MAESTRÍA EN INGENIERÍA ESTRUCTURAL Y DE CONSTRUCCIÓN

U N I V E R S I T A T P O L I T É C N I C A D E C A T A L U N Y A

TRABAJO N°05:
Isoparametric Representation
Convergence Rerquirements

Student:

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"Isoparametric representation"

Problem 5.1

Consider a three-node bar element referred to the natural coordinate ξ. The two end nodes and the mid node are identified as 1, 2 and 3 respectively. The natural coordinates of nodes 1, 2 and 3 are $\xi = -1$, $\xi = 1$ and $\xi = 0$, respectively. The variation of the shape functions $N_1(\xi)$, $N_2(\xi)$ and $N_3(\xi)$ is sketched in the figure below. These functions must be quadratic polynomials in ξ:

$$N_1^e(\xi) = a_0 + a_1\xi + a_2\xi^2 \quad N_2^e(\xi) = b_0 + b_1\xi + b_2\xi^2 \quad N_3^e(\xi) = c_0 + c_1\xi + c_2\xi^2$$

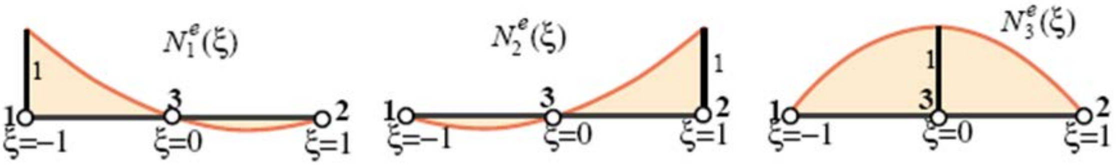


Figure.- Isoparametric shape functions for 3-node bar element (sketch).
Node 3 has been drawn at the 1-2 midpoint but it may be moved away from it.

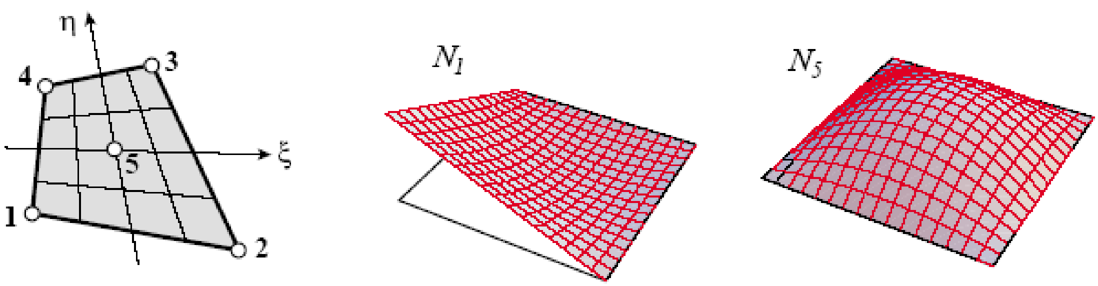
a) Determine the coefficients $a_0,\ldots,c_2$ using the node value conditions depicted in figure. For exemple $N^e_1 = 1$ for $\xi=1$ and 0 for the rest of natural coordinates. The rest of the nodes follow the same scheme.

b) Verify that their sum is identically one.

c) Calculate their derivatives respect to the natural coordinates.

Problem 5.2

A five node quadrilateral element has the nodal configuration shown if the figure with two perspective views of N^e_1 and N^e_5. Find five shape functions N^e_i, i=1,…,5 that satisfy compatibility and also verify that their sum is unity.



Hint: develop $N_5(\xi,\eta)$ first for the 5-node quad using the line-product method. Then the corner shape functions $\underline{N}_i(\xi,\eta)$, i=1,2,3,4, for the 4-node quad (already given in the notes). Finally combine $N_i = \underline{N}_i + \alpha N_5$ determining α so that all N_i vanish node 5. Check that $N_1 + N_2 + N_3 + N_4 + N_5 = 1$ identically.

“Convergence rerquirements”

Problem 5.3

Which minimum integration rules of Gauss-product type gives a rank sufficient stiffness matrix for these elements:

1. the 8-node hexahedron
2. the 20-node hexahedron
3. the 27-node hexahedron
4. the 64-node hexahedron

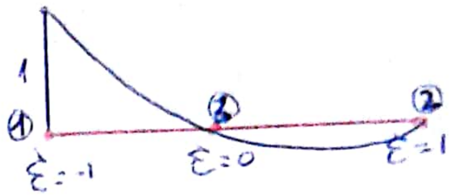
Date of Assignment: **5 / 03 / 2018**
Date of Submission: **12 / 03 / 2018**

The assignment must be submitted as a pdf file named **As5-Surname.pdf** to the CIMNE virtual center.

