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Assignment 3.1

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \quad -(1); \quad \mu = \frac{E}{2(1+\nu)} \Rightarrow E = \mu 2(1+\nu) \quad -(2)$$

~~from~~ Substituting (2) in (1) we get,

$$\lambda = \frac{2\cancel{(1+\nu)}\nu\mu}{\cancel{(1+\nu)}(1-2\nu)}$$

$$\lambda = \frac{2\nu\mu}{1-2\nu}$$

$$\Rightarrow \frac{\mu}{\lambda} = \frac{1-2\nu}{2\nu}$$

$$\Rightarrow \frac{\mu}{\lambda} = \frac{1}{2\nu} - 1$$

$$\Rightarrow \frac{\mu}{\lambda} + 1 = \frac{1}{2\nu}$$

$$\Rightarrow \frac{(\mu+\lambda)2}{\lambda} = \frac{1}{\nu}$$

$$\Rightarrow \boxed{\nu = \frac{\lambda}{2(\mu+\lambda)}} \quad -(3)$$

from (2) we have,

$$E = 2\mu(1+\nu)$$

$$= 2\mu\left(1 + \frac{\lambda}{2(\mu+\lambda)}\right)$$

$$= \cancel{2}\mu\left(\frac{2\mu + 2\lambda + \lambda}{\cancel{2}(\mu+\lambda)}\right)$$

$$\therefore \boxed{E = \mu\left(\frac{2\mu + 3\lambda}{\mu+\lambda}\right)} \quad — (4)$$

2. Plane Stress.

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \end{bmatrix}$$

where $\dfrac{E}{1-\nu^2} = \dfrac{E}{(1+\nu)(1-\nu)}$ — (5)

Substituting values of (3) & (4) we get

$$\frac{E}{(1+\nu)(1-\nu)} = \frac{\mu\left(\frac{2\mu + 3\lambda}{\mu+\lambda}\right)}{\left(1 + \frac{\lambda}{2(\mu+\lambda)}\right)\left(1 - \frac{\lambda}{2(\mu+\lambda)}\right)}$$

$$= \frac{\mu\left(\frac{2\mu + 3\lambda}{\mu+\lambda}\right)}{\frac{(2\mu+2\lambda+\lambda)(2\mu+\lambda)}{4(\mu+\lambda)^2}} = \frac{\mu\left(\frac{\cancel{2\mu+3\lambda}}{\cancel{\mu+\lambda}}\right)}{\frac{\cancel{(2\mu+3\lambda)}(2\mu+\lambda)}{4(\mu+\lambda)^{\cancel{2}}}}$$

$$\Rightarrow \boxed{\frac{E}{1-\nu^2} = \frac{4\mu(\mu+\lambda)}{2\mu+\lambda}} \quad — (6)$$

$$\frac{1-\nu}{2} = \frac{1 - \frac{\lambda}{2(\mu+\lambda)}}{2}$$

$$= \frac{2\mu + 2\lambda - \lambda}{4(\mu+\lambda)} \Rightarrow \boxed{\frac{1-\nu}{2} = \frac{2\mu+\lambda}{4(\mu+\lambda)}}$$

$\therefore$ From Plane Stress eq^n for 2D problem we have,

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{4\mu(\mu+\lambda)}{2\mu+\lambda} \begin{bmatrix} 1 & \frac{\lambda}{2(\mu+\lambda)} & 0 \\ \frac{\lambda}{2(\mu+\lambda)} & 1 & 0 \\ 0 & 0 & \frac{2\mu+\lambda}{4(\mu+\lambda)} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \end{bmatrix}$$

