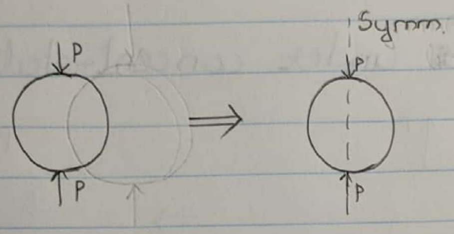
Assignment 2

→ Assignment 2.1 (FEM Modelling: Introduction)

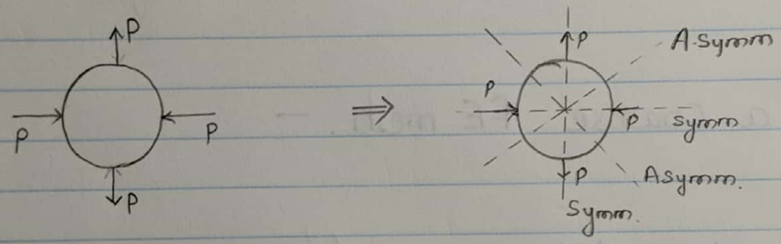
Q1) Symmetry and Antisymmetry lines.

[Symm - Symmetric line
A.Symm - AntiSymmetric line]

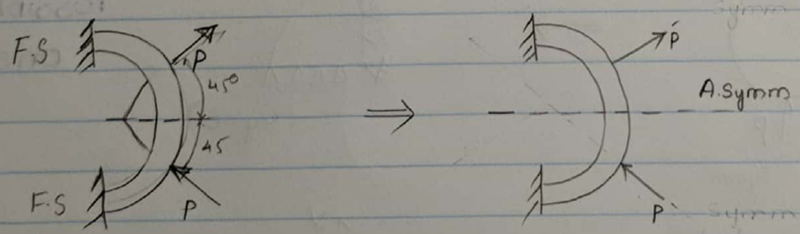
→ (a) A circular disk -



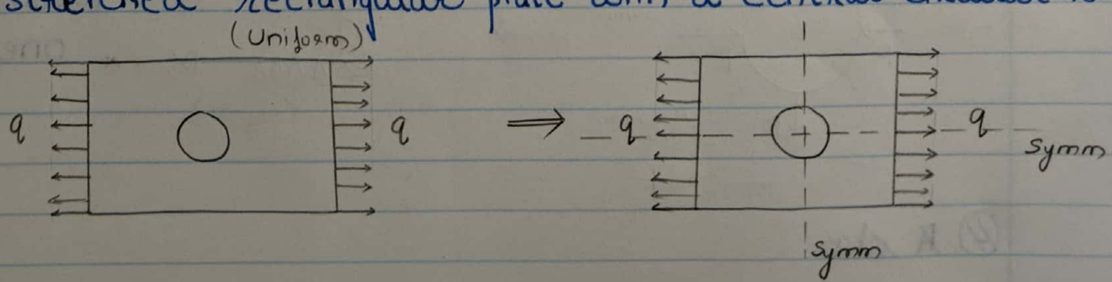
(b) A circular disk -



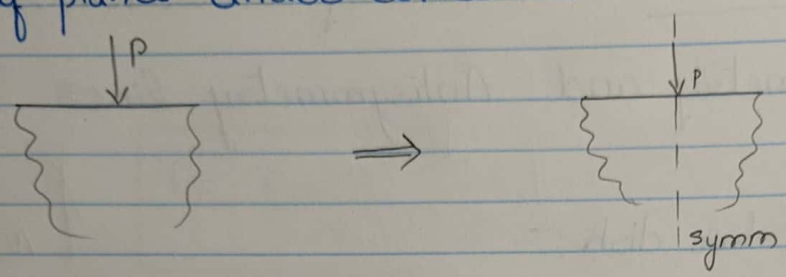
(c) A clamped semiannulus -



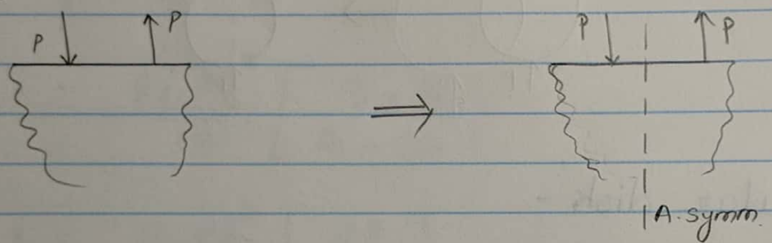
(d) A stretched rectangular plate with a central circular hole



(e) Half-planes under concentrated loads -



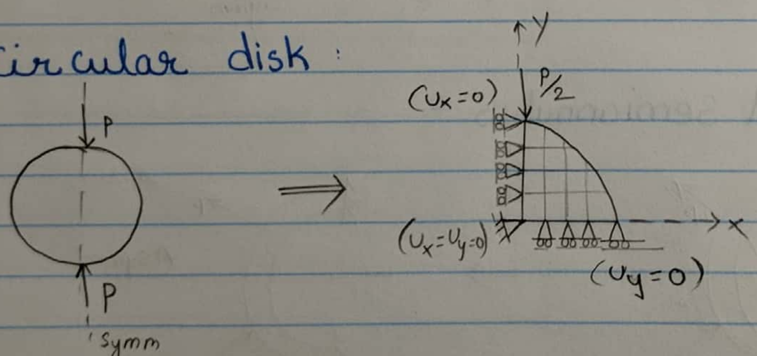
(f) Half-planes under concentrated loads -