Problema 5.1

i) Calculo de los coeficientes a_0 ... c_2:

* Para $N_1^e(\xi) = a_0 + a_1\xi + a_2\xi^2$



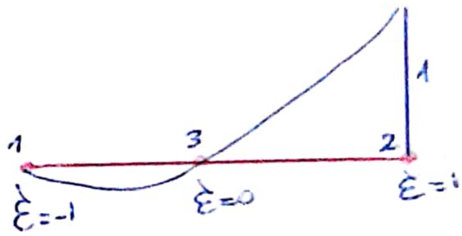
Para nodo 1: $N_1^e(\xi) = 1 \rightarrow 1 = a_0 + a_1(-1) + a_2(-1)^2 = a_0 - a_1 + a_2$

Para nodo 3: $N_1^e(\xi) = 0 \rightarrow 0 = a_0 + a_1(0) + a_2(0)^2 = a_0$

Para nodo 2: $N_1^e(\xi) = 0 \rightarrow 0 = a_0 + a_1(1) + a_2(1)^2 = a_0 + a_1 + a_2$

Asi tendremos:
$$\left.\begin{aligned} 1 &= a_0 - a_1 + a_2 \\ 0 &= a_0 \\ 0 &= a_0 + a_1 + a_2 \end{aligned}\right\} \left.\begin{aligned} 1 + a_1 &= a_2 \\ -a_1 &= a_2 \end{aligned}\right\} a_0 = 0,\ a_1 = -0.5,\ a_2 = 0.5$$

* Para $N_2^e(\xi) = b_0 + b_1\xi + b_2\xi^2$



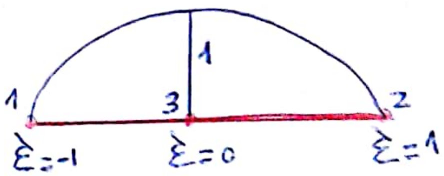
Para nodo 1: $N_2^e(\xi) = 0 \rightarrow 0 = b_0 + b_1(-1) + b_2(-1)^2 = b_0 - b_1 + b_2$

Para nodo 3: $N_2^e(\xi) = 0 \rightarrow 0 = b_0 + b_1(0) + b_2(0)^2 = b_0$

Para nodo 2: $N_2^e(\xi) = 1 \rightarrow 1 = b_0 + b_1(1) + b_2(1)^2 = b_0 + b_1 + b_2$

Asi tendremos:
$$\left.\begin{aligned} 0 &= b_0 - b_1 + b_2 \\ 0 &= b_0 \\ 1 &= b_0 + b_1 + b_2 \end{aligned}\right\} b_0 = 0,\ b_1 = 0.5,\ b_2 = 0.5$$

* Para $N_3^e(\xi) = c_0 + c_1\xi + c_2\xi^2$



Para nodo 1: $N_3^e(\xi) = 0 \quad 0 = c_0 + c_1(-1) + c_2(-1)^2 = c_0 - c_1 + c_2$

Para nodo 3: $N_3^e(\xi) = 1 \quad 1 = c_0 + c_1(0) + c_2(0)^2 = c_0$

Para nodo 2: $N_3^e(\xi) = 0 \quad 0 = c_0 + c_1(1) + c_2(1)^2 = c_0 + c_1 + c_2$

Asi tendremos:
$$\left.\begin{aligned} 0 &= c_0 - c_1 + c_2 \\ 1 &= c_0 \\ 0 &= c_0 + c_1 + c_2 \end{aligned}\right\} c_0 = 1,\ c_1 = 0,\ c_2 = -1$$

ii) Verificación:

