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Master of Science in Computational Mechanics 2017-2018

Computational Structural Mechanics and Dynamics

Assignment 1

On "The Direct Stiffness Method":

Consider the truss problem defined in the Figure. All geometric and material properties: L, α, E and A, as well as the applied forces P and H, are to be kept as variables. This truss has 8 degrees of freedom, with six of them removable by the fixed-displacement conditions at nodes 2, 3 and 4. This structure is statically indeterminate as long as $\alpha \neq 0$.

(a) Show that the master stiffness equations are

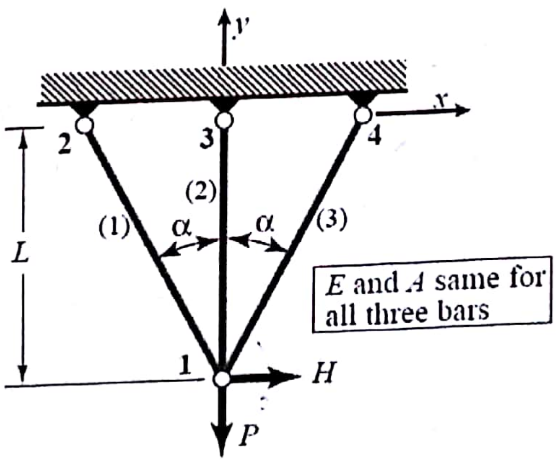
$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ & & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & & 1 & 0 & 0 \\ & & & & & & cs^2 & c^2s \\ \text{symm} & & & & & & & c^3 \end{bmatrix}\begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix} = \begin{bmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

in which $c = \cos\alpha$ and $s = \sin\alpha$. Explain from physics why the 5th row and column contain only zeros.

(b) Apply the BC's and show the 2-equation modified stiffness system.

(c) Solve for the displacements u_{x1} and u_{y1}. Check that the solution makes physical sense for the limit cases $\alpha \to 0$ and $\alpha \to \pi/2$. Why does u_{x1} "blow up" if $H \neq 0$ and $\alpha \to 0$?

(d) Recover the axial forces in the three members. Partial answer: $F^{(3)} = -H/(2s) + Pc^2/(1+2c^3)$. Why do $F^{(1)}$ and $F^{(3)}$ "blow up" if $H \neq 0$ and $\alpha \to 0$?

