CSMD Assignment 9

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Axisymmetric Shells

1. Describe how non-symmetric loads can be applied to axi-symmetric structures

Solution

Non-symmetric loads will cause tangential displacements to be non-zero. this will generate non-symmetric terms when computing the stresses.

One approach is to use a full 3D formulation. Another is to use the axi-symmetric formulation but express the displacements and force components using a Fourier expansion, splitting symmetric ~~and~~ non-symmetric terms

$$f(\theta) = \frac{a_0}{2} + \sum' \left(\frac{\cos(n\theta)}{\pi} \int_{-\infty}^{\infty} f(\theta) \cos(n\theta)\, d\theta + \ldots \right.$$

$$\left. \frac{\sin(n\theta)}{\pi} \int_{-\infty}^{\infty} f(\theta) \cos(n\theta)\, d\theta \right)$$

For the formulation of axi-symmetric thin shells, strains and stresses must be modified to account for the displacements and loads in 3D:

$$\bar{\varepsilon} = \begin{bmatrix} \varepsilon_s \\ \varepsilon_\theta \\ \gamma_{s\theta} \\ \chi_s \\ \chi_\theta \\ \chi_{s\theta} \end{bmatrix} = \begin{bmatrix} \partial u / \partial s \\ \frac{1}{r}\frac{\partial v}{\partial \theta} + (u \cos\theta - \frac{1}{r} w \sin\theta) \\ \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial s} - \frac{1}{r} v \cos\theta \\ -\frac{\partial^2 w}{\partial s^2} \\ -\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} - \frac{1}{r}\frac{\partial w}{\partial s}\cos\theta + \frac{1}{r}\frac{\partial v}{\partial \theta}\sin\theta \\ \frac{2}{r}\left(-\frac{1}{r}\frac{\partial^2 w}{\partial \theta^2} + \frac{1}{r^2}\frac{\partial w}{\partial \theta} + \frac{1}{r}\frac{\partial v}{\partial s} - \frac{1}{r^2} v \sin\theta \cos\theta\right) \end{bmatrix}$$

2. Using the thin beams, describe the shape of the B^e matrix and comment on the integration rule.

<u>Solution</u>: In the Kirchoff formulation for axi-symmetric shell all shear effects are neglected. The B matrix considers only the membrane strain and the bending strain matrices.

therefore, B becomes

$$\bar{B}^e = \begin{bmatrix} B^e_m \\ B^e_b \end{bmatrix} = \begin{bmatrix} \frac{\partial N^e_u}{\partial s} & 0 & 0 \\ \frac{N^e_u \cos\theta}{r} & -\frac{N^e_w \sin\theta}{r} & -\frac{N_w \sin\theta}{r} \\ 0 & \frac{\partial^2 N_w}{\partial s^2} & \frac{\partial^2 N^e_w}{\partial s^2} \\ 0 & \frac{\cos\theta}{r}\frac{\partial N^e_w}{\partial s} & \frac{\cos\theta}{r}\frac{\partial N^e}{\partial s} \end{bmatrix}$$

Since there are many elements in B which depend on $1/r$, care must be put not to evaluate the functions near $r=0$

Gaussian quadrature does not require evaluation at the extremes of the range and provides excellent approximations to the numerical integral minimizing the required operations.

It is therefore the recommended method!