UPC - BARCELONA TECH
MSc Computational Mechanics
Spring 2018

Computational Structural Mechanics and Dynamics

Assignment 1

Due 12/02/2018

Prasad ADHAV

Assignment 1.1

On “The Direct Stiffness Method”

Consider the truss problem defined in the figure 1.1. All geometric and material properties: L, α, E and A, as well as the applied forces P and H are to be kept as variables. This truss has 8 degrees of freedom, with six of them removable by the fixed-displacement conditions at nodes 2, 3 and 4. This structure is statically indeterminate as long as $\alpha \neq 0$.

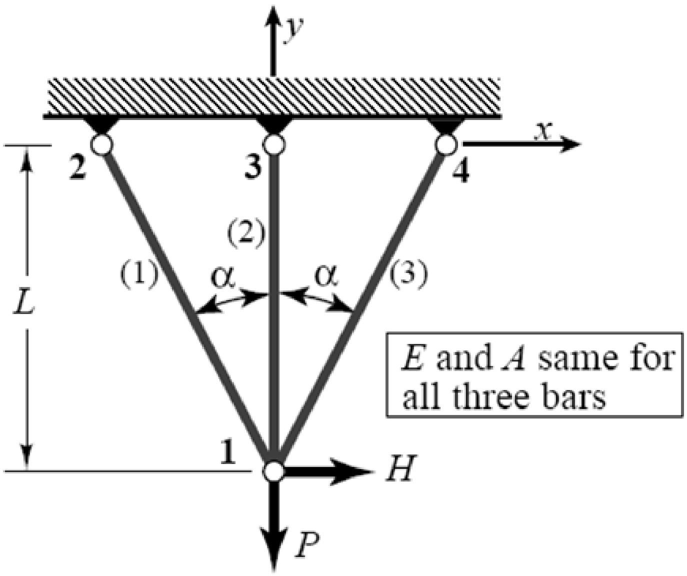


Figure 1.1.- Truss structure. Geometry and mechanical features

1. Show that the master stiffness equations are,

$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ & & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & & 1 & 0 & 0 \\ & & & & & & cs^2 & c^2s \\ \text{symm} & & & & & & & c^3 \end{bmatrix}\begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix} = \begin{bmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

in which c = cosα and s = sinα. Explain from physics why the 5th row and column contain only zeros.

2. Apply the BC's and show the 2-equation modified stiffness system.

3. Solve for the displacements u_{x1} and u_{y1}. Check that the solution makes physical sense for the limit cases $\alpha \to 0$ and $\alpha \to \pi/2$. Why does u_{x1} "blow up" if $H\neq0$ and $\alpha\to0$?

4. Recover the axial forces in the three members. Partial answer: $F^{(3)} = -H/(2s) + Pc^2/(1+2c^3)$. Why do $F^{(1)}$ and $F^{(3)}$ "blow up" if $H\neq0$ and $\alpha\to0$?

5. Dr. Who proposes "improving" the result for the example truss of the 1st lesson by putting one extra node, 4 at the midpoint of member (3) 1-3, so that it is subdivided in two different members: (3) 1-4 and (4) 3-4. His "reasoning" is that more is better. Try Dr. Who's suggestion by hand computations and verify that the solution "blows up" because the modified master stiffness is singular. Explain physically.

Assignment 1.2

Dr. Who proposes "improving" the result for the example truss of the 1st lesson by putting one extra node, 4 at the midpoint of member (3) 1-3, so that it is subdivided in two different members: (3) 1-4 and (4) 3-4. His "reasoning" is that more is better. Try Dr. Who's suggestion by hand computations and verify that the solution "blows up" because the modified master stiffness is singular. Explain physically.