2. Plane Strain

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \end{bmatrix}$$

where, $$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)} = \frac{\mu\left(\frac{2\mu+3\lambda}{\mu+\lambda}\right)\left(1-\frac{\lambda}{2(\mu+\lambda)}\right)}{\left(1+\frac{\lambda}{2(\mu+\lambda)}\right)\left(1-\frac{2\lambda}{2(\mu+\lambda)}\right)}$$

$$= \frac{\mu\cancel{\left(\frac{2\mu+3\lambda}{\mu+\lambda}\right)}\left(\frac{2\mu+2\lambda-\lambda}{\cancel{2(\mu+\lambda)}}\right)}{\cancel{\left(\frac{2\mu+2\lambda+\lambda}{2(\mu+\lambda)}\right)}\left(\frac{2\mu+\cancel{2\lambda}-\cancel{2\lambda}}{\cancel{2(\mu+\lambda)}}\right)} = \frac{\cancel{\mu}(2\mu+\lambda)}{\cancel{\mu}}$$

$$\therefore \quad \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} = 2\mu+\lambda$$

and,
$$\frac{\nu}{1-\nu} = \frac{\dfrac{\lambda}{2(\mu+\lambda)}}{1-\dfrac{\lambda}{2(\mu+\lambda)}}$$

$$= \frac{\dfrac{\lambda}{\cancel{2(\mu+\lambda)}}}{\dfrac{2\mu+\cancel{2}\lambda-\cancel{\lambda}}{\cancel{2(\mu+\lambda)}}}$$

$$\boxed{\frac{\nu}{1-\nu} = \frac{\lambda}{2\mu+\lambda}}$$

and,
$$\frac{1-2\nu}{2(1-\nu)} = \frac{1-\dfrac{2\lambda}{2(\mu+\lambda)}}{2\left(\dfrac{2\mu+\lambda}{2(\mu+\lambda)}\right)}$$

$$= \frac{\cancel{2}\mu}{\cancel{2}\cancel{(\mu+\lambda)}} \times \frac{\cancel{(\mu+\lambda)}}{2\mu+\lambda}$$

$$= \frac{\mu}{2\mu+\lambda}$$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = (2\mu+\lambda)\begin{bmatrix} 1 & \dfrac{\lambda}{2\mu+\lambda} & 0 \\ \dfrac{\lambda}{2\mu+\lambda} & 1 & 0 \\ 0 & 0 & \dfrac{\mu}{2\mu+\lambda} \end{bmatrix}\begin{bmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \end{bmatrix}$$

The Stress Strain Matrix E

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} 2\mu+\lambda & \lambda & 0 \\ \lambda & 2\mu+\lambda & 0 \\ 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \left(\underbrace{\begin{bmatrix} 2\mu & 0 & 0 \\ 0 & 2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix}}_{E_\mu} + \underbrace{\begin{bmatrix} \lambda & \lambda & 0 \\ \lambda & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{E_\lambda} \right) \begin{bmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \end{bmatrix}$$

$$\{\sigma\} = \left([E_\mu] + [E_\lambda] \right) \{e\}$$

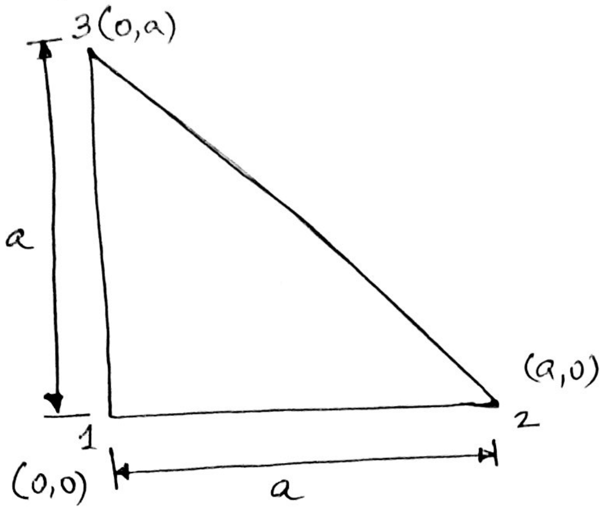
4) E_λ in terms of E & ν

$$E_\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$E_\mu = \frac{E}{2(1+\nu)} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Assignment 3.2.

