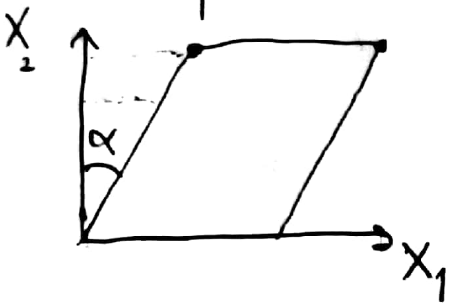
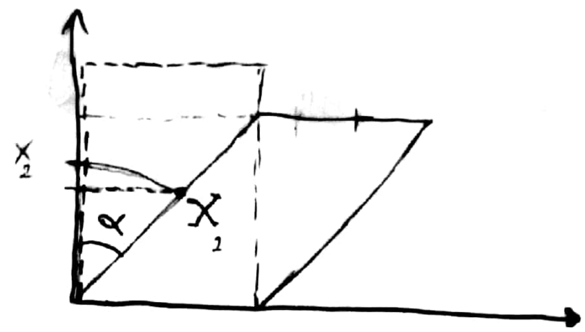
Ovid Folip





$$\begin{cases} x_1 = X_1 + \sin\alpha\, X_2 \\ x_2 = \cos\alpha\, X_2 \end{cases}$$

$$\boxed{\vec{x} = \vec{\varphi}(\vec{X}, t) = \begin{bmatrix} X_1 + \sin\alpha\, X_2 \\ \cos\alpha\, X_2 \end{bmatrix} \quad \text{deformation map}}$$

2) Deformation gradient reads: $F = \text{Grad}\ \vec{\varphi}$

$$F = \begin{pmatrix} \frac{\partial \varphi_x}{\partial X} & \frac{\partial \varphi_x}{\partial Y} \\ \frac{\partial \varphi_y}{\partial X} & \frac{\partial \varphi_y}{\partial Y} \end{pmatrix} = \begin{pmatrix} 1 & \sin\alpha \\ 0 & \cos\alpha \end{pmatrix}$$

right Cauchy-Green deformation tensor C reads:

$$C = F^T F = \begin{pmatrix} 1 & 0 \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} 1 & \sin\alpha \\ 0 & \cos\alpha \end{pmatrix} = \begin{pmatrix} 1 & \sin\alpha \\ \sin\alpha & \sin^2\alpha + \cos^2\alpha \end{pmatrix} = \boxed{\begin{pmatrix} 1 & \sin\alpha \\ \sin\alpha & 1 \end{pmatrix}}$$

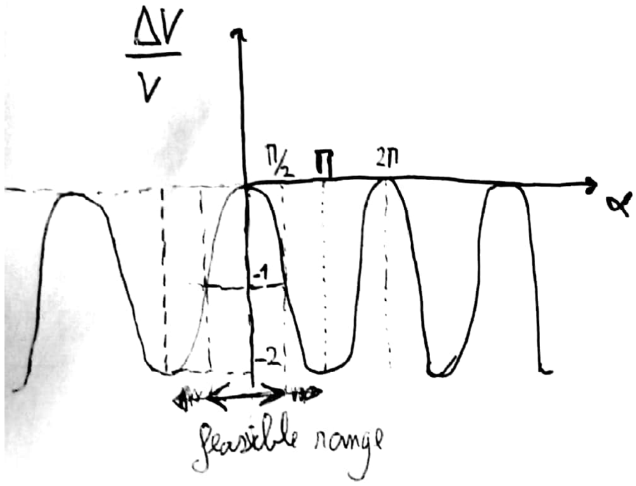
3]

$dv = J\,dV \rightarrow$ given that J is homogeneous then

$v = J\,V$ where J is defined as $J \equiv \det F$

So $\boxed{v = \det F\, V = \cos\alpha\, V}$

$$\Delta V = \cos\alpha V - V = V(\cos\alpha - 1)$$

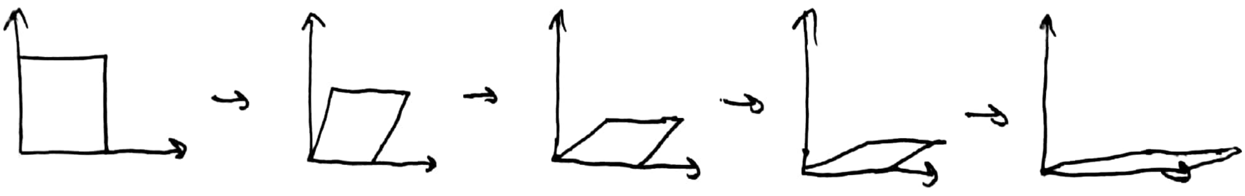


we see that volume always decreases with respect to the initial volume.