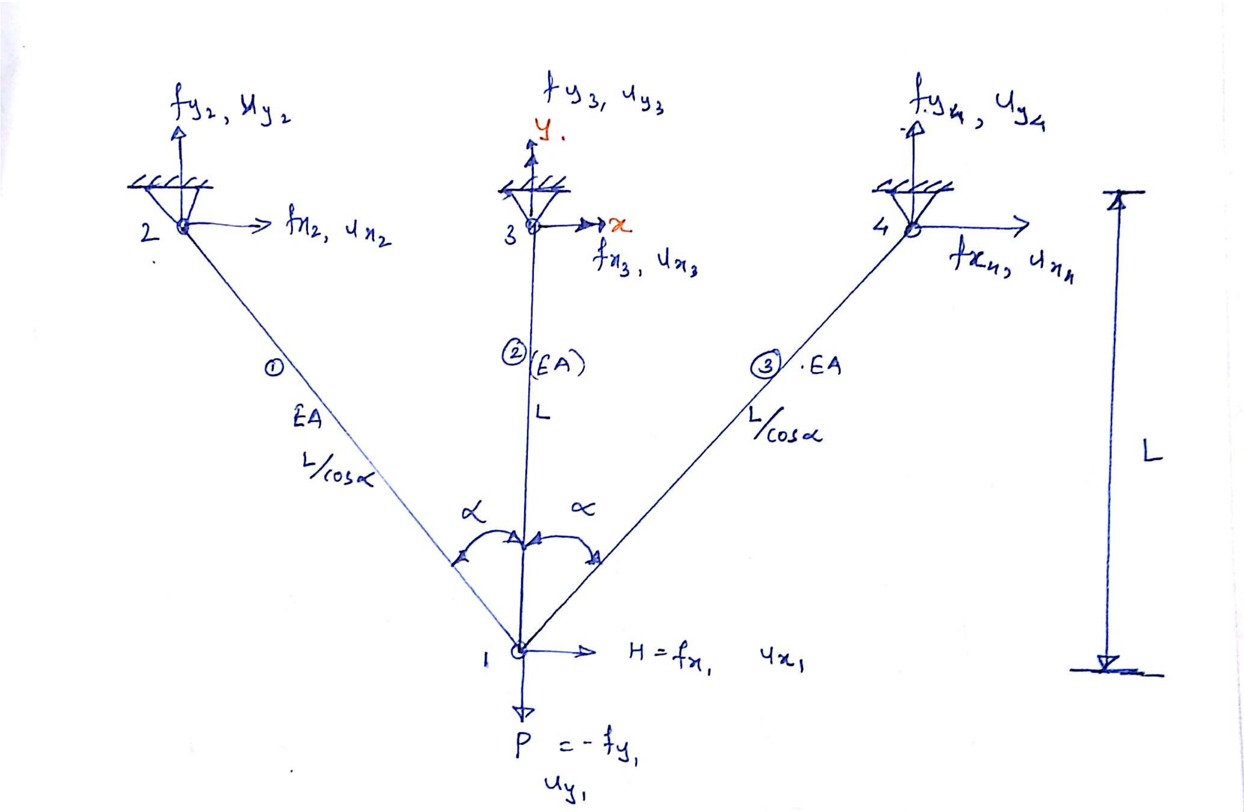
Date of Assignment: **5 / 02 / 2018**
Date of Submission: **12 / 02 / 2018**

The assignment must be submitted as a pdf file named **As1-Surname.pdf** to the CIMNE virtual center.

1

1.a

Data Given:



The general stiffness matrix of any element can be written as follows (as derived in the lecture.)
Here, φ is the angle at which the element is inclined with respect to the x-axis.
To avoid confusion, the notation is followed
$a = cos\varphi, b = sin\varphi, s = sin \propto, c = cos \propto$

$$\mathbf{K}^e = \frac{E^eA^e}{L^e}\begin{bmatrix} b^2 & ab & -b^2 & -ab \\ ab & a^2 & -ab & -a^2 \\ -b^2 & -ab & b^2 & ab \\ -ab & -a^2 & ab & a^2 \end{bmatrix}$$