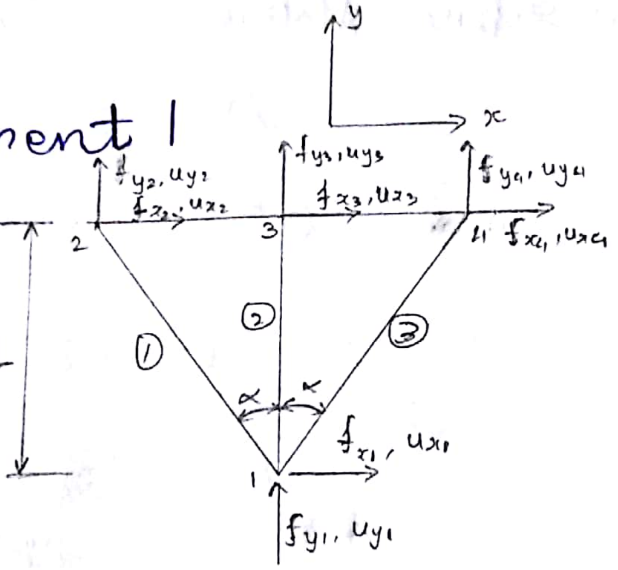


Assignment 2

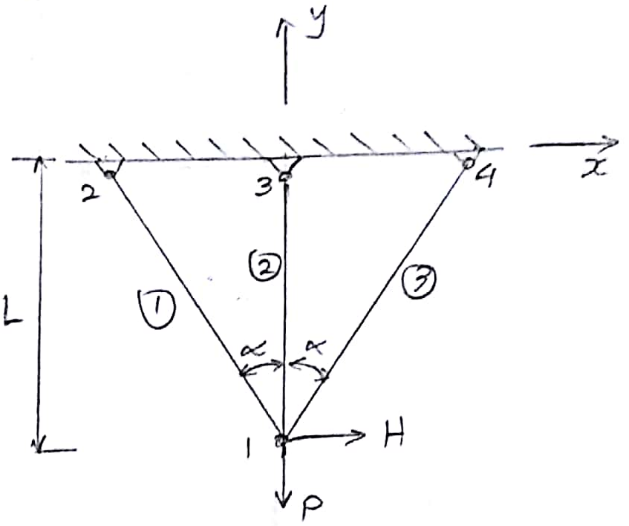
Dr. Who proposes "improving" the result for the example truss of the 1st lesson by putting one extra node, 4 at the midpoint of member (3) 1-3, so that it is subdivided in two different members: (3) 1-4 and (4) 3-4. His "reasoning" is that more is better. Try Dr. Who's suggestion by hand computations and verify that the solution "blows up" because the modified master stiffness is singular. Explain physically.

Date of Assignment: 05/02/2018
Date of Submission: 12/02/2018

Assignment 1



Simplified figure
with forces and displacement
vector



Given

⇒ E and A same for all three bars.

⇒ L, α, E, A geometric and material properties kept as variables, for all element

⇒ the applied forces P and H is also kept as variables.

⇒ 8 degree of Freedom

⇒ 2, 3 and 4 nodes are fixed, ~~six~~ so six degree of freedom removable

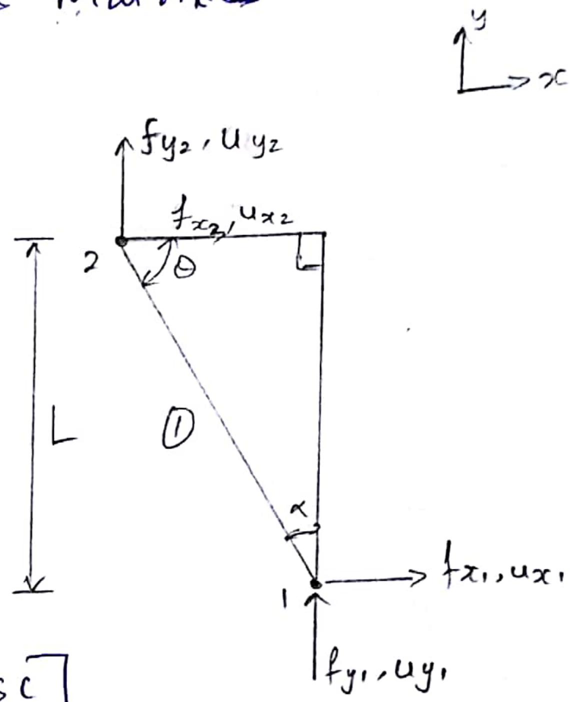
⇒ $\alpha \neq 0$

(a) ⇒ $C = \cos\alpha$ and $s = \sin\alpha$

We know, for element stiffness Matrices.

$$K^e = \frac{E^e A^e}{L^e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

For element ①, Element Stiffness Matrix



$$\theta = -(90-\alpha)$$

$$\sin(-(90-\alpha)) = -\sin(90-\alpha) = -\cos\alpha = -c$$

$$\cos(-(90-\alpha)) = \cos(90-\alpha) = \sin\alpha = s$$

$$\boxed{L^{(1)} = \frac{L}{\cos\alpha} = \frac{L}{c}}$$

⇒ Stiffness matrix

$$K^{(1)} = \frac{E^{(1)}A^{(1)}}{L^{(1)}} \begin{bmatrix} s^2 & -sc & -s^2 & sc \\ -sc & c^2 & sc & -c^2 \\ -s^2 & sc & s^2 & -sc \\ sc & -c^2 & -sc & c^2 \end{bmatrix}$$

$$= \frac{EA}{L} \begin{bmatrix} cs^2 & -sc^2 & -s^2c & sc^2 \\ -sc^2 & c^3 & sc^2 & -c^3 \\ -s^2c & sc^2 & s^2c & -sc^2 \\ sc^2 & -c^3 & -sc^2 & c^3 \end{bmatrix}$$

