $u_{y_1} = \frac{-PL}{3EA}$

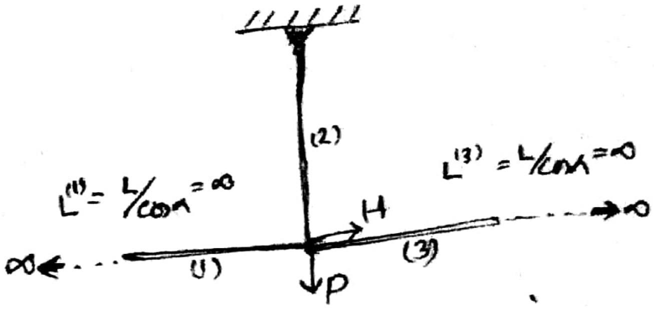
If $H = 0$, then the ~~[illegible]~~ system acts as a vertical spring with vertical loading at point 1, as follows:-



∴ Displacement, $x = \frac{-P}{\frac{3EA}{L}} = \frac{-PL}{3EA}$

However, if $H \neq 0$, then the system will undergo bending deformation as the system doesnot allow rotation at point '3'. In this case the ~~[illegible]~~ solution would blow up as our model only includes axial forces & doesnot include bending effects.

(ii) For $\alpha = \pi/2$



$c = \cos\alpha = 0$

$s = \sin\alpha = 1$

$u_{x_1} \to \infty, \quad u_{y_1} = \dfrac{-PL}{EA}$

The vertical displacement makes physical sense for a spring loaded system with spring constant $\frac{EA}{L}$ & a ~~boo~~ vertical load 'P' acting downward.

For ~~vertical~~ horizontal displacement, it can also be considered as a spring loaded system with zero stiffness as length is infinite. So, horizontal displacement $= H/K = H/0 = \infty$.

1(c)

Axial forces

~~Element 1~~

From displacement transformation we have,

$$\bar{u}^e = T^e u^e$$

$$\Rightarrow \begin{bmatrix} \bar{u}_{x_i} \\ \bar{u}_{y_i} \\ \bar{u}_{x_j} \\ \bar{u}_{y_j} \end{bmatrix} = \begin{bmatrix} c' & s' & 0 & 0 \\ -s' & c' & 0 & 0 \\ 0 & 0 & c' & s' \\ 0 & 0 & -s' & c' \end{bmatrix} \begin{bmatrix} u_{x_i} \\ u_{y_i} \\ u_{x_j} \\ u_{y_j} \end{bmatrix} \quad — (4)$$

For element 1

$$\bar{u}^{(1)} = \begin{bmatrix} \bar{u}_{x_1} \\ \bar{u}_{y_1} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{u}_{x_1} \\ \bar{u}_{y_1} \\ \bar{u}_{x_2} \\ \bar{u}_{y_2} \end{bmatrix}$$

From (4),

$$\therefore \begin{bmatrix} -s & c & 0 & 0 \\ -c & -s & 0 & 0 \\ 0 & 0 & -s & c \\ 0 & 0 & -s & -c \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{u}_{x_1} = -u_{x_1} s^{\circ} + u_{y_1} c^{\circ} \\ \bar{u}_{y_1} = -u_{x_1} c^{\circ} - u_{y_1} s^{\circ} \\ 0 \\ 0 \end{bmatrix}$$

Elongation, $d^{(1)} = \bar{u}_{x_2} - \bar{u}_{x_1}$

$$= 0 - (-u_{x_1}s + u_{y_1}c)$$

$$= \frac{HL}{2ACSE} + \frac{PLc}{EA(1+2c^3)}$$

$\therefore$ Axial force, $F^{(1)} = \frac{EA}{L}d^{(1)} = \frac{EA}{L/c}d^{(1)} = \frac{EAc}{L} \times \left(\frac{HL}{2AECS} + \frac{PLc}{EA(1+2c^3)}\right)$

$$= \frac{H}{2S} + \frac{Pc^2}{1+2c^3}$$

Element - 2

$$\bar{u}^{(2)} = \begin{bmatrix} \bar{u}_{x_1} \\ \bar{u}_{y_1} \\ \bar{u}_{x_3} \\ \bar{u}_{y_3} \end{bmatrix} = \begin{bmatrix} \bar{u}_{x_1} \\ \bar{u}_{y_1} \\ 0 \\ 0 \end{bmatrix}$$

