*Universitat Politecnica De Catalunya, BarcelonaTech*
*Masters in Computational Mechanics*

*Course*
*Computational Structural Mechanics and Dynamics*

# Assignment 9
# on
# Axisymmetric Shells

by

KIMEY K WAZARE

Exercise: Axisymmetric Shell

Ques 1. Describe in extension how can be applied a non-symmetric load on this formulation?

**Solution:** In Axisymmetric shells under any arbitrary loading the length of the structure is a whole circumference (i.e. angle α is replaced by 2π). The displacements are expanded in Fourier series along circumferential direction. Therefore, displacement field is split in symmetric and non-symmetric components.
The displacement vector (u') is

$$\mathbf{U}' = \sum_{\mathbf{l=0}}^{\mathbf{m}} \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{N_i}\{(\bar{s}\ \overline{a_i'^l}) + (\bar{\bar{s}}\overline{\overline{a_i'^l}})\}$$

Where, single bar represents symmetric component and double bar represents non-symmetric component.
The displacement components U', W' and $\theta_s$ contained in the symmetry plane are zero for an anti-symmetric loading. This zero harmonic term corresponds to its deformation where β is constant which have same displacement components.

The loads are expanded in *Fourier series using harmonic functions for displacements*,

$$\boldsymbol{t} = \sum_{\boldsymbol{l=0}}^{\boldsymbol{m}} \{(\bar{\boldsymbol{s}}\overline{\boldsymbol{t}'}) + (\bar{\bar{\boldsymbol{s}}}\overline{\overline{\boldsymbol{t}'}})\}$$

Where t' are load amplitude.
The local stiffness matrix for an axisymmetric strip element is

$$[\mathbf{K}'^{ll}_{ij}]^{(e)} = C\int_{a^{(e)}} [\mathbf{B}'^l_i]^T \hat{\mathbf{D}}' \mathbf{B}'^l_j r ds$$

Where D is constitute matrix, $C = \begin{cases} 2\pi\ for\ \ l = 0 \\ \pi\ \ for\ l \neq 0 \end{cases}$, r is radius of revolution shell.

$$\hat{\boldsymbol{\varepsilon}}' = \sum_{l=1}^{m}\sum_{i=1}^{n} \hat{\mathbf{S}}^l\ \mathbf{B}'^l_i\ \mathbf{a}^l_i$$

$$\hat{\boldsymbol{\varepsilon}}' = \begin{Bmatrix} \hat{\boldsymbol{\varepsilon}}'_m \\ \hat{\boldsymbol{\varepsilon}}'_b \\ \hat{\boldsymbol{\varepsilon}}'_s \end{Bmatrix};\quad \begin{cases} \hat{\boldsymbol{\varepsilon}}'_m = \begin{Bmatrix} \dfrac{\partial u'_0}{\partial x'} \\ \dfrac{\partial v'_0}{\partial y'} \\ \dfrac{\partial u'_0}{\partial y'} + \dfrac{\partial v'_0}{\partial x'} \end{Bmatrix} \\ \hat{\boldsymbol{\varepsilon}}'_f = \begin{Bmatrix} \dfrac{\partial \theta x'}{\partial x'} \\ \dfrac{\partial \theta y'}{\partial y'} \\ \left(\dfrac{\partial \theta x'}{\partial y'} + \dfrac{\partial \theta y'}{\partial x'}\right) \end{Bmatrix} \\ \hat{\boldsymbol{\varepsilon}}'_s = \begin{Bmatrix} \dfrac{\partial w'_0}{\partial x'} - \theta x' \\ \dfrac{\partial w'_0}{\partial y'} - \theta y' \end{Bmatrix} \end{cases}$$

$$\mathrm{B}'^l_i = \begin{Bmatrix} \mathrm{B}'^l_{m_i} \\ \mathrm{B}'^l_{b_i} \\ \mathrm{B}'^l_{s_i} \end{Bmatrix};\quad \begin{cases} \mathrm{B}'^l_{m_i} = \begin{bmatrix} \dfrac{\partial N_i}{\partial x'} & 0 & 0\,0\,0 \\ 0 & -N_i\gamma & 0\,0\,0 \\ N_i\gamma & \dfrac{\partial N_i}{\partial x'} & 0\,0\,0 \end{bmatrix} \\ \mathrm{B}'^l_{b_i} = \begin{bmatrix} 0\,0\,0 & \dfrac{\partial N_i}{\partial x'} & 0 \\ 0\,0\,0 & 0 & N_i\gamma \\ 0\,0\,0 & N_i\gamma & \dfrac{\partial N_i}{\partial x'} \end{bmatrix} \\ \mathrm{B}'^l_{s_i} = \begin{bmatrix} 0\,0 & \dfrac{\partial N_i}{\partial x'} & -N_i & 0 \\ 0\,0 & N_i\gamma & 0 & -N_i \end{bmatrix} \end{cases}$$

The global stiffness matrix for an axisymmetric strip element is

$$[\mathbf{K}^{ll}_{ij}]^{(e)} = C\int_{a^{(e)}} [\mathbf{B}^l_i]^T \hat{\mathbf{D}}' \mathbf{B}^l_j r ds$$

In this way, a revolution shell formulation can be expanded on the nonsymmetric loading using Fourier Series.