we know

$$f^{(1)} = K^{(1)} u^{(1)}$$

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \end{bmatrix} = \frac{EA}{L} \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} cs^2 & -sc^2 & -s^2c & sc^2 \\ -sc^2 & c^3 & sc^2 & -c^3 \\ -s^2c & sc^2 & s^2c & -sc^2 \\ sc^2 & -c^3 & -sc^2 & c^3 \end{bmatrix} \end{array} \begin{bmatrix} u_{x_1}^{①} \\ u_{y_1}^{①} \\ u_{x_2}^{①} \\ u_{y_2}^{①} \end{bmatrix}$$

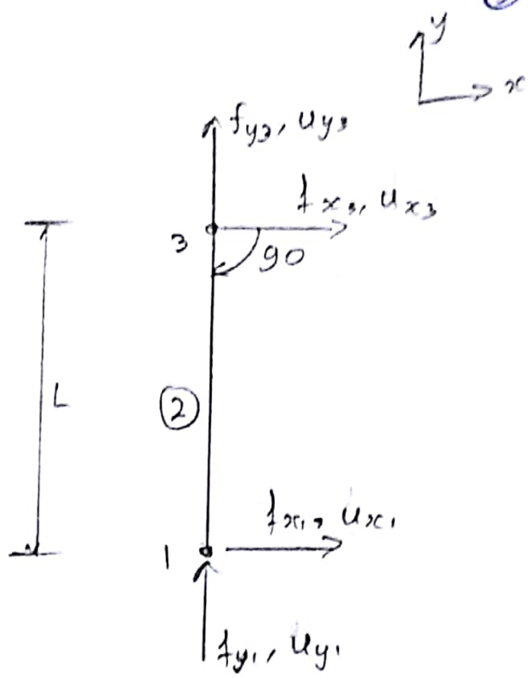
For element 2

$\theta = -90$

$\sin(-90) = -1$

$\cos(-90) = 0$

$L^{(2)} = L$



$\Rightarrow$ stiffness matrix

$$K^{(2)} = \frac{E^{(2)} A^{(2)}}{L^{(2)}} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

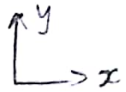
we know

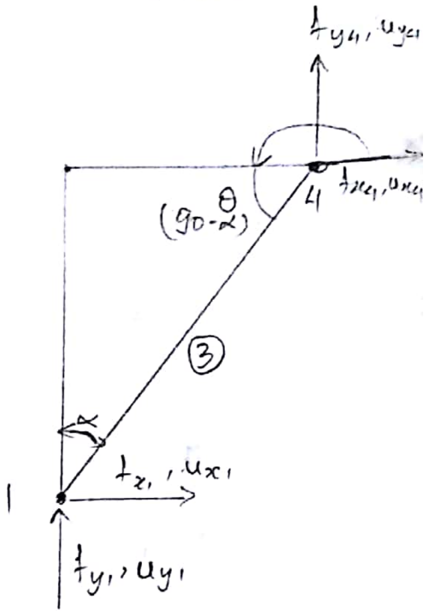
$$\begin{bmatrix} f_{x_1}^{(2)} \\ f_{y_1}^{(2)} \\ f_{x_3}^{(2)} \\ f_{y_3}^{(2)} \end{bmatrix} = \frac{EA}{L} \begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} u_{x_1}^{(2)} \\ u_{y_1}^{(2)} \\ u_{x_3}^{(2)} \\ u_{y_3}^{(2)} \end{bmatrix}$$