From (4),

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{u}_{x_1} = -u_{y_1} \\ \bar{u}_{y_1} = -u_{x_1} \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore d^{(2)} = \bar{u}_{x_3} - \bar{u}_{x_1} = -\bar{u}_{y_1} = \frac{PL}{(1+2c^3)EA}$$

$$F^{(2)} = \frac{P}{1+2c^3}$$

Element 3

$$\bar{u}^{(3)} = \begin{bmatrix} \bar{u}_{x_1} \\ \bar{u}_{y_1} \\ \bar{u}_{x_4} \\ \bar{u}_{y_4} \end{bmatrix} = \begin{bmatrix} \bar{u}_{x_1} \\ \bar{u}_{y_1} \\ 0 \\ 0 \end{bmatrix}$$

From (4),

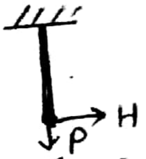
$$\begin{bmatrix} s & c & 0 & 0 \\ -c & -s & 0 & 0 \\ 0 & 0 & s & c \\ 0 & 0 & -c & -s \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{u}_{x_1} = u_{x_1}s + u_{y_1} \\ \bar{u}_{y_1} = -u_{x_1}c + u_{y_1}s \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ Elongation, $d^{(3)} = u_{x_2}^{(3)} - u_{x_1}^{(3)} = -u_{x_1}s - u_{y_1}c$

$$= \frac{-HLs}{2cs^2EA} + \frac{PLc}{(1+2c^3)EA}$$

$$\therefore F^{(3)} = \frac{-H}{2s} + \frac{Pc^2}{(1+2c^3)}$$

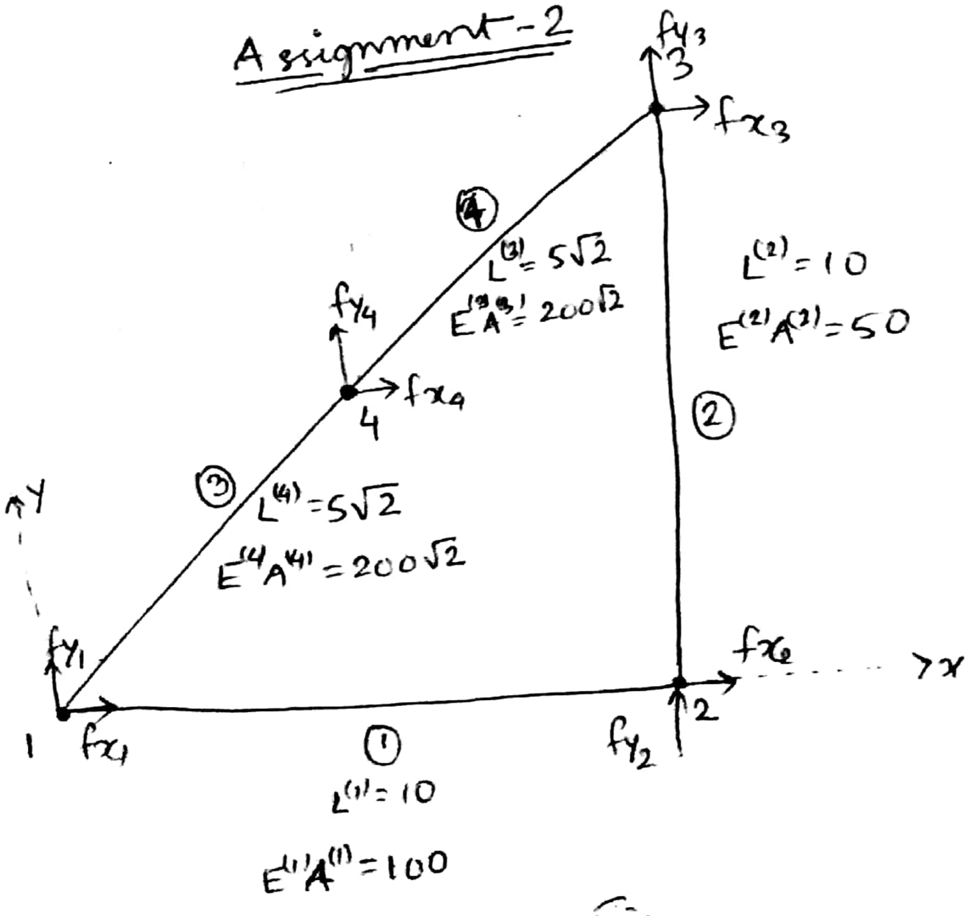
When $\alpha \to 0$ & $H \neq 0$



$F_1 \to \infty$, $F_2 \to -\infty$

Since our model ~~only~~ doesnot account for bending effects the solution blows up