For element 1 $\varphi = \pi/2 + \propto$
$\therefore a = cos(\pi/2 + \propto) = -sin \propto = -s, b = sin(\pi/2 + \propto) = cos \propto = c$

$$\mathbf{K}^{(1)} = \frac{EA}{L}\begin{bmatrix} s^2c & -sc^2 & -s^2c & sc^2 \\ -sc^2 & c^3 & sc^2 & -c^3 \\ -s^2c & sc^2 & s^2c & -sc^2 \\ sc^2 & -c^3 & -sc^2 & c^3 \end{bmatrix}$$

For element 2 $\varphi = \pi/2$
$\therefore a = cos\varphi = 0, b = sin\varphi = 1$

$$\mathbf{K}^{(2)} = \frac{EA}{L}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

For element 3 $\varphi = \pi/2 - \propto$

$\therefore a = cos(\pi/2 - \propto) = sin \propto = s, b = sin(\pi/2 - \propto) = cos \propto = c$

$$\mathbf{K}^{(3)} = \frac{EA}{L}\begin{bmatrix} s^2c & sc^2 & -s^2c & -sc^2 \\ sc^2 & c^3 & -sc^2 & -c^3 \\ -s^2c & -sc^2 & s^2c & sc^2 \\ -sc^2 & -c^3 & sc^2 & c^3 \end{bmatrix}$$

Computing Global Stiffness Matrix

$\underline{\mathrm{f}} = \underline{\mathrm{f}}_{(1)} + \underline{f}_{(2)} + \underline{f}_{(3)} = [\underline{K}^{(1)} + \underline{K}^{(2)} + \underline{K}^{(3)}]\underline{u} = \underline{Ku}$

Expanding elemental stiffness matrices

$$\underline{\mathrm{K}}^{(1)} = \frac{EA}{L}\begin{bmatrix} s^2c & -sc^2 & -s^2c & sc^2 & 0 & 0 & 0 & 0 \\ -sc^2 & c^3 & sc^2 & -c^3 & 0 & 0 & 0 & 0 \\ -s^2c & sc^2 & s^2c & -sc^2 & 0 & 0 & 0 & 0 \\ sc^2 & -c^3 & -sc^2 & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underline{\mathrm{K}}^{(2)} = \frac{EA}{L}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underline{\mathrm{K}}^{(3)} = \frac{EA}{L}\begin{bmatrix} s^2c & sc^2 & 0 & 0 & 0 & 0 & -s^2c & -sc^2 \\ sc^2 & c^3 & 0 & 0 & 0 & 0 & -sc^2 & -c^3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -s^2c & -sc^2 & 0 & 0 & 0 & 0 & s^2c & sc^2 \\ -sc^2 & -c^3 & 0 & 0 & 0 & 0 & sc^2 & c^3 \end{bmatrix}$$

Global system of equations

$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ 0 & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -cs^2 & 0 & 0 & 0 & 0 \\ c^2s & -c^3 & -c^2s & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -cs^2 & -c^2s & 0 & 0 & 0 & 0 & cs^2 & c^2s \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 \end{bmatrix}\begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix} = \begin{bmatrix} H \\ -P \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \\ f_{x4} \\ f_{y4} \end{bmatrix}$$

All coefficient of $\underline{\mathrm{K}}$ involving x_3 are zero. Therefore, it can be said that no horizontal force would appear on node 3 upon prescribing a displacement in any direction at any node of the structure. In fact, its easy to see that $f_{x3} = 0$ and horizontal reaction forces only appear at node 2 and 4. This fact is directly related withe symmetry of the structure. Also, there is no reaction at node 3 as in our model it is assumed that only axial forces are acting on the elements.

1.b

Applying boundary conditions and obtaining modified stiffness matrix.

Boundary conditions are as follows
$u_{x2} = u_{y2} = u_{x3} = u_{x3} = u_{y3} = u_{x4} = u_{y4} = 0$

$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix}\begin{bmatrix} u_{x1} \\ u_{y1} \end{bmatrix} = \begin{bmatrix} H \\ -P \end{bmatrix}$$

1.c

Solving the above equations for u_{x1} and u_{y1}

$$u_{x1} = \frac{HL}{2cs^2EA} \quad u_{y1} = \frac{PL}{(1+2c^3)EA}$$

To check if the solution makes physical sense for given limit cases.