$\because E^{(2)} = E$

$\because A^{(2)} = A$

$$f^{(2)} = K^{(2)} \; u^{(2)}$$





For element ③

$$\theta = 180 + (90-\alpha)$$

$$\sin(180+(90-\alpha)) = -\sin(90-\alpha)$$
$$= -\cos\alpha$$
$$= -c$$

$$\cos(180+(90-\alpha)) = -\cos(90-\alpha)$$
$$= -\sin\alpha$$
$$= -s$$

$$L^{(3)} = \frac{L}{\cos\alpha} = \frac{L}{c}$$

$\Rightarrow$ stiffness matrix

$$k^{(3)} = \frac{E^{(3)}A^{(3)}}{L^{(3)}} \begin{bmatrix} s^2 & cs & -s^2 & -cs \\ cs & c^2 & -cs & -c^2 \\ -s^2 & -cs & s^2 & cs \\ -cs & -c^2 & cs & c^2 \end{bmatrix}$$

we know

$$\begin{bmatrix} f_{x1}^{(3)} \\ f_{y1}^{(3)} \\ f_{x4}^{(3)} \\ f_{y4}^{(3)} \end{bmatrix} = \frac{EA}{L} \begin{array}{c|cccc} & 1 & 2 & 7 & 8 \\ \hline 1 & & & & \\ 2 & & & & \\ 7 & & & & \\ 8 & & & & \end{array} \begin{bmatrix} cs^2 & c^2s & -cs^2 & -c^2s \\ c^2s & c^3 & -c^2s & -c^3 \\ -cs^2 & -c^2s & cs^2 & c^2s \\ -c^2s & -c^3 & c^2s & c^3 \end{bmatrix} \begin{bmatrix} u_{x1}^{(3)} \\ u_{y1}^{(3)} \\ u_{x4}^{(3)} \\ u_{y4}^{(3)} \end{bmatrix}$$

$$f^{(2)} = k^{(3)} u^{(3)}$$

The globalized stiffness matrix

$$K = K^{(1)} + K^{(2)} + K^{(3)}$$

$$K = \frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ 0 & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ c^2s & -c^3 & -c^2s & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -cs^2 & -c^2s & 0 & 0 & 0 & 0 & cs^2 & c^2s \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 \end{bmatrix}$$

The we have

$$f = K\,u$$

$$\begin{bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \\ f_{x4} \\ f_{y4} \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ 0 & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ c^2s & -c^3 & -c^2s & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -cs^2 & -c^2s & 0 & 0 & 0 & 0 & cs^2 & c^2s \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 \end{bmatrix}\begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix}$$

It can be observed that irrospective of the displacements of all the nodes, f_{x_3} is always zero. To depict this behaviour, we can find that in the master stiffness matrix all the elements of the 5th row are zero.

The solutions of the given problem are independent of the value of the u_{x_3}. As all the elements in the 5th column in the master stiffness matrix are zero; It shows all the reaction forces are independent of the u_{x_3}

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(6) Applying the boundary condition.

(1) Displacement boundary conditions.

$$u_{x_2} = u_{y_2} = u_{x_3} = u_{y_3} = u_{x_4} = u_{y_4} = 0$$

(2) Given forces, boundary conditions.

$$f_{x_1} = H \quad , \quad f_{y_1} = -P$$

The remaining forces at the remaining nodes are the reaction forces.

After apply boundary conditions we get

$$\begin{bmatrix} H \\ -P \\ f_{x_2} \\ f_{y_2} \\ f_{x_3} \\ f_{y_3} \\ f_{x_4} \\ f_{y_4} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ 0 & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ c^2s & -c^3 & -c^2s & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -cs^2 & -c^2s & 0 & 0 & 0 & 0 & cs^2 & c^2s \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ The modified stiffness system

$$\begin{bmatrix} H \\ -P \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \end{bmatrix}$$

(C)

$$\begin{bmatrix} u_{x1} \\ u_{y1} \end{bmatrix} = \frac{L}{EA} \begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix}^{-1} \begin{bmatrix} H \\ -P \end{bmatrix}$$

$$= \frac{L}{EA\, 2cs^2 (1+2c^3)} \begin{bmatrix} 1+2c^3 & 0 \\ 0 & 2cs^2 \end{bmatrix} \begin{bmatrix} H \\ -P \end{bmatrix}$$

For the limit case $\alpha \to 0$

$$\Rightarrow u_{x1} = \lim_{\alpha \to 0} \frac{LH}{EA(2cs^2)}$$

$\therefore$ $u_{x1} \to \infty$

$$\Rightarrow u_{y1} = \lim_{\alpha \to 0} \frac{-LP}{EA(1+2c^3)} = \frac{-LP}{3EA}$$