(1)



$$K_{tri} = A h B^T E B$$

$$= \frac{h}{4A}\begin{bmatrix} y_{23} & 0 & x_{32} \\ 0 & x_{32} & y_{23} \\ y_{31} & 0 & x_{13} \\ 0 & x_{13} & y_{31} \\ y_{12} & 0 & x_{21} \\ 0 & x_{21} & y_{12} \end{bmatrix} \times \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{12} & E_{22} & E_{23} \\ E_{13} & E_{23} & E_{33} \end{bmatrix}$$

$$\times \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{13} & x_{21} & y_{21} \end{bmatrix}$$

$x_1 = 0\ ,\ y_1 = 0$

$x_2 = a,\ y_2 = 0$

$x_3 = 0 \quad y_3 = a$

$y_{23} = y_2 - y_3 = 0 - a = a \ ; \quad x_{32} = x_3 - x_2 = -a$

$y_{31} = y_3 - y_1 = a \ ; \quad x_{13} = x_1 - x_3 = 0$

$y_{12} = y_1 - y_2 = 0 \ ; \quad x_{21} = x_2 - x_1 = a$

$$\therefore K_{tri} = \frac{Eh}{4A(1-\nu^2)} \begin{bmatrix} -a & 0 & -a \\ 0 & -a & -a \\ a & 0 & 0 \\ 0 & 0 & a \\ 0 & 0 & a \\ 0 & a & 0 \end{bmatrix} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} -a & 0 & a & 0 & 0 & a \\ 0 & -a & 0 & 0 & 0 & a \\ -a & -a & 0 & a & a & 0 \end{bmatrix}$$

$$= \frac{Eh}{4A(1-\nu^2)} \begin{bmatrix} -a & 0 & -a \\ 0 & -a & -a \\ a & 0 & 0 \\ 0 & 0 & a \\ 0 & 0 & a \\ 0 & a & 0 \end{bmatrix} \begin{bmatrix} -a & -\nu a & a & 0 & 0 & \nu a \\ -\nu a & -a & \nu a & 0 & 0 & a \\ \frac{-a+\nu a}{2} & \frac{-a+\nu a}{2} & 0 & \frac{a-\nu a}{2} & \frac{a-\nu a}{2} & 0 \end{bmatrix}$$

$$= \frac{Eh}{2a^2(1-\nu^2)} \begin{bmatrix} a^2 + \frac{a^2-\nu a^2}{2} & \nu a^2 + \frac{a^2-\nu a^2}{2} & -a^2 & \frac{-a^2+\nu a^2}{2} & \frac{-a+\nu a^2}{2} & -\nu a^2 \\ \nu a^2 + \frac{a^2-\nu a^2}{2} & a^2 + \frac{a^2-\nu a^2}{2} & -\nu a^2 & \frac{-a^2+\nu a^2}{2} & \frac{-a+\nu a^2}{2} & -a^2 \\ -a^2 & -\nu a^2 & a^2 & 0 & 0 & \nu a^2 \\ \frac{-a^2+\nu a^2}{2} & \frac{-a^2+\nu a^2}{2} & 0 & \frac{a^2-\nu a^2}{2} & \frac{a^2-\nu a^2}{2} & 0 \\ \frac{-a^2+\nu a^2}{2} & \frac{-a^2+\nu a^2}{2} & 0 & \frac{a^2-\nu a^2}{2} & \frac{a^2-\nu a^2}{2} & 0 \\ -\nu a^2 & -a^2 & \nu a^2 & 0 & 0 & a^2 \end{bmatrix}$$