4] The deformation cease to be admissible when $J < 0$

given that $J = \det F = \cos\alpha$ when $\alpha > 90°$ the deformation is not admissible.

Geometrically means



So we see that when $\alpha \Rightarrow 0$

means that "all" points are in the same line, which is physically impossible

when $\alpha > 90°$ is not possible because it would mean we deformed beyond $\alpha = 90°$ which is impossible!!!

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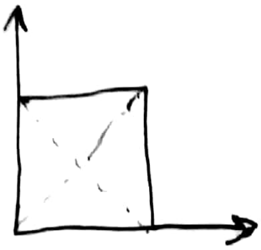
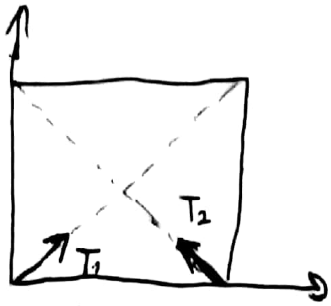
5)





$$\lambda = (T \cdot CT)^{1/2} \qquad \text{being} \quad \left[\begin{array}{l} T_1 = \frac{1}{\sqrt{2}}(1,1) \\ T_2 = \frac{1}{\sqrt{2}}(-1,1) \end{array}\right.$$

$$\lambda_1 = \frac{1}{2}(1,1)\begin{pmatrix} 1 & \sin\alpha \\ \sin\alpha & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2}(1,1)\begin{pmatrix} 1+\sin\alpha \\ 1+\sin\alpha \end{pmatrix} = \frac{1}{2}\,2(1+\sin\alpha) = \boxed{1+\sin\alpha = \lambda_1}$$

$$\lambda_2 = \frac{1}{2}(-1,1)\begin{pmatrix} 1 & \sin\alpha \\ \sin\alpha & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \frac{1}{2}(-1,1)\begin{pmatrix} -1+\sin\alpha \\ 1+\sin\alpha \end{pmatrix} = \frac{1}{2}2(1-\sin\alpha) = \boxed{1-\sin\alpha = \lambda_2}$$

Then,

$$ds_1 = \lambda\, dS_1 \Rightarrow \boxed{s_1 = \lambda_1 S_1 = (1+\sin\alpha) S_1}$$ where S_1 is the initial ~~length~~ lenght

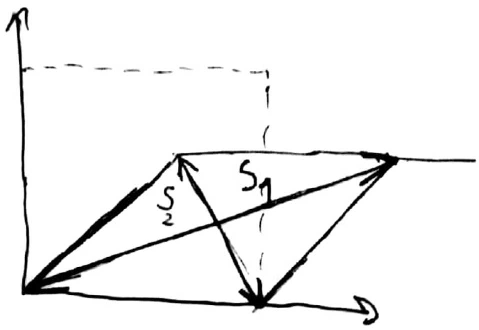
$$\boxed{s_2 = \lambda_2 S_2 = (1-\sin\alpha) S_2}$$ and S_2 the initial lenght of the other diagonal

The previous result has sense since the deformed shape:

s_1 increases due to $(1+\sin\alpha)$

s_2 decreases due to $(1-\sin\alpha)$

as it can be seen in the scheme!!