For the limit case $\alpha \to \pi/2$

$$\Rightarrow u_{x1} = \lim_{\alpha \to \frac{\pi}{2}} \frac{LH}{2EAcs^2}$$

$u_{x1} \to \infty$

$$\Rightarrow u_{y1} = \lim_{\alpha \to \frac{\pi}{2}} \frac{-LP}{EA(1+2c^3)} = \frac{-LP}{EA}$$

Reason: Since the element ② form a oscillating system after merging with element ① & ③, the element ② then rotate about the node 3.

Because u_{x1} blows up as $\alpha \to 0$ & $H \neq 0$.

d) $\Rightarrow$
For Axial forces in the 3 members

$$[f_L] = [K_L][T][U_g]$$

For element 1

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \end{bmatrix} = \frac{EAc}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} s & -c & 0 & 0 \\ c & s & 0 & 0 \\ 0 & 0 & s & -c \\ 0 & 0 & c & s \end{bmatrix}\begin{bmatrix} \frac{LH}{2EAcs^2} \\ -\frac{LP}{EA(1+2c^3)} \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \end{bmatrix} = \begin{bmatrix} \frac{H}{2s} + \frac{Pc^2}{1+2c^3} \\ 0 \\ -\frac{H}{2s} - \frac{Pc^2}{1+2c^3} \\ 0 \end{bmatrix}$$

For Element 2

$$\begin{bmatrix} f_{x_1}^{(2)} \\ f_{y_1}^{(2)} \\ f_{x_3}^{(2)} \\ f_{y_3}^{(2)} \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} \frac{LH}{2EAcs^2} \\ \frac{-LP}{EA(1+2c^3)} \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} f_{x_1}^{(2)} \\ f_{y_1}^{(2)} \\ f_{x_3}^{(2)} \\ f_{y_3}^{(2)} \end{bmatrix} = \begin{bmatrix} \frac{-P}{1+2c^3} \\ 0 \\ \frac{P}{1+2c^3} \\ 0 \end{bmatrix}$$

10

for Element 3

$$\begin{bmatrix} f_{x_1}^{(3)} \\ f_{y_1}^{(3)} \\ f_{x_4}^{(3)} \\ f_{y_4}^{(3)} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{bmatrix} \begin{bmatrix} \frac{LH}{2EAcs^2} \\ \frac{-LP}{EA(1+2c^3)} \\ 0 \\ 0 \end{bmatrix}$$

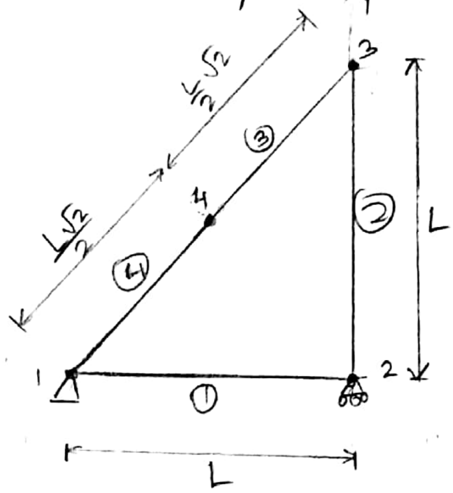
$$\begin{bmatrix} f_{x_1}^{(3)} \\ f_{y_1}^{(3)} \\ f_{x_4}^{(3)} \\ f_{y_4}^{(3)} \end{bmatrix} = \begin{bmatrix} -\frac{H}{2s} - \frac{Pc^2}{1+2c^3} \\ 0 \\ \frac{H}{2s} + \frac{Pc^2}{1+2c^3} \\ 0 \end{bmatrix}$$

Reason: $f^{(1)}$ & $f^{(3)}$ blows up because when α tends to zero, the structure couldn't sustain the force in x direction. And starts spinning at node 3, which concludes in giving up the ~~sta~~ stability. The structure becomes unstable & collapses.