Limit case $\propto \rightarrow 0 \Rightarrow c \rightarrow 1 \ s \rightarrow 0$

Thus $u_y = \dfrac{-PL}{3EA}$ and this makes sense as this displacement corresponds with the case of a single ba fixed at one end, with a vertical load applied at th free end and with axial stiffness $\dfrac{3AE}{L}$.

If $H \neq 0$, the solution blowsup as $u_x \rightarrow \infty$. As $s \rightarrow 0$, the reduced stiffness matrix is singular. In this case the structure is in fact a mechanism and as the support allows free rotation, the solution blows up.
Even though the support didn't allow any rotation, the solution would still fail as the truss model used for the analysis does not account for any bending resistance. It just includes axial effects.

Limit case $\propto \rightarrow \pi/2 \Rightarrow c \rightarrow 0 \ s \rightarrow 1$

Thus, $u_y = \dfrac{-PL}{EA}$ and this makes sense as it corresponds with the case of an unique ba with axial stiffness $\dfrac{EA}{L}$, fixed at one end and free at the other end.
For this case, the length of bars 1 and 3 tends to the ∞ ($L_{1,3} = \dfrac{L}{c}$) and thus they don't show any axial stiffness.

1.d

Recovering axial forces in the three members.

Element 1

Global displacement $\underline{u}^{(1)} = \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \end{bmatrix} = \begin{bmatrix} u_{x1} \\ u_{y1} \\ 0 \\ 0 \end{bmatrix}$

Local Displacement $u^{(1)} = \underline{T}^{(1)}\underline{u}^{(1)} = \begin{bmatrix} -s & c & 0 & 0 \\ -c & -s & 0 & 0 \\ 0 & 0 & -s & c \\ 0 & 0 & -c & -s \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -u_{x1}.s + u_{y1}.c \\ -u_{x1}.c - u_{y1}.s \\ 0 \\ 0 \end{bmatrix}$

On simplification we get,

Elongation $d^{(1)} = \dfrac{HL}{2csEA} + \dfrac{PLc}{(1+2c^3)EA}$

Axial force $F^{(1)} = \dfrac{H}{2s} + \dfrac{Pc^2}{1+2c^3}$

Element 2

Global displacement $\underline{u}^{(2)} = \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x3} \\ u_{y3} \end{bmatrix} = \begin{bmatrix} u_{x1} \\ u_{y1} \\ 0 \\ 0 \end{bmatrix}$

Local Displacement $u^{(2)} = \underline{T}^{(2)}\underline{u}^{(2)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} u_{y1} \\ -u_{x1} \\ 0 \\ 0 \end{bmatrix}$

On simplification we get,

Elongation $d^{(2)} = \underline{u}_{x3} - \underline{u}_{x1} = -u_{y1} = \dfrac{PL}{(1+2c^3)EA}$

Axial force $F^{(2)} = \dfrac{P}{1+2c^3}$

Element 3

Global displacement $\underline{u}^{(3)} = \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x4} \\ u_{y4} \end{bmatrix} = \begin{bmatrix} u_{x1} \\ u_{y1} \\ 0 \\ 0 \end{bmatrix}$

Local Displacement $u^{(3)} = \underline{T}^{(3)}\underline{u}^{(3)} = \begin{bmatrix} s & c & 0 & 0 \\ -c & -s & 0 & 0 \\ 0 & 0 & s & c \\ 0 & 0 & -c & -s \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} u_{x1}.s + u_{y1}.c \\ -u_{x1}.c + u_{y1}.s \\ 0 \\ 0 \end{bmatrix}$