2) Assignment-2

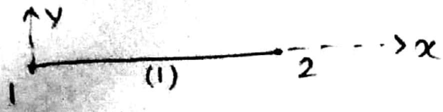


General elemental stiffness matrix,

$$K^{(e)} = \frac{E^eA^e}{L^e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix} \quad \text{—— (1)}$$

~~For element 1,~~
~~From eq^n (4),~~
~~$K^{(4)}$~~

For element 1



$\psi = 0$

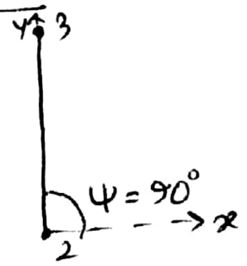
$\therefore\ c = \cos\psi = 1$

$s = \sin\psi = 0$

$\therefore$ From eq^n (1),

$$K^{(1)} = \frac{100}{10}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 \\ -10 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Element 2



$\psi = \pi/2$

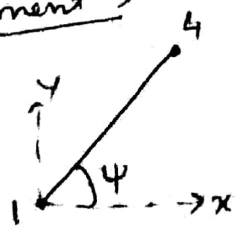
$\therefore\ c = \cos\psi = 0$

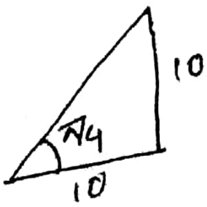
$s = \sin\psi = 1$

From (1),

$$\therefore K^{(2)} = \frac{50}{10}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & -5 & 0 & 5 \end{bmatrix}$$

Element 3





$\psi = \pi/4$

$\therefore\ c = \cos\psi = 1/\sqrt{2}$

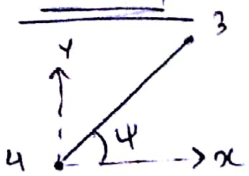
$s = \sin\psi = 1/\sqrt{2}$

From ①,

$$\therefore K^{(3)} = \frac{200\sqrt{2}}{5\sqrt{2}} \begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 20 & -20 & -20 \\ 20 & 20 & -20 & -20 \\ -20 & -20 & 20 & 20 \\ -20 & -20 & 20 & 20 \end{bmatrix}$$

Element 4



$\psi = \pi/4$

$\therefore c = \cos\psi = 1/\sqrt{2}$

$s = \sin\psi = 1/\sqrt{2}$

$$\therefore K^{(4)} = K^{(3)} = \begin{bmatrix} 20 & 20 & -20 & -20 \\ 20 & 20 & -20 & -20 \\ -20 & -20 & 20 & 20 \\ -20 & -20 & 20 & 20 \end{bmatrix}$$

Now, Expanding Elemental stiffness equations

$$K^{(1)} = \begin{bmatrix} 10 & 0 & -10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K_2^{2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K^{(3)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & 0 & -20 & -20 & 20 & 20 \\ 0 & 0 & 0 & 0 & -20 & -20 & 20 & 20 \end{bmatrix}$$

$$K^{(4)} = \begin{bmatrix} 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -20 & -20 & 0 & 0 & 0 & 0 & 20 & 20 \\ -20 & -20 & 0 & 0 & 0 & 0 & 20 & 20 \end{bmatrix}$$

Master stiffness matrix equation

$$KU = f$$

$$K = K^1 + K^2 + K^3 + K^4 \quad , \quad f = f^1 + f^2 + f^3 + f^4$$

$$\begin{bmatrix} 30 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ -10 & 0 & -10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 20 & 25 & -20 & -20 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix} = \begin{bmatrix} f_{x_1} \\ f_{y_1} \\ f_{x_2} \\ f_{y_2} \\ f_{x_3} \\ f_{y_3} \\ f_{x_4} \\ f_{y_4} \end{bmatrix}$$

Considering the structure,

Boundary condition,

$$u_{x_1} = u_{y_1} = u_{y_2} = 0$$

$$f_{x_2} = f_{x_4} = f_{y_4} = 0$$

$$f_{x_3} = 2$$

$$f_{y_3} = 1$$

Now, Neglecting known displacements,

$$\therefore \begin{bmatrix} -10 & 0 & 0 & 0 & 0 \\ 0 & 20 & 20 & -20 & -20 \\ 0 & 20 & 25 & -20 & -20 \\ 0 & -20 & -20 & 40 & 40 \\ 0 & -20 & -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} u_{x_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Here, we see that the master stiffness matrix is singular. Since there is no support provided at node '4', the structure becomes unstable. Since our model doesnot accomodate such complexities, the solution blows up.

A constraint has to be put on the node '4' to make the structure stable.