CSMD

Assignment 9

Lei Pan

a) When the loading is no longer symmetric, we need to apply Fourier series in a circumferential direction. Then we obtain the results for each Fourier component and sum them using proper symmetric and anti-symmetric component multiplication to get the solution on the desired angle.

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b) when using thin beams formulation, we have implemented the Kirchhoff assumption that the normals to the generatrix remain straight and orthogonal to the generatrix after deformation. So the normal rotation coincides with the slope of the generatrix

$$\theta = \frac{\partial w'}{\partial s}\Big|_{z'=0} \quad ①$$

Because $\frac{\partial w'}{\partial s} = \frac{\partial w_0'}{\partial s} + \frac{u_0'}{R_s} - z'\frac{\theta}{R_s}$ ②. we replace ② into ① and get:

$$\theta = \frac{\partial w_0'}{\partial s} + \frac{u_0'}{R_s} \quad ③$$

We have $\gamma_{x'z'} = \frac{1}{C_s}\left(\frac{\partial w_0'}{\partial s} + \frac{u_0'}{R_s} - \theta\right)$ ④. substituting ③ into ④:

$$\gamma_{x'z'} = \frac{1}{C_s}\left(\frac{\partial w_0'}{\partial s} + \frac{u_0'}{R_s} - \left(\frac{\partial w_0'}{\partial s} + \frac{u_0'}{R_s}\right)\right) = 0$$ (the shear effects is neglected).

Hence, the local displacement is:

$$\vec{u}^e = \left[u_0', w_0', \frac{\partial w_0}{\partial s}\right]^T$$

When not considering thin beams formulation

$$\varepsilon' = \begin{Bmatrix} \hat{\varepsilon}_m' \\ 0 \end{Bmatrix} + \begin{Bmatrix} z'\hat{\varepsilon}_b' \\ \hat{\varepsilon}_s' \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0'}{\partial s} - \frac{w_0'}{R} \\ \frac{u_0'\cos\phi - w_0'\sin\phi}{r} \\ 0 \end{Bmatrix} + \begin{Bmatrix} -z'\frac{\partial\theta}{\partial s} \\ -z'\frac{\theta\cos\phi}{r} \\ \frac{\partial w_0'}{\partial s} + \frac{w_0'}{R} - \theta \end{Bmatrix} \quad ⑤$$

Becaus $\hat{\varepsilon}'_s = 0$, so we replacting $\hat{\varepsilon}'_s = 0$ and $\gamma = \frac{\partial w'_0}{\partial s} + \frac{u'_0}{R_s}$ into ①:

$$\hat{\varepsilon}'_m = \begin{Bmatrix} \frac{\partial u'_0}{\partial s} - \frac{\partial w'_0}{R_s} \\ \frac{u'_0 \cos\phi - w'_0 \sin\phi}{r} \end{Bmatrix} ; \quad \hat{\varepsilon}'_b = \begin{Bmatrix} \frac{\partial^2 w'_0}{\partial s^2} + \frac{\partial}{\partial s}\left(\frac{u'_0}{R_s}\right) \\ \frac{\cos\phi}{r}\left(\frac{\partial w'_0}{\partial s} + \frac{u'_0}{R_s}\right) \end{Bmatrix}$$

We consider $R_s = \infty$. So:

$$\hat{\varepsilon}'_m = \begin{Bmatrix} \frac{\partial u'_0}{\partial s} \\ \frac{u'_0 \cos\phi - w'_0 \sin\phi}{r} \end{Bmatrix} ; \quad \hat{\varepsilon}'_b = \begin{Bmatrix} \frac{\partial^2 w'_0}{\partial s^2} \\ \frac{\cos\phi}{r}\frac{\partial w'_0}{\partial s} \end{Bmatrix}$$

And: $\vec{u}' = \sum_{i=1}^{n} \tilde{N}_i \vec{a}_i^{(e)}$; $\quad \hat{\varepsilon}' = \sum_{i=1}^{n} \tilde{B}'_i \vec{a}_i'^{(e)}$

$$\Rightarrow B'_i = \begin{Bmatrix} B'_{mi} \\ \hline B'_{bi} \end{Bmatrix} = \begin{bmatrix} \frac{\partial N_i^u}{\partial s} & 0 & 0 \\ \frac{N_i^u \cos\phi}{r} & \frac{-N_i^w \sin\phi}{r} & \frac{-\bar{N}_i^w \sin\phi}{r} \\ \hline 0 & \frac{\partial^2 N_i^w}{\partial s^2} & \frac{\partial^2 \bar{N}_i^w}{\partial s^2} \\ 0 & \frac{\cos\phi}{r}\frac{\partial N_i^w}{\partial s} & \frac{\cos\phi}{r}\frac{\partial \bar{N}_i^w}{\partial s} \end{bmatrix}$$

A two-point quadrature is recommended. ~~for~~ for the integration.