On simplification we get,

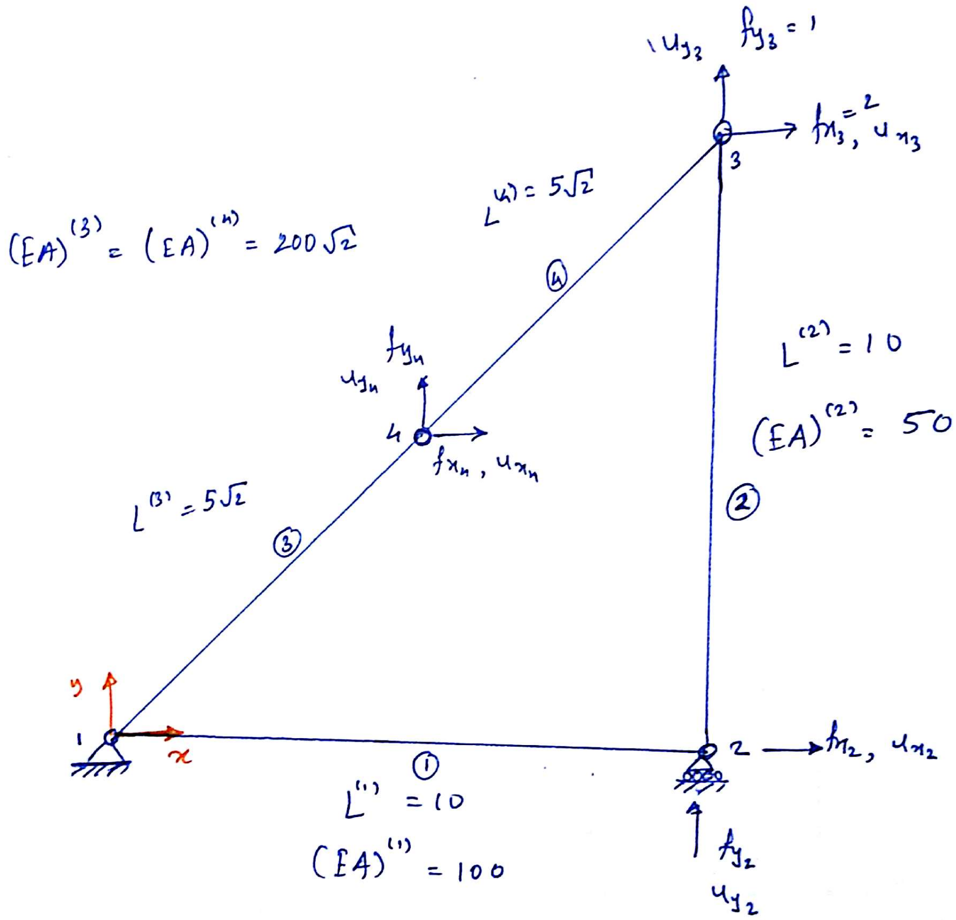
Elongation $d^{(3)} = \underline{u}_{x2}^{(3)} - \underline{u}_{x1}^{(3)} = -u_{x1}s - u_{y1}c = \dfrac{-HLs}{2cs^2EA} + \dfrac{PLc}{(1+2c^3)EA}$

Axial force $F^{(1)} = \dfrac{-H}{2s} + \dfrac{Pc^2}{1+2c^3}$

Limit case $\propto \rightarrow 0$, $H \neq 0$
Again, as $\propto \rightarrow 0 \Rightarrow c \rightarrow 1$ $s \rightarrow 0$

This case is like considering a single bar with axial stiffness $\dfrac{3EA}{L}$. In fact, the value of the inertial force would be F = P
As $\propto \rightarrow 0$, both $F^{(1)}$ and $F^{(3)}$ tend to infinity.
For the case $H \neq 0$
Due to the idealized truss model is too simple as it only considers axial loads on truss members with no bending resistance. This case is in fact some kind of mechanism.