Reemplazando los valores de los coeficientes en las funciones de forma:

$$N_1^e = a_0 + a_1\xi + a_2\xi^2 = 0 + (-0.5)(\xi) + (0.5)(\xi)^2 = -0.5\xi + 0.5\xi^2$$

$$N_2^e = b_0 + b_1\xi + b_2\xi^2 = 0 + (0.5)(\xi) + (0.5)(\xi)^2 = 0.5\xi + 0.5\xi^2$$

$$N_3^e = c_0 + c_1\xi + c_2\xi^2 = 1 + 0(\xi) + (-1)\xi^2 = 1 - \xi^2$$

$$N_1^e + N_2^e + N_3^e = (-0.5\xi + 0.5\xi^2) + (0.5\xi + 0.5\xi^2) + 1 - \xi^2$$

$$N_1^e + N_2^e + N_3^e = 1$$

iii) Derivadas de funciones de forma respecto a coordenadas naturales:

$$N_1^e = -0.5\xi + 0.5\xi^2 \qquad \frac{\partial N_1^e}{\partial \xi} = -0.5 + \xi$$

$$N_2^e = 0.5\xi + 0.5\xi^2 \qquad \frac{\partial N_2^e}{\partial \xi} = 0.5 + \xi$$

$$N_3^e = 1 - \xi^2 \qquad \frac{\partial N_3^e}{\partial \xi} = -2\xi$$

Problema 5.2

- Funcion de Forma del nodo 5:

Conforme el grafico, se deberá cumplir que $N_5^e = 0$ para los bordes, osea para $\xi = 1$; $\xi = -1$, $\eta = 1$, $\eta = -1$, por lo que podremos plantear:

$$N_5^e = c_0(\xi+1)(\xi-1)(\eta+1)(\eta-1)$$
$$N_5^e = c_0(\xi^2-1)(\eta^2-1)$$

Luego, se cumplirá que pará $N_5^e = 1$; $\xi = 0$ y $\eta = 0$ $\rightarrow$ $1 = c_0(0-1)(0-1) \rightarrow c_0 = 1$

Luego: $N_5^e = (\xi^2-1)(\eta^2-1)$

* Funcion de Forma de los nodos 1, 2, 3, 4:

Nos basaremos en la ecuación: $N_i^e = N_i + \alpha N_5^e$

donde: $N_i = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)$

Luego para nodo 1: $N_1 = \frac{1}{4}(1+\xi(-1))(1+\eta(-1))$

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)$$

para nodo 2: $N_2 = \frac{1}{4}(1+\xi(1))(1+\eta(-1))$

$$N_2 = \frac{1}{4}(1+\xi)(1-\eta)$$

para nodo 3: $N_3 = \frac{1}{4}(1+\xi(1))(1+\eta(1))$

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)$$

para nodo 4: $N_4 = \frac{1}{4}(1+\xi(-1))(1+\eta(1))$

$$N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

Ahora reemplazamos en la ecuacion inicial:

$$N_1^e = N_1 + \alpha N_5^e = \frac{1}{4}(1-\xi)(1-\eta) + \alpha(\xi^2-1)(\eta^2-1)$$

Se cumplirá que: para $N_1^e = 0$; $\xi = 0$ y $\eta = 0$

Reemplazando:

$$0 = \frac{1}{4}(1-0)(1-0) + \alpha(0-1)(0-1) \longrightarrow \alpha = -\frac{1}{4}$$

Luego observamos que $\alpha = -\frac{1}{4}$ se cumplirá para N_2^e; N_3^e; N_4^e.

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Luego conoceremos las funciones de forma:

$$N_1^e = \frac{1}{4}(1-\varepsilon)(1-n) - \frac{1}{4}(\varepsilon^2-1)(n^2-1)$$
$$N_2^e = \frac{1}{4}(1+\varepsilon)(1-n) - \frac{1}{4}(\varepsilon^2-1)(n^2-1)$$
$$N_3^e = \frac{1}{4}(1+\varepsilon)(1+n) - \frac{1}{4}(\varepsilon^2-1)(n^2-1)$$
$$N_4^e = \frac{1}{4}(1-\varepsilon)(1+n) - \frac{1}{4}(\varepsilon^2-1)(n^2-1)$$
$$N_5^e = (\varepsilon^2-1)(n^2-1)$$

Comprobación:

$$N_1^e + N_2^e + N_3^e + N_4^e + N_5^o$$
$$\frac{1}{4}(1-\varepsilon)[(1-n)+(1+n)] + \frac{1}{4}(1+\varepsilon)[(1-n)+(1+n)] - (4)\left(\frac{1}{4}\right)(\varepsilon^2-1)(n^2-1) + (\varepsilon^2-1)(n^2-1)$$
$$\frac{1}{2}(1-\varepsilon) + \frac{1}{2}(1+\varepsilon) = \frac{1}{2} - \frac{\varepsilon}{2} + \frac{1}{2} + \frac{\varepsilon}{2} = 1$$