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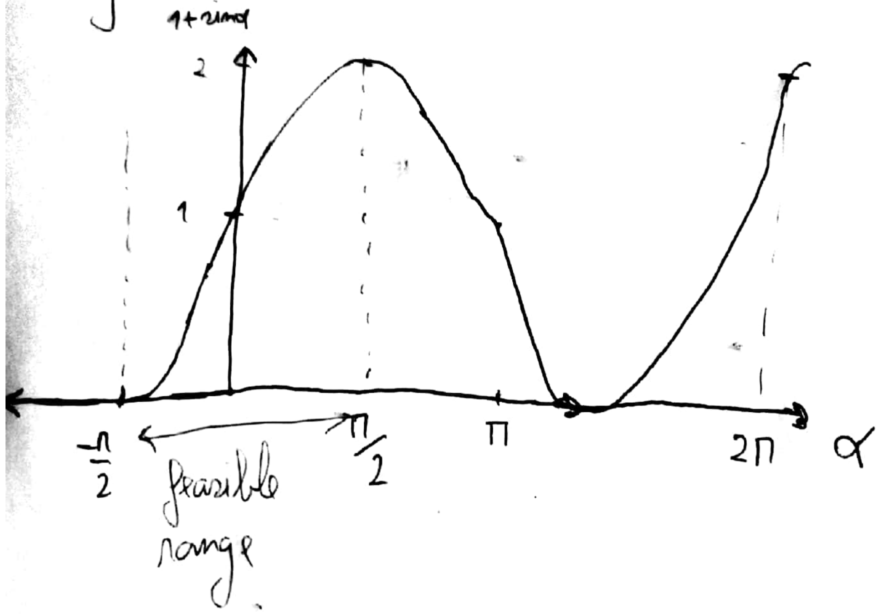
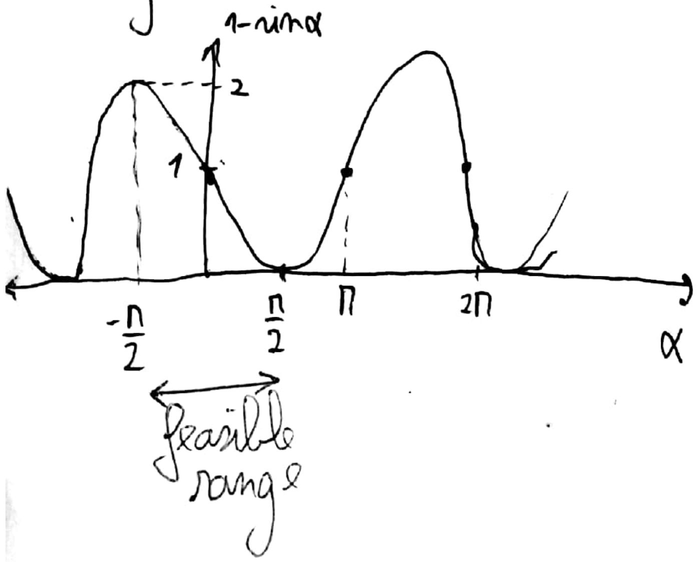
Diagonal AC:

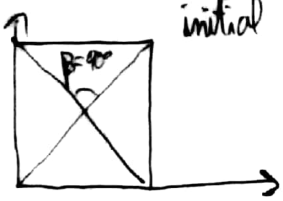


Diagonal BD

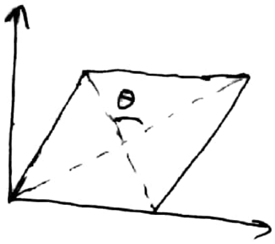


Regarding the angle:

initial



final



$$\cos\theta = \frac{T^{(1)} \cdot (\mathbb{I} + 2E)\, T^{(2)}}{\sqrt{1 + 2T^{(1)} \cdot E\, T^{(1)}}\ \sqrt{1 + 2T^{(2)} \cdot E\, T^{(2)}}}$$

with

$$E = \frac{1}{2}(C - 1) = \frac{1}{2}\begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix} \qquad \left[\begin{array}{l} T_1 = \frac{1}{\sqrt{2}}(1, 1) \\ \\ T_2 = \frac{1}{\sqrt{2}}(-1, 1) \end{array}\right.$$

Then

$$\sqrt{1 + 2T^{(1)} \cdot E\, T^{(1)}} = \sqrt{1 + \frac{1}{2}(1,1)\begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}} = \sqrt{1 + \frac{1}{2}(1,1)\begin{pmatrix} \sin\alpha \\ \sin\alpha \end{pmatrix}} = \sqrt{1 + \sin\alpha}$$

$$\sqrt{1 + 2T^{(2)} \cdot E\, T^{(2)}} = \sqrt{1 + \frac{1}{2}(-1,1)\begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix}} = \sqrt{1 + \frac{1}{2}(-1,1)\begin{pmatrix} \sin\alpha \\ -\sin\alpha \end{pmatrix}} = \sqrt{1 - \sin\alpha}$$

$$T^{(1)} \cdot (\mathbb{I} + 2E)\, T^{(2)} = \frac{1}{2}(1,1)\begin{pmatrix} 1 & \sin\alpha \\ \sin\alpha & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \frac{1}{2}(1,1)\begin{pmatrix} \sin\alpha - 1 \\ 1 - \sin\alpha \end{pmatrix} = \frac{1}{2}(\sin\alpha - 1 + 1 - \sin\alpha) = \boxed{0}$$

then

$\cos\theta = 0 \quad \forall \alpha \rightarrow \boxed{\theta = 90^\circ}$ which means there is no variation in the angle!!