2



As per Dr. Who, for 'improving' the result of the example from lesson 1, one extra node has been added midway between node 1 and 3.So that it is subdivided into two different members $1-4$ (element 3) and $3-4$ (element 4).
Member stiffness matrix in global coordinates
$c = cos\varphi, s = sin\varphi$

$$\underline{K}^{(e)} = \frac{E^{(e)}A^{(e)}}{L^{(e)}}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

Element 1 $\varphi = 0 \rightarrow cos\varphi = c = 1; sin\varphi = s = 0$

$$\underline{\mathrm{K}}^{(1)} = \frac{100}{10}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 \\ -10 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Element 2 $\varphi = \pi/2 \rightarrow cos\varphi = c = 0; sin\varphi = s = 1$

$$\underline{\mathrm{K}}^{(2)} = \frac{50}{10}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & -5 & 0 & 5 \end{bmatrix}$$

Element 3 $\varphi = \pi/4 \rightarrow cos\varphi = c = 1/\sqrt{2} = \sqrt{2}/2; sin\varphi = s = 1/\sqrt{2} = \sqrt{2}/2$

$$\underline{\mathrm{K}}^{(3)} = \frac{200\sqrt{2}}{5\sqrt{2}}\begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix} = \begin{bmatrix} 20 & 20 & -20 & -20 \\ 20 & 20 & -20 & -20 \\ -20 & -20 & 20 & 20 \\ -20 & -20 & 20 & 20 \end{bmatrix}$$

Element 4 $\varphi = 3\pi/4 \rightarrow cos\varphi = c = -1/\sqrt{2} = -\sqrt{2}/2; sin\varphi = s = -1/\sqrt{2} = -\sqrt{2}/2$

$$\underline{\mathrm{K}}^{(3)} = \frac{200\sqrt{2}}{5\sqrt{2}}\begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix} = \begin{bmatrix} 20 & 20 & -20 & -20 \\ 20 & 20 & -20 & -20 \\ -20 & -20 & 20 & 20 \\ -20 & -20 & 20 & 20 \end{bmatrix}$$

$$\underline{\mathrm{f}} = \underline{\mathrm{f}}_{(1)} + \underline{f}_{(2)} + \underline{f}_{(3)} + \underline{f}_{(4)} = [\underline{K}^{(1)} + \underline{K}^{(2)} + \underline{K}^{(3)} + \underline{K}_{(4)}]\underline{u} = \underline{Ku}$$

Expanding the elemental stiffness matrices for obtaining the global stiffness matrix.

$$\underline{\mathrm{K}}^{(1)} = \begin{bmatrix} 10 & 0 & -10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underline{\mathrm{K}}^{(2)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underline{K}^{(3)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & 0 & -20 & -20 & 20 & 20 \\ 0 & 0 & 0 & 0 & -20 & -20 & 20 & 20 \end{bmatrix}$$

$$\underline{K}^{(4)} = \begin{bmatrix} 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & -20 & -20 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -20 & -20 & 0 & 0 & 0 & 0 & 20 & 20 \\ -20 & -20 & 0 & 0 & 0 & 20 & 20 & \end{bmatrix}$$

Global system of equation $\underline{K}u = f$

$$\begin{bmatrix} 30 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ -10 & 0 & -10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 20 & 25 & -20 & -20 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix} = \begin{bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \\ f_{x4} \\ f_{y4} \end{bmatrix}$$

Boundary conditions are as follows:

$u_{x1} = u_{y1} = u_{y1} = 0$

$f_{x2} = f_{x4} = f_{y4} = 0$

$f_{x3} = 2$

$f_{y3} = 1$

After Introducing the boundary condition, the global syste of equations is reduced as follows:

$$\begin{bmatrix} -10 & 0 & 0 & 0 & 0 \\ 0 & 20 & 20 & -20 & -20 \\ 0 & 20 & 25 & -20 & -20 \\ 0 & -20 & -20 & 40 & 40 \\ 0 & -20 & -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} u_{x2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

It is easy to realize that the stiffness matrix of reduced system is singular, as the fourth and fifth columns are equal. Therefore det$[\underline{K}^{red}] = 0$ and the system of equations is not compatible and there is no solution. That is there is no way for the mathematical model to react to the applied forces.

Physically, this is due to the fact that the structure is not fully constrained in space. It is like the structure is 'floating' in the x-y plane. Or we can even say this is a four bar chain mechanism. In order to solve this a compatibility equation at joint 4 may be imposed. Also, another element between 2 and 4 can be added to make the truss stable.