Q2) Draw a Coarse FE mesh. —

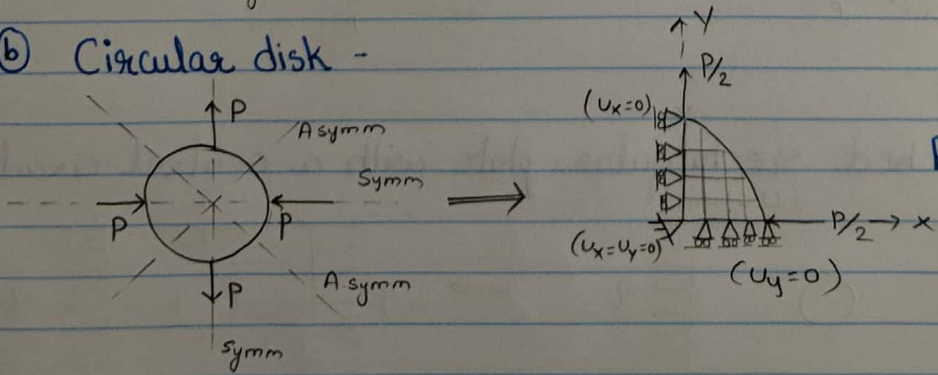
→

(a) Circular disk :



Possible to cut to one quarter

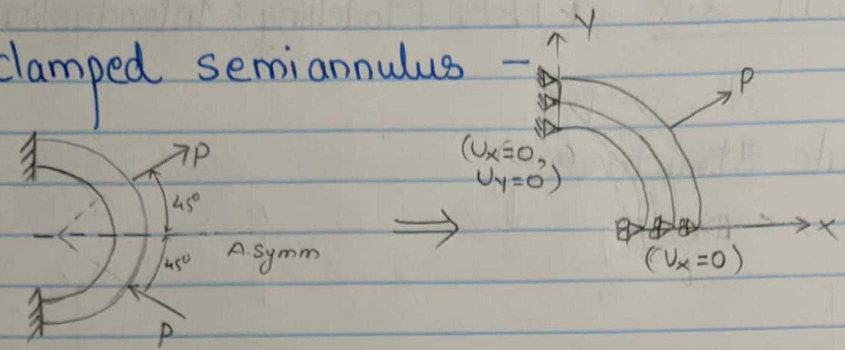
(b) Circular disk -



Possible to cut to one quarter.

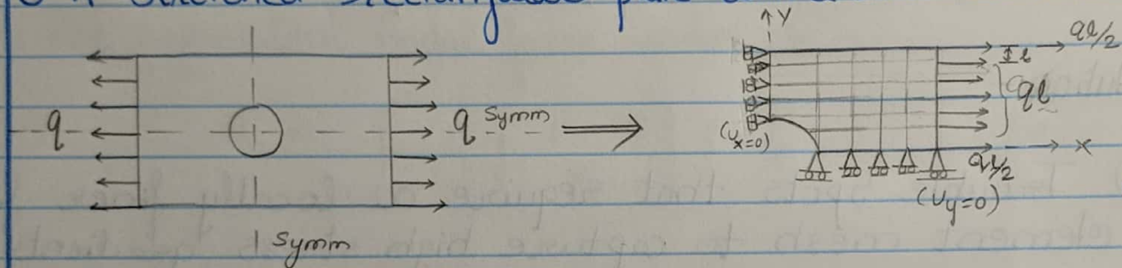
~~(c) A cla~~

(c) A clamped semiannulus –



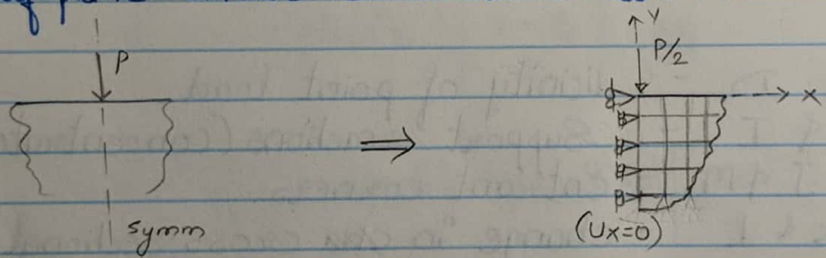
Possible to cut to one quarter half.

(d) A stretched rectangular plate with a central circular hole.