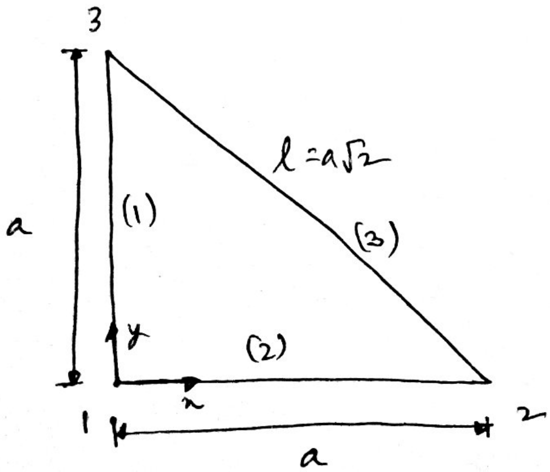
$$= \frac{Eh}{2(1-\nu^2)} \begin{bmatrix} 1 + \frac{1-\nu}{2} & \nu + \frac{1-\nu}{2} & -1 & \frac{-1+\nu}{2} & \frac{-1+\nu}{2} & -\nu \\ \nu + \frac{1-\nu}{2} & 1 + \frac{1-\nu}{2} & -\nu & \frac{-1+\nu}{2} & \frac{-1+\nu}{2} & -1 \\ -1 & -\nu & 1 & 0 & 0 & \nu \\ \frac{-1+\nu}{2} & \frac{-1+\nu}{2} & 0 & \frac{1-\nu}{2} & \frac{1-\nu}{2} & 0 \\ \frac{-1+\nu}{2} & \frac{-1+\nu}{2} & 0 & \frac{1-\nu}{2} & \frac{1-\nu}{2} & 0 \\ -\nu & -1 & \nu & 0 & 0 & 1 \end{bmatrix}$$

(2) ~~Therefore~~

Taking $a=1$, $h=1$ & $\nu=0$, we have,

$$K_{Tri} = \frac{E}{2}\begin{bmatrix} \frac{3}{2} & \frac{1}{2} & -1 & -\frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{3}{2} & 0 & -\frac{1}{2} & -\frac{1}{2} & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

For K_{bar}



For element (2)

$$K^{(2)} = \frac{A_2 E}{a}\begin{bmatrix} c_2^2 & c_2 s_2 & -c_2^2 & -c_2 s_2 \\ c_2 s_2 & s_2^2 & -c_2 s_2 & -s_2^2 \\ -c_2^2 & -c_2 s_2 & c_2^2 & c_2 s_2 \\ -c_2 s_2 & -s_2^2 & c_2 s_2 & s_2^2 \end{bmatrix}$$

where $c_2 = \cos 0° = 1$
$s_2 = \sin 0° = 0$

Hence, $$K^{(2)} = \frac{A_2 E}{a}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For Element 1.

$$\theta = 90^\circ, \quad \cancel{c_I =} \; c_1 = \cos\theta = \cos 90^\circ = 0$$
$$s_1 = \sin\theta = \cancel{\cos}\, \sin 90^\circ = 1$$

$$K^{(1)} = \frac{A_1 E}{a} \begin{bmatrix} c_1^2 & c_1 s_1 & -c_1^2 & -c_1 s_1 \\ c_1 s_1 & s_1^2 & -c_1 s_1 & -s_1^2 \\ -c_1^2 & -c_1 s_1 & c_1^2 & c_1 s_1 \\ -c_1 s_1 & -s_1^2 & c_1 s_1 & s_1^2 \end{bmatrix}$$

$$K^{(1)} = \frac{A_1 E}{a} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

For element 3

$$\theta = \cancel{8}\, 135^\circ \qquad c_3 = \cos 135^\circ = -\frac{1}{\sqrt{2}} \qquad c_3^2 = \frac{1}{2} \; ; \; -c_3^2 = -\frac{1}{2}$$
$$s_3 = \sin 135^\circ = \frac{1}{\sqrt{2}} \qquad c_3 s_3 = -\frac{1}{2} \; ; \; -c_3 s_3 = \frac{1}{2}$$
$$s_3^2 = \frac{1}{2} \; ; \; -s_3^2 = -\frac{1}{2}$$

$$K^{(3)} = \frac{A_3 E}{a\sqrt{2}} \begin{bmatrix} c_3^2 & c_3 s_3 & -c_3^2 & -c_3 s_3 \\ c_3 s_3 & s_3^2 & -c_3 s_3 & -s_3^2 \\ -c_3^2 & -c_3 s_3 & c_3^2 & c_3 s_3 \\ -c_3 s_3 & -s_3^2 & c_3 s_3 & s_3^2 \end{bmatrix}$$