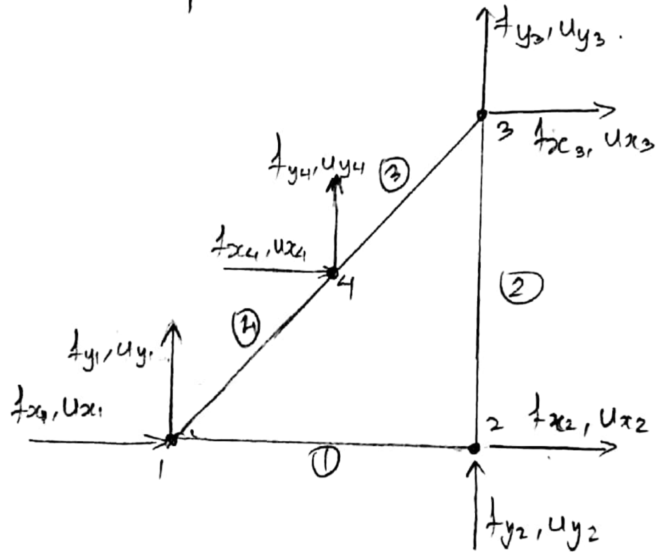
Assignment 2

Here,

The given example of truss with node ④ at mid point of member ③



The simplified figure with force and displacement vector.



$L^{①} = 10$

$L^{②} = 10$

$L^{③} = 5\sqrt{2}$

$L^{④} = 5\sqrt{2}$

$E^{①} A^{①} = 100$

$E^{②} A^{②} = 50$

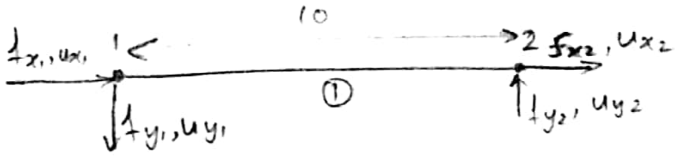
$E^{③} A^{③} = E^{④} A^{④} = 200\sqrt{2}$

Here, we are calculating the element stiffness matrix for each element, using the standard formula.

$$K^e = \frac{E^e A^e}{L^e} \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

For element 1

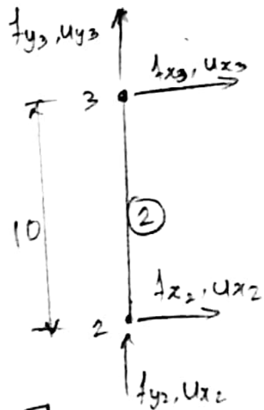
We know,



$$f^{(1)} = k_1 u^{(1)}$$

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \end{bmatrix} = 10 \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} u_{x_1}^{(1)} \\ u_{y_1}^{(1)} \\ u_{x_2}^{(1)} \\ u_{y_2}^{(1)} \end{bmatrix}$$

$$\left(\frac{EA}{L}\right)^{(1)} = \frac{100}{10} = 10$$



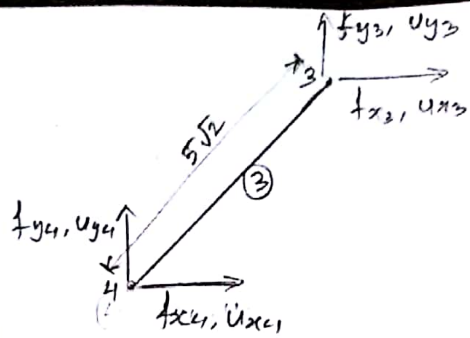
For element 2

$$f^{(2)} = k_2 u^{(2)}$$

$$\begin{bmatrix} f_{x_2}^{(2)} \\ f_{y_2}^{(2)} \\ f_{x_3}^{(2)} \\ f_{y_3}^{(2)} \end{bmatrix} = 5 \begin{array}{c} \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccc} 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} u_{x_2}^{(2)} \\ u_{y_2}^{(2)} \\ u_{x_3}^{(2)} \\ u_{y_3}^{(2)} \end{bmatrix}$$

