<u>Computational Structural Mechanics and Dynamics</u>

Assignment 2

FEM

By

Domingo Eugenio Cattoni Correa

Master in numerical method in engineering

Assignment 2.1:

Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in the figure. They are:

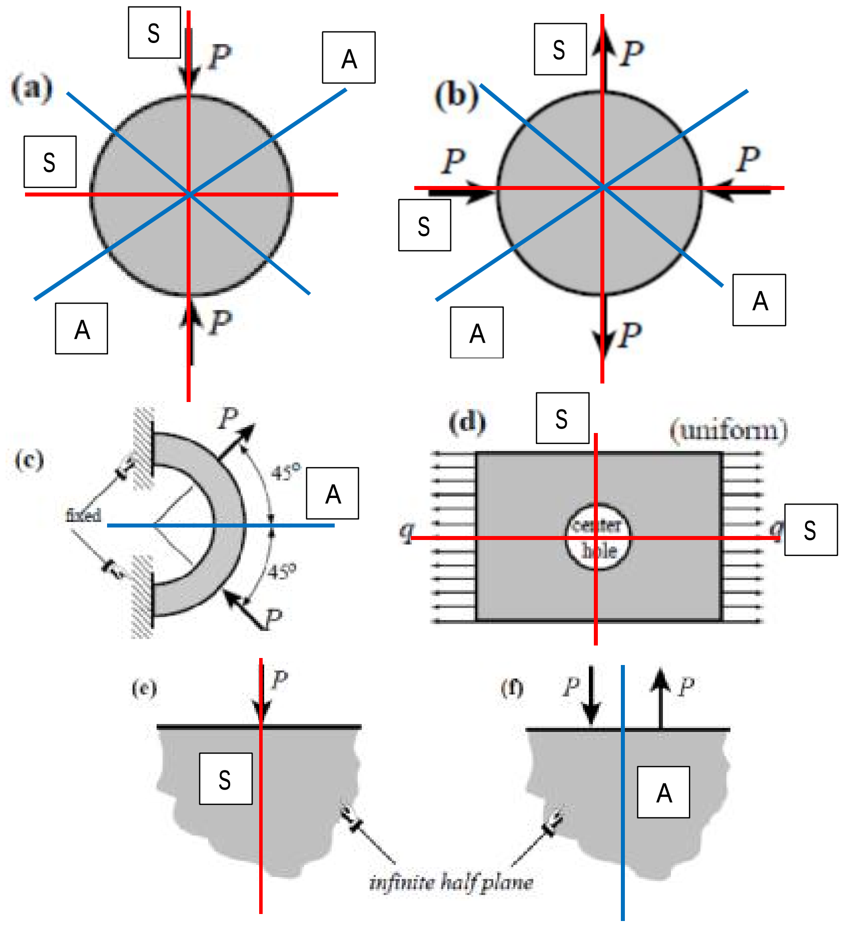


Figure 1: Different geometries associated to different symmetric or antisymmetric axis.

Where
S: Symmetry axis
A: Antisymmetry axis

Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.

In the figure above can be seen that it is possible to cut them to one half or one quarter. For instance, Figure 1 a, b and d can be cut to one quarter. Figure 1 c and f can be cut to one half considering its antisymmetric axis. Finally, Figure 1 e can be cut to one half taking into account its antisymmetric axis.

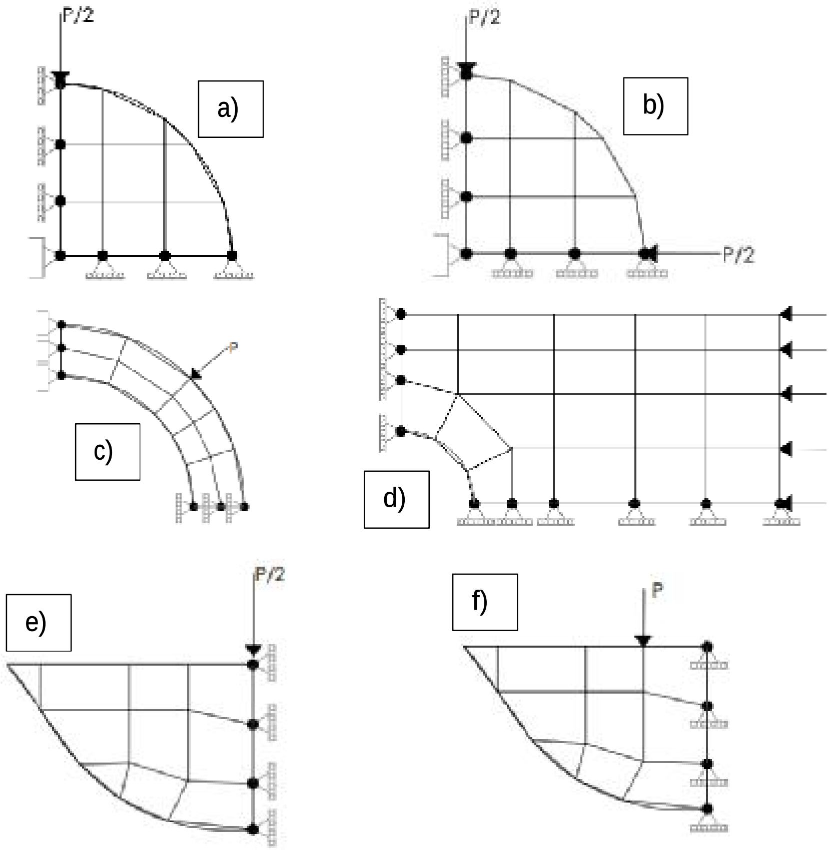
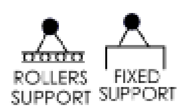


Figure 2: Mesh and BC's. (1)

(1) LEGEND OF THE FIGURE:



The Table 1 shows the type of support and displacement BC for each figure.