$$K^{(3)} = \frac{A_3 E}{a\sqrt{2}} \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$K_{bar} = \begin{bmatrix} \frac{A_2E}{a} & 0 & -\frac{A_2E}{a} & 0 & 0 & 0 \\ 0 & \frac{A_1E}{a} & 0 & 0 & 0 & -\frac{A_1E}{a} \\ -\frac{A_2E}{a} & 0 & \frac{A_2E}{a} + \frac{A_3E}{a2\sqrt{2}} & -\frac{A_3E}{a2\sqrt{2}} & -\frac{A_3E}{2\sqrt{2}a} & \frac{A_3E}{2\sqrt{2}a} \\ 0 & 0 & -\frac{A_3E}{a2\sqrt{2}} & \frac{A_3E}{a2\sqrt{2}} & \frac{A_3E}{2\sqrt{2}a} & -\frac{A_3E}{2\sqrt{2}a} \\ 0 & 0 & -\frac{A_3E}{a2\sqrt{2}} & \frac{A_3E}{2\sqrt{2}a} & \frac{A_3E}{2\sqrt{2}a} & -\frac{A_3E}{2\sqrt{2}a} \\ 0 & -\frac{A_1E}{a} & \frac{A_3E}{2\sqrt{2}a} & -\frac{A_3E}{2\sqrt{2}a} & -\frac{A_3E}{2\sqrt{2}a} & \frac{A_1E}{a} + \frac{A_3E}{2\sqrt{2}a} \end{bmatrix}$$

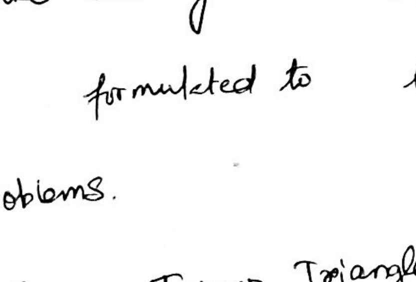
2. There are <u>no</u> values of crossection for which ~~$A_1 = A_2$~~ both the stiffness matrix are equivalent. i.e. $K_{bar} = K_{Tri}$

Values with $A_3 = \sqrt{2}a$ and $A_1 = A_2 = \frac{a}{2}$ may make the matrices more similar, but not equal

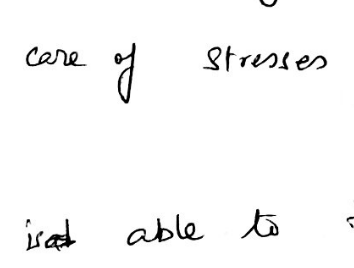
3. Where as the (K_{bar}) bar matrixes are specifically formulated to take care of Trusses, Turner ~~Traingle~~ Triangle matrices are formulated to take care of stresses in Plane Stress problems.

The Turner Triangle is able to take into account the strains somewhere in the middle of the boundary of the element.

That is not possible for the Bar (K_{bar}) element



Strains and displacements taken into account due to the type of Shape function considered



strains and displacement not taken into account due to the Shape function taking into account displacement only along the axis of each bar element

(4) For $\nu \neq 0$

we will have

$$K_{Tri} = \frac{E}{2(1-\nu^2)} \begin{bmatrix} 1+\frac{1-\nu}{2} & \nu+\frac{1-\nu}{2} & -1 & -\frac{1+\nu}{2} & -\frac{1+\nu}{2} & -\nu \\ \nu+\frac{1-\nu}{2} & 1+\frac{1-\nu}{2} & -\nu & \frac{-1+\nu}{2} & \frac{-1+\nu}{2} & -1 \\ -1 & -\nu & 1 & 0 & 0 & \nu \\ \frac{-1+\nu}{2} & \frac{-1+\nu}{2} & 0 & \frac{1-\nu}{2} & \frac{1-\nu}{2} & 0 \\ \frac{-1+\nu}{2} & \frac{-1+\nu}{2} & 0 & \frac{1-\nu}{2} & \frac{1-\nu}{2} & 0 \\ -\nu & -1 & \nu & 0 & 0 & 1 \end{bmatrix}$$

When $\nu \neq 0$, the Element matrix takes into account the lateral displacement that may take place which is not the case when $\nu = 0$.

~~when~~