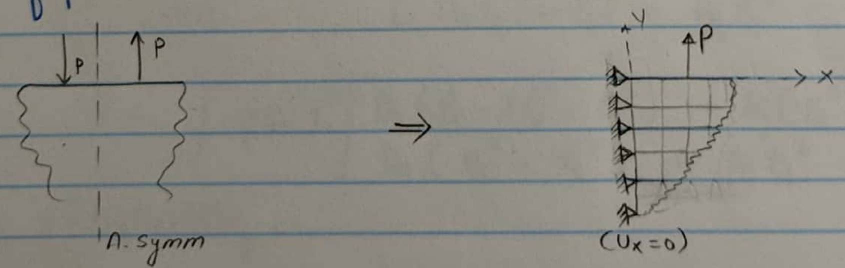
Possible to cut to one quarter.

(e) Half plane under concentrated load.



Possible to cut to half

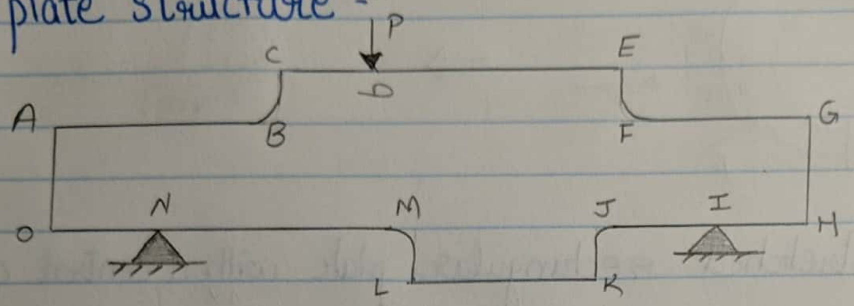
(f) Half plane under concentrated load -



Possible to cut to half.

Assignment 2.2 (FEM Modelling : Introduction)

Q The plate structure -



Solution -

ⓐ Trouble Spots that require a locally finer finite element mesh to capture high stress gradients are points B, C, D, E, F, I, J, K, L, M & N.

ⓑ

(i) ~~Point~~ D - Vicinity of point load.
(ii) N & I - Support reactions (Concentrated loads)
(iii) B, F, J & M - Entrant corners.
(iv) C, E, K & L - change in ~~the~~ cross-sectional area as well as thickness. ~~& increase~~.

Assignment 2.3 (Variational formulation)

Q. Tapered bar element

$$A = A_i(1-\xi) + A_j\xi \quad , \quad \xi = \left(\frac{x - x_i}{l}\right)$$

$$q(x) = \rho A \omega^2 x$$

Solution:

(a)

The consistent nodal force vector is -

$$f = \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l\, d\xi \quad \ldots \left(\xi = \frac{x - x_1}{l}\right)$$

Substituting and solving -

$$f = \int_0^1 (\rho A \omega^2 x) \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l\, d\xi$$

$$F = \int_0^1 (\rho\,(A_i(1-\xi) + A_j\xi)\,\omega^2 \xi l) \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l\, d\xi \quad \cdot\cdot (x = \xi l)$$

$$F = \int_0^1 \rho\omega^2 l^2 \begin{bmatrix} A_i\xi(1-\xi)^2 + A_j\xi^2(1-\xi) \\ A_i\xi^2(1-\xi) + A_j\xi^3 \end{bmatrix} d\xi$$

$$F = \int_0^1 \rho\omega^2 l^2 \begin{bmatrix} A_i(\xi - 2\xi^2 + \xi^3) + A_j(\xi^2 - \xi^3) \\ A_i(\xi^2 - \xi^3) + A_j\xi^3 \end{bmatrix} d\xi$$

on integrating,

$$F = \rho\omega^2 l^2 \begin{bmatrix} A_i\left(\frac{\xi^2}{2} - \frac{2\xi^3}{3} + \frac{\xi^4}{4}\right) + A_j\left(\frac{\xi^3}{3} - \frac{\xi^4}{4}\right) \\ A_i\left(\frac{\xi^3}{3} - \frac{\xi^4}{4}\right) + A_j\frac{\xi^4}{4} \end{bmatrix}_0^1$$

$$F = \rho\omega^2 l^2 \begin{bmatrix} \frac{A_i}{12} + \frac{A_j}{12} \\ \frac{A_i}{12} + \frac{A_j}{4} \end{bmatrix}$$

$$F = \rho\omega^2 l^2 \begin{bmatrix} \frac{1}{12}(A_i + A_j) \\ \frac{1}{12}(A_i + 3A_j) \end{bmatrix}$$

$$\boxed{F = \frac{\rho\omega^2 l^2}{12} \begin{bmatrix} A_i + A_j \\ A_i + 3A_j \end{bmatrix}}$$

(b) The consistent nodal force vector for prismatic bar $A = A_i = A_j$.

$$F = \frac{\rho\omega^2 l^2}{12} \begin{bmatrix} A_i + A_j \\ A_i + 3A_j \end{bmatrix}$$

$$F = \frac{\rho\omega^2 l^2}{12} \begin{bmatrix} 2A \\ 4A \end{bmatrix} \quad \ldots \; (A = A_i = A_j)$$

$$\boxed{F = \frac{\rho\omega^2 l^2 A}{12} \begin{bmatrix} 2 \\ 4 \end{bmatrix}}$$

—— x —— x ——