$$\left(\frac{EA}{L}\right)^{(2)} = \frac{50}{10} = 5$$

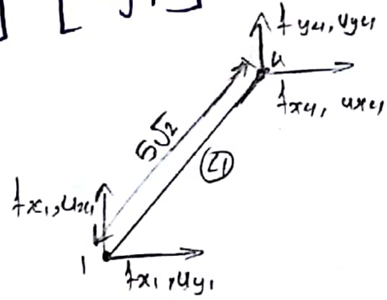
Element 3



$$f^{③} = k_3 u^{③}$$

$$\begin{bmatrix} f_{x_3}^{③} \\ f_{y_3}^{③} \\ f_{x_4}^{③} \\ f_{y_4}^{③} \end{bmatrix} = 40 \begin{array}{c} \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix} \end{array} \begin{bmatrix} u_{x_3}^{③} \\ u_{y_3}^{③} \\ u_{x_4}^{③} \\ u_{y_4}^{③} \end{bmatrix}$$

$$\because \left(\frac{EA}{L}\right)^{③} = \frac{200\sqrt{2}}{5\sqrt{2}} = 40$$



Element 4

$$f^{④} = k_4 u^{④}$$

$$\begin{bmatrix} f_{x_1}^{④} \\ f_{y_1}^{④} \\ f_{x_4}^{④} \\ f_{y_4}^{④} \end{bmatrix} = 40 \begin{array}{c} \\ 1 \\ 2 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 7 & 8 \end{array} \\ \begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix} \end{array} \begin{bmatrix} u_{x_1}^{④} \\ u_{y_1}^{④} \\ u_{x_4}^{④} \\ u_{y_4}^{④} \end{bmatrix}$$

$$\left(\frac{EA}{L}\right)^{④} = 40$$

The Global stiffness matrix

$$K = k_1 + k_2 + k_3 + k_4$$

$$K = \begin{bmatrix} 30 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & -5 & 20 & 25 & -20 & -20 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \end{bmatrix}$$

Now,

we know.

$$f = K\, u$$

$$\begin{bmatrix} f_{x_1} \\ f_{y_1} \\ f_{x_2} \\ f_{y_2} \\ f_{x_3} \\ f_{y_3} \\ f_{x_4} \\ f_{y_4} \end{bmatrix} = \begin{bmatrix} 30 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & -5 & 20 & 25 & -20 & -20 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix}$$

Now, Applying the boundary Conditions.

Displacement BCs:

$$u_{x_1} = u_{y_1} = u_{y_2} = 0$$

Force BCs:

$$f_{x_2} = 0 \;,\quad f_{x_4} = f_{y_4} = 0 \;,\quad f_{x_3} = 2 \quad f_{y_3} = 1$$

$\cancel{f_{x_1} = R_{x_1}}$, $\cancel{f_{y_1} = R_{y_1}}$, $\cancel{f_{y_2} = R_{y_2}}$ ~~(suppose)~~

implementing the boundary conditions:

$$\begin{bmatrix} f_{x1} \\ f_{y1} \\ 0 \\ f_{y2} \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 30 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & -5 & 20 & 25 & -20 & -20 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ u_{x2} \\ 0 \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix}$$

Reduced stiffness equation

$$\begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 20 & 20 & -20 & -20 \\ 0 & 20 & 25 & -20 & -20 \\ 0 & -20 & -20 & 40 & 40 \\ 0 & -20 & -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} u_{x2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix}$$

⇒ Physical explanation

⇒ Since the members of the truss are not properly placed, the truss will change its shape. Hence the truss cannot maintain its ~~sha~~ original shape against randomly applied external forces at the nodes, the truss is then said to be internally unstable. When the constrained global stiffness matrix is ~~sing~~ singular the truss is ~~said~~ can be said kinematically unstable. The structure is not internally connected properly although it is externally stable. The total ~~no~~ number of forces unknown that is (3 support reactions + 4 member forces) < 2N (N=4). This depicts the structure is internally ~~ins~~ unstable. So the solutions Blow up due to the unstable nature of the truss, kinamatically. So, Dr. Who's proposal is not beneficial in this case.