**Ques 2. Using thin beams formulation, describe the shape of the B(e) matrix and comment the integration rule.**

Solution:

a. Shape of the B(e) matrix:

The Thin Beam formulation (Kirchhoff theory) for element can be derived by introducing normal orthogonality condition in the kinetic field (i.e., neglecting the effect of transverse shear strain in the analysis). This formulation is applicable to the Thin Shell problems only.

$$\hat{\varepsilon}_s = 0; \quad \theta_s = \frac{\partial w_0'}{\partial s}; \quad \theta_t = \frac{1}{r}\frac{\partial w_0'}{\partial \beta} + \frac{v_0'}{r} s$$

Taking into account above terms, strain matrix and B matrix are

$$\hat{\boldsymbol{\varepsilon}}' = \begin{Bmatrix} \hat{\boldsymbol{\varepsilon}}_m' \\ \hat{\boldsymbol{\varepsilon}}_b' \end{Bmatrix} \quad ; \quad \begin{cases} \hat{\boldsymbol{\varepsilon}}_m' = \begin{Bmatrix} \dfrac{\partial u_0'}{\partial s} \\ \dfrac{1}{r}\dfrac{\partial v_0'}{\partial \beta} + \dfrac{u_0'}{r}C - \dfrac{w_0'}{r}S \\ \dfrac{\partial v_0'}{\partial s} + \dfrac{1}{r}\dfrac{\partial u_0'}{\partial \beta} - \dfrac{v_0'}{r}C \end{Bmatrix} \\ \hat{\boldsymbol{\varepsilon}}_b' = \begin{Bmatrix} \dfrac{\partial^2 w_0'}{\partial s^2} \\ \dfrac{1}{r^2}\dfrac{\partial^2 w_0'}{\partial \beta^2} + \dfrac{S}{r^2}\dfrac{\partial v_0'}{\partial \beta} + \dfrac{C}{r}\dfrac{\partial w_0'}{\partial s} \\ \dfrac{2}{r}\dfrac{\partial^2 w_0'}{\partial s \partial \beta} - \dfrac{2CS}{r^2}v_0' - \dfrac{2C}{r^2}\dfrac{\partial w_0'}{\partial \beta} + \dfrac{S}{r}\dfrac{\partial v_0'}{\partial s} \end{Bmatrix} \end{cases}$$

$$\mathbf{B}_i'^l = \begin{Bmatrix} \mathbf{B}_{m_i}'^l \\ \mathbf{B}_{b_i}'^l \end{Bmatrix}; \begin{cases} \mathbf{B}_{m_i}'^l = \begin{bmatrix} \dfrac{\partial N_i}{\partial s} & 0 & 0 & 0 \\ \dfrac{N_i}{r}C & -\dfrac{N_i}{r}\gamma & -\dfrac{H_i'}{r}S & -\dfrac{\bar{H}_i}{r}S \\ \dfrac{N_i}{r}\gamma & \left(\dfrac{\partial N_i}{\partial s} - \dfrac{N_i}{r}C\right) & 0 & 0 \end{bmatrix} \\ \mathbf{B}_{b_i}'^l = \begin{bmatrix} 0 & 0 & \dfrac{\partial^2 H_i}{\partial s^2} & \dfrac{\partial^2 \bar{H}_i}{\partial s^2} \\ 0 & \dfrac{N_i}{r^2}S\gamma & \left[\dfrac{C}{r}\dfrac{\partial H_i}{\partial s} - \left(\dfrac{\gamma}{r}\right)^2 H_i\right] & \left[\dfrac{C}{r}\dfrac{\partial \bar{H}_i}{\partial s} - \left(\dfrac{\gamma}{r}\right)^2 \bar{H}_i\right] \\ 0 & \left(\dfrac{S}{r}\dfrac{\partial N_i}{\partial s} - \dfrac{2N_i}{r^2}CS\right) & \left(\dfrac{2\gamma}{r}\dfrac{\partial H_i}{\partial s} - \dfrac{2H_i}{r^2}C\gamma\right) & \left(\dfrac{2\gamma}{r}\dfrac{\partial \bar{H}_i}{\partial s} - \dfrac{2\bar{H}_i}{r^2}C\gamma\right) \end{bmatrix} \end{cases}$$

Where, $N_i$ - Lagrange Shape Function, $H_i$ ; $\overline{H_i}$ – Hermite Shape Function.

b. Integration Rule: Two-point quadrature is highly recommended for computing the following integrals. But results also can be obtained using simplest reduced one-point quadrature.

----x----x----x----