Table 1: Type of support and displacement BC.

Figure	Type os Support	Displacement BC
A and B	Roller in X dir	Ty = 0
	Roller in Y dir	Tx = 0
	Fixed over one node	Tx = Ty = 0
C	Roller in Y dir	Tx = 0
	Fixed	Tx = Ty = 0
D	Roller in X dir	Ty = 0
	Roller in Y dir	Tx = 0
E	Roller in Y dir	Tx = 0
F	Roller in X dir	Ty = 0

Assignment 2.2:

The plate structure shown in the figure is loaded and deforms in the plane of the paper. The applied load at *D* and the supports at *I* and *N* extend over a fairly narrow area. List what you think are the likely "trouble spots" that would require a locally finer finite element mesh to capture high stress gradients. Identify those spots by its letter and a reason.

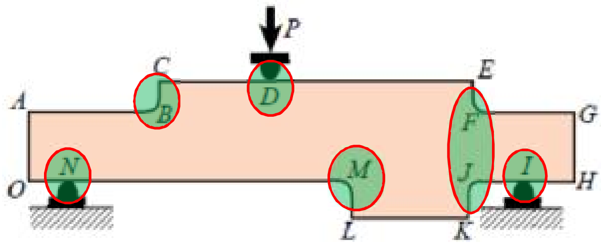


Figure 3: Trouble spots.

In Figure 3 can be seen the most likely “trouble spots” in which will be necessary refine the mesh in order to capture high stress gradients.

In Table 2 is explained why these points should be taken into account for a mesh refinement process.

Table 2: Trouble spots explanation.

Point/ Region	Reason of refinement
Points: B, F J, M	These points are considered near to an entrant corner or sharply curved edges
Points: D, N, I	These points are in vicinity of concentrated load and concentrated reaction.
Region F-J	This region produces an abrupt change of section of the structure.

Assignment 2.3:

A tapered bar element of length l and areas A_i and A_j with A interpolated as

$$A = A_i(1-\xi) + A_j\xi$$

and constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node i. Taking axis x along the rotating bar with origin at node i, the centrifugal axial force is $q(x)=\rho A\omega^2 x$ along the length in which x is the longitudinal coordinate $x=x^e$.

Find the consistent node forces as functions of ρ, A_i, A_j, ω and l, and specialize the result to the prismatic bar $A = A_i = A_j$.

For a given discretized domain Ω and a given body or volumetric force it can be possible to compute that force over each node belong to the discretized domain as:

$$\int_{\Omega} \underset{\sim}{N}^T f_{vol}\, d\Omega \quad [1]$$

Where:

N^T : Set of shape functions

f_{vol} : Volumetric or body force

For this assignment the body force is the given axial force $f_v = q(x)$. Taking into account that the domain is discretized with one 1D linear element, then, the set of shape functions in terms of a natural coordinate system are:

$$\underset{\sim}{N}^T = \begin{bmatrix} 1-x \\ x \end{bmatrix}$$

The domain can be discretized using a natural coordinate system as show in Figure 4.

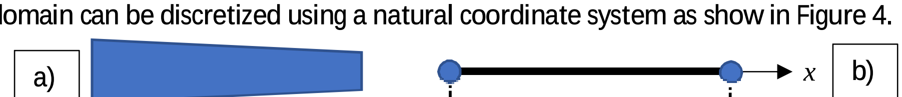


Figure 4: Domain. a) Tapered Bar, b) Discretized domain with 1D linear element.

So, the problem is find the consistence nodal forces in term of some given parameter:

First, to express the integral $\int_{\Omega} \underset{\sim}{N}^T f_{vol} d\Omega$ [1] in terms of the natural coordinate system, it will be necessary to do a change of variable.

$x^e = N_i x_i + N_j x_j = x L$

Where:

$N_i = 1 - x$

$N_j = x$

$x_i = 0$

$x_j = L$

$$\int_0^1 \underset{\sim}{N}^T q(x) |J| dx$$

Where:

$q(x) = r A w^2 L x$

$A = A_i(1-x) + A_j x$

$\underset{\sim}{N}^T = \begin{bmatrix} 1-x \\ x \end{bmatrix}$

$|J| = \left| \frac{dx}{dx} \right| = L$ (Jacobian of the transformation)

$$\int_0^1 \begin{bmatrix} 1-x \\ x \end{bmatrix} \left[r(A_i(1-x) + A_j x) w^2 L x \right] L dx = \begin{bmatrix} \dfrac{Lrw^2(A_i + A_j)}{12} \\ \dfrac{Lrw^2(A_i + 3A_j}{12} \end{bmatrix}$$

The force computed is:

$$f = \begin{bmatrix} \dfrac{Lrw^2(A_i + A_j)}{12} \\ \dfrac{Lrw^2(A_i + 3A_j)}{12} \end{bmatrix}$$

Where:

$f_j = \dfrac{Lrw^2(A_j + A_i)}{12}$ force at node j

$f_i = \dfrac{Lrw^2(A_i + 3A_j)}{12}$ force at node i

In the case of prismatic bar where $A=A_j=A_i$ the result is:

$f_j = \dfrac{Lrw^2 A}{6}$ force at node j

$f_i = \dfrac{Lrw^2 A}{